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OSCILLATION IN SECOND ORDER LINEAR DELAY DIFFERENTIAL EQUATIONS WITH NONLINEAR IMPULSES

ZHIMIN HE* — WEIGAO GE**

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ABSTRACT. In this paper, the second order linear delay differential equation with nonlinear impulses

$$x''(t) + P(t)x(t-\tau) = 0\,, \qquad t \ge t_0\,, \quad t \ne t_k\,, \ k = 1,2,\dots\,,$$
$$x(t_k^+) = g_k\big(x(t_k)\big)\,, \quad x'(t_k^+) = h_k\big(x'(t_k)\big)\,, \qquad k = 1,2,\dots,$$

is considered, where $0 \le t_0 < t_1 < \dots < t_k < \dots$ with $\lim\limits_{k\to+\infty} t_k = +\infty$, and τ is a positive constant. Some sufficient conditions are obtained ensuring that all solutions of this equation oscillate.

1. Introduction and preliminaries

Recently there has been an extensive studies in the oscillatory theory of first order impulsive delay differential equations, see [4]–[8]. However, there are not much concerning the oscillatory properties of the second order impulsive delay differential equations and the second order impulsive ordinary differential equations, which is an important mathematical model of many evolutionary processes, see [9]–[12]. In this paper, we consider the following second order linear delay differential equation with nonlinear impulses

$$\begin{aligned} x''(t) + P(t)x(t-\tau) = 0\,, \qquad t \ge t_0\,, \quad t \ne t_k\,, \ k = 1,2,\dots\,,\\ x(t_k^+) = g_k\big(x(t_k)\big)\,, \quad x'(t_k^+) = h_k\big(x'(t_k)\big)\,, \qquad k = 1,2,\dots\,, \end{aligned} \tag{1}$$

where $0 \le t_0 < t_1 < \dots < t_k < \dots$ with $\lim\limits_{k\to+\infty} t_k = +\infty$, and τ is a positive constant.

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Throughout this paper, we always assume that

(i) $P \in C\big((0,+\infty),[0,+\infty)\big)$,

(ii) $g_k, h_k \in C(\mathbb{R},\mathbb{R})$ and there exist positive numbers $a_k, \overline{a}_k, b_k, \overline{b}_k$ such that

$$\overline{a}_k \le \frac{g_k(x)}{x} \le a_k\,, \quad \overline{b}_k \le \frac{h_k(x)}{x} \le b_k \qquad \text{for all} \quad x \neq 0\,, \quad k = 1,2,\ldots\,.$$

Let $J \subset \mathbb{R}$ be an interval, we define

$$\begin{aligned} PC(J,\mathbb{R}) = \big\{x\colon J \to \mathbb{R}:\ & x(t) \text{ is continuous everywhere} \\ & \text{except some } t_k\text{'s at which} \\ & x(t_k^-) \text{ and } x(t_k^+) \text{ exist and } x(t_k^-) = x(t_k)\big\}\,; \\ PC'(J,\mathbb{R}) = \big\{x \in PC(J,\mathbb{R}) :\ & x(t) \text{ is continuously differentiable everywhere} \\ & \text{except some } t_k\text{'s at which} \\ & x'(t_k^-) \text{ and } x'(t_k^+) \text{ exist and } x'(t_k^-) = x'(t_k)\big\}\,. \end{aligned}$$

Let $t_0 \ge 0$, $\phi \in PC\big([t_0-\tau,t_0],\mathbb{R}\big)$. By a *solution* of (1) we mean a real valued function $x \in PC\big([t_0-\tau,+\infty),\mathbb{R}\big) \cap PC'\big([t_0,+\infty),\mathbb{R}\big)$ which satisfies

(iii) for any $t \in [t_0-\tau,t_0]$, $x(t) = \phi(t)$, $x(t_0^+) = x_0$, $x'(t_0^+) = x'_0$,

(iv) for any $t \in [t_0,+\infty)$, $t \neq t_k$, $k = 1,2,\ldots$, $x(t)$ satisfies

$$x''(t) + P(t)x(t-\tau) = 0\,,$$

(v) for any $k = 1,2,\ldots$, $x(t_k^+) = g_k\big(x(t_k)\big)$, $x'(t_k^+) = h_k\big(x'(t_k)\big)$.

Let t_0 be a given initial point and let $\phi \in PC\big([t_0-\tau,t_0],\mathbb{R}\big)$ be a given initial function, then one can show by using the method of steps that (1) has a unique solution on $[t_0,+\infty)$ satisfying the initial condition $x(t) = \phi(t)$ for $t \in [t_0-\tau,t_0]$.

A solution of (1) is said to be *nonoscillatory* if this solution is eventually positive or eventually negative. Otherwise this solution is said to be *oscillat*

LEMMA 1. ([1]) *Assume that*

(a1) *The sequence $\{t_k\}$ satisfies $0 \le t_0 < t_1 < t_2 < \cdots < t_k < \ldots$ and* $\lim\limits_{k\to\infty} t_k = +\infty$.

(a2) $m \in PC'(\mathbb{R}_+,\mathbb{R})$ *is left continuous at t_k for $k = 1,2,\ldots$*

(a3) *For $k = 1,2,\ldots$, $t \ge t_0$,*

$$\begin{aligned} m'(t) &\le p(t)m(t) + q(t)\,, \qquad t \neq t_k\,, \\ m(t_k^+) &\le d_k m(t_k) + b_k \end{aligned} \tag{2}$$

where $p,q \in C(\mathbb{R}_+,\mathbb{R})$, $d_k \ge 0$, and b_k are real constants.

Then

$$m(t) \le m(t_0) \prod_{t_0<t_k<t} d_k \exp\left(\int_{t_0}^{t} p(s)\,\mathrm{d}s\right) + \int_{t_0}^{t} \prod_{s<t_k<t} d_k \exp\left(\int_{s}^{t} p(\sigma)\,\mathrm{d}\sigma\right) q(s)\,\mathrm{d}s + \sum_{t_0<t_k<t} \prod_{t_k<t_j<t} d_j \exp\left(\int_{t_k}^{t} p(s)\,\mathrm{d}s\right) b_k . \tag{4}$$

Remark 1. If the inequalities (2) and (3) are reversed, then in the conclusion the inequality (4) is also reversed.

2. Main results

LEMMA 2. *Let $x(t)$ be a solution of* (1). *Assume that there exists some $T \ge t_0$ such that $x(t) > 0$ for $t \ge T$ and the following conditions hold*

(h1) *conditions* (i) *and* (ii) *are satisfied,*

(h2) $\lim\limits_{t\to+\infty} \int_{t_0}^{t} \prod_{t_0<t_k<s} \frac{\bar{b}_k}{a_k}\,\mathrm{d}s = +\infty$.

Then $x'(t) \ge 0$ for $t \in [T, t_l] \cup \left(\bigcup_{k=l}^{+\infty} (t_k, t_{k+1}]\right)$, where $l = \min\{k:\ t_k \ge T\}$.

Proof. At first, we shall prove that $x'(t_k) \ge 0$ for any $k \ge l$. If it is not true, then there exists some j such that $j \ge l$ and $x'(t_j) \le 0$. From (1) and (ii), we have

$$x'(t_j^+) = h_j\big(x'(t_j)\big) \le \bar{b}_j x'(t_j) < 0 .$$

Let $x'(t_j^+) = -\alpha$ $(\alpha > 0)$. By (1) and (i), for $t \in \bigcup_{i=1}^{+\infty} \big(t_{j+i-1}, t_{j+i}\big]$ we have

$$x''(t) = -P(t)x(t-\tau) \le 0 .$$

Hence, $x'(t)$ is monotonically nonincreasing in $\big(t_{j+i-1}, t_{j+i}\big]$, $i = 1, 2, \dots$.

So,

$$\begin{aligned} &x'(t_{j+1}) \le x'(t_j^+) = -\alpha < 0 , \\ &x'(t_{j+2}) \le x'(t_{j+1}^+) = h_{j+1}\big(x'(t_{j+1})\big) \le \bar{b}_{j+1} x'(t_{j+1}) \le -\bar{b}_{j+1}\alpha < 0 , \\ &x'(t_{j+3}) \le x'(t_{j+2}^+) \le \bar{b}_{j+2} x'(t_{j+2}) \le -\bar{b}_{j+2}\bar{b}_{j+1}\alpha < 0 . \end{aligned}$$

It is easy to show that, for any positive integer $n \geq 2$.

$$x'(t_{j+n}) \leq -\left(\prod_{i=1}^{n-1} \bar{b}_{j+i}\right)\alpha < 0.$$

Consider the following impulsive differential inequalities

$$\begin{aligned} x''(t) \leq 0, \qquad t > t_j, \quad t \neq t_k, \quad k = j+1, j+2, \dots, \\ x'(t_k^+) \leq \bar{b}_k x'(t_k), \qquad k = j+1, j+2, \dots . \end{aligned}$$

Let $m(t) = x'(t)$. Then

$$\begin{aligned} m'(t) \leq 0, \qquad t > t_j, \quad t \neq t_k, \quad k = j+1, j+2, \dots, \\ m(t_k^+) \leq \bar{b}_k m(t_k), \qquad k = j+1, j+2, \dots . \end{aligned}$$

From Lemma 1, we have

$$m(t) \leq m(t_j^+) \prod_{t_j<t_k<t} \bar{b}_k, \tag{5}$$

i.e.,

$$x'(t) \leq x'(t_j^+) \prod_{t_j<t_k<t} \bar{b}_k. \qquad (6$$

Then, using the facts that $x(t_k^+) \leq a_k x(t_k)$ $(k = j+1, j+2, \dots)$ holds, by Lemma 1 we get

$$\begin{aligned} x(t) &\leq x(t_j^+) \prod_{t_j<t_k<t} a_k + \int_{t_j^+}^{t} \prod_{s<t_k<t} a_k \left(x'(t_j^+) \prod_{t_j<t_k \quad s} b_k \right) \mathrm{d}s \\ &= \prod_{t_j<t_k<t} a_k \left[x(t_j^+) - \alpha \int_{t_j^+}^{t} \prod_{t_j<t_k \quad s} \frac{\bar{b}_k}{a_k} \, \mathrm{d}s \right]. \end{aligned} \qquad (7$$

Since $x(t) > 0$ for $t \geq T$, the last inequality (7) contradicts (h2) of Lemma 2 Therefore, $x'(t_k) \geq 0$ for $k \geq l$. The condition (ii) implies $x'(t_k^+) \geq \bar{b}_k x'(t_k)$ _ 0 for any $k \geq l$. Because $x'(t)$ is nonincreasing in $(t_k, t_{k+1}]$, it i clear that $x'(t) \geq x'(t_{k+1}) \geq 0$ for $t \in (t_k, t_{k+1}]$, $k \geq l$, and $x'(t)$ x' t_l ≥ 0 for $t \in [T, t_l]$. Thus the proof of Lemma 2 is complete

Remark 2. In the cas that $x(t)$ is event ially negativ , ur d the c idit (h1) and (h2), it can be prov d s'milar y th it $'(t) < 0$ $\in$ T, $\left(\bigcup_{l-l}^{+\infty} (t_k, t_{k+1}]\right)$, wh r l n in$\{k$ $\}$

THEOREM 1. *Assume that the conditions* (h1) *and* (h2) *of Lemma* 2 *hold and there exists a positive integer* k_0 *such that* $\overline{a}_k \ge 1$ *for* $k \ge k_0$. *If*

$$\lim_{t\to+\infty} \int_{t_0}^{t} \prod_{t_0<t_k<s} \frac{1}{b_k} P(s)\, \mathrm{d}s = +\infty\,, \tag{8}$$

then every solution of (1) *is oscillatory.*

P r o o f. Without loss of generality, we can assume $k_0 = 1$. Let $x(t)$ be a nonoscillatory solution of (1), say $x(t) > 0$ for $t \ge t_0$. From Lemma 2, we can find $x'(t) \ge 0$ for $t \in [t_0, t_1] \cup \left(\bigcup_{k=1}^{+\infty} (t_k, t_{k+1}] \right)$. It is clear that $x'(t-\tau) \ge 0$ for $t \ge t_0 + \tau$.

Set

$$u(t) = \frac{x'(t)}{x(t-\tau)}\,.$$

Then, $u(t_k^+) \ge 0$ for $k = 1, 2, \dots$, $u(t) \ge 0$ for $t \ge t_0$. By (i) and (1), we get

$$u'(t) = \frac{x''(t)}{x(t-\tau)} - \frac{x'(t)x'(t-\tau)}{x^2(t-\tau)} \le -P(t)\,, \qquad t \ne t_k\,.$$

If $t_k - \tau \notin \{t_k :\ k = 1, 2, \dots\}$, then $x(t_k^+ - \tau) = x(t_k - \tau)$. Condition (ii) yields that,

$$u(t_k^+) = \frac{x'(t_k^+)}{x(t_k^+ - \tau)} \le \frac{b_k x'(t_k)}{x(t_k - \tau)} = b_k u(t_k)\,. \tag{9}$$

If $t_k - \tau \in \{t_k :\ k = 1, 2, \dots\}$, we assume $t_k - \tau = t_j$ for some positive integer j. From condition (ii),

$$x(t_k^+ - \tau) = x(t_j^+) = g_j\big(x(t_j)\big) \ge \overline{a}_j x(t_j) = \overline{a}_j x(t_k - \tau)\,.$$

Since $\overline{a}_j \ge 1$, we obtain

$$u(t_k^+) = \frac{x'(t_k^+)}{x(t_k^+ - \tau)} \le \frac{b_k x'(t_k)}{\overline{a}_j x(t_k - \tau)} \le \frac{b_k x'(t_k)}{x(t_k - \tau)} = b_k u(t_k)\,. \tag{10}$$

If $t_k + \tau \notin \{t_k :\ k = 1, 2, \dots\}$, then $x(t_k^+ + \tau) = x(t_k + \tau)$. By (ii),

$$u(t_k^+ + \tau) = \frac{x'(t_k^+ + \tau)}{x(t_k^+)} \le \frac{x'(t_k + \tau)}{x(t_k)} = u(t_k + \tau)\,.$$

If $t_k + \tau \in \{t_k :\ k = 1, 2, \dots\}$, we assume $t_k + \tau = t_j$ for some positive integer j. From (ii),

$$u(t_k^+ + \tau) = \frac{x'(t_k^+ + \tau)}{x(t_k^+)} \le \frac{b_j x'(t_k + \tau)}{x(t_k)} = b_j u(t_k + \tau)\,.$$

We construct the sequences

$$\{t'_k:\ k\in N\}=\{t_k:\ k\in N\}\cup\{t_k+\tau:\ k\in N\},$$

where $0<t'_1<t'_2<\cdots<t'_k<\dots$ with $\lim\limits_{k\to+\infty} t'_k=+\infty$. Set

$$e_k=\begin{cases} b_i & \text{if } t'_k=t_i,\ k=1,2,\dots,\\ 1 & \text{if } t'_k=t_j+\tau,\ k=1,2,\dots.\end{cases}$$

Consider the following impulsive differential inequalities

$$\begin{aligned} u'(t)&\le -P(t), \qquad t\ge t_0,\quad t\ne t'_k,\ k=1,2,\dots,\\ u\big((t'_k)^+\big)&\le e_k u(t'_k), \qquad k=1,2,\dots. \end{aligned} \tag{11}$$

By Lemma 1, we obtain

$$\begin{aligned} u(t)&\le u(t_0)\prod_{t_0<t'_k<t} b_k-\int_{t_0}^{t}\prod_{s<t'_k<t} b_kP(s)\,\mathrm{d}s\\ &=u(t_0)\prod_{t_0<t_k<t} b_k-\int_{t_0}^{t}\prod_{s<t_k<t} b_kP(s)\,\mathrm{d}s\\ &=\prod_{t_0<t_k<t} b_k\left[u(t_0)-\int_{t_0}^{t}\prod_{t_0<t_k<s}\frac{1}{b_k}P(s)\,\mathrm{d}s\right]. \end{aligned} \tag{12}$$

The last inequality (12) and $u(t)\ge 0$ contradict (8) of Theorem 1. Hence every solution of (1) is oscillatory. The proof of Theorem 1 is complete. □

THEOREM 2. *Assume that the conditions* (h1) *and* (h2) *of Lemma* 2 *hold and* $t_{k+1}-t_k=\tau$ *for all* $k=1,2,\dots$. *If*

$$\lim_{t\to+\infty}\int_{t_0}^{t}\prod_{t_0<t_k<s}\frac{1}{c_k}P(s)\,\mathrm{d}s=+\infty, \tag{13}$$

where

$$c_k=\begin{cases} b_1 & \text{if } k=1,\\ \frac{b_k}{a_{k-1}} & \text{if } k=2,3,\dots,\end{cases}$$

then every solution of (1) *is oscillatory.*

Proof. Let $x(t)$ be a nonoscillatory solution of equation (1), say $x(t)>0$ for $t\ge t_0$. From Lemma 2, $x'(t)\ge 0$ for $t\ge t_0$. It is clear that $x'(t-\tau)\ge 0$ for $t\ge t_0+\tau$.

Set

$$u(t) = \frac{x'(t)}{x(t-\tau)}.$$

Then, $u(t_k^+) \geq 0$ for $k = 1, 2, \ldots$, $u(t) \geq 0$ for $t \geq t_0$. Using condition (i), by (1), we have

$$u'(t) \leq -P(t), \qquad t \neq t_k.$$

If $k = 1$,

$$u(t_1^+) = \frac{x'(t_1^+)}{x(t_1^+ - \tau)} \leq \frac{b_1 x'(t_1)}{x(t_1 - \tau)} = b_1 u(t_1) = c_1 u(t_1). \tag{14}$$

If $k = 2, 3, \ldots$,

$$u(t_k^+) = \frac{x'(t_k^+)}{x(t_k^+ - \tau)} \leq \frac{b_k x'(t_k)}{x(t_{k-1}^+)} \leq \frac{b_k x'(t_k)}{\overline{a}_{k-1} x(t_{k-1})} = \frac{b_k x'(t_k)}{\overline{a}_{k-1} x(t_k - \tau)} = c_k u(t_k). \tag{15}$$

Consider the following impulsive differential inequalities

$$\begin{aligned} u'(t) \leq -P(t), \qquad t \geq t_0, \quad t \neq t_k, \; k = 1, 2, \ldots, \\ u(t_k^+) \leq c_k u(t_k), \qquad k = 1, 2, \ldots. \end{aligned} \tag{16}$$

By Lemma 1, we have

$$\begin{aligned} u(t) &\leq u(t_0) \prod_{t_0 < t_k < t} c_k - \int_{t_0}^{t} \prod_{s < t_k < t} c_k P(s) \, \mathrm{d}s \\ &= \prod_{t_0 < t_k < t} c_k \left[u(t_0) - \int_{t_0}^{t} \prod_{t_0 < t_k < s} \frac{1}{c_k} P(s) \, \mathrm{d}s \right]. \end{aligned} \tag{17}$$

The last inequality (17) and $u(t) \geq 0$ contradict (13) of Theorem 2. Hence every solution of (1) is oscillatory. The proof of Theorem 2 is complete. □

From Theorem 1 and Theorem 2, we can immediately obtain the following corollaries.

COROLLARY 1. *Assume that the conditions* (h1) *and* (h2) *of Lemma* 2 *hold and there exists a positive integer* k_0 *such that* $\overline{a}_k \geq 1$, $b_k \leq 1$ *for* $k \geq k_0$. *If*

$$\lim_{t \to +\infty} \int_{t_0}^{t} P(s) \, \mathrm{d}s = +\infty,$$

then every solution of (1) *is oscillatory.*

Proof. Without loss of generality, let $k_0 = 1$. Since $b_k \le 1$, we have

$$\begin{aligned}\lim_{t\to+\infty}\int_{t_0}^{t}\prod_{t_0<t_k<s}\frac{1}{b_k}P(s)\,\mathrm{d}s &= \lim_{n\to+\infty}\int_{t_0}^{t_{n+1}}\prod_{t_0<t_k<s}\frac{1}{b_k}P(s)\,\mathrm{d}s\\ &= \lim_{n\to+\infty}\sum_{i=0}^{n}\int_{t_i^+}^{t_{i+1}}\prod_{t_0<t_k<s}\frac{1}{b_k}P(s)\,\mathrm{d}s\\ &= \lim_{n\to+\infty}\sum_{i=0}^{n}\prod_{t_0<t_k<t_{i+1}}\frac{1}{b_k}\int_{t_i^+}^{t_{i+1}}P(s)\,\mathrm{d}s\\ &\ge \lim_{n\to+\infty}\sum_{i=0}^{n}\int_{t_i^+}^{t_{i+1}}P(s)\,\mathrm{d}s\\ &= \lim_{n\to+\infty}\int_{t_0^+}^{t_{n+1}}P(s)\,\mathrm{d}s = +\infty\,.\end{aligned}$$

In view of Theorem 1, we find that every solution of (1) is oscillatory. □

COROLLARY 2. *Assume that the conditions* (h1) *and* (h2) *of Lemma* 2 *hold and there exist a positive integer* k_0 *and a constant* $\alpha > 0$ *such that* $\bar{a}_k \ge 1$. $\frac{1}{b_k} \ge \left(\frac{t_{k+1}}{t_k}\right)^{\alpha}$ *for* $k \ge k_0$. *If*

$$\lim_{t\to+\infty}\int_{t_0}^{t} s^{\alpha}P(s)\,\mathrm{d}s = +\infty\,,$$

then every solution of (1) *is oscillatory.*

Proof. Without loss of generality, let $k_0 = 1$. We have

$$\begin{aligned}\lim_{t\to+\infty}\int_{t_0}^{t}\prod_{t_0<t_k<s}\frac{1}{b_k}P(s)\,\mathrm{d}s &= \lim_{n\to+\infty}\sum_{i=0}^{n}\prod_{t_0<t_k<t_{i+1}}\frac{1}{b_k}\int_{t_i^+}^{t_{i+1}}P(s)\,\mathrm{d}s\\ &\ge \lim_{n\to+\infty}\frac{1}{t_1^{\alpha}}\sum_{i=1}^{n}t_{i+1}^{\alpha}\int_{t_i^+}^{t_{i+1}}P(s)\,\mathrm{d}s\end{aligned}$$

$$\geq \lim_{n\to+\infty} \frac{1}{t_1^\alpha} \sum_{i=1}^{n} \int_{t_i^+}^{t_{i+1}} s^\alpha P(s)\, \mathrm{d}s$$

$$= \lim_{n\to+\infty} \frac{1}{t_1^\alpha} \int_{t_1^+}^{t_{n+1}} s^\alpha P(s)\, \mathrm{d}s = +\infty\,.$$

In view of Theorem 1, we can see that every solution of (1) is oscillatory. □

COROLLARY 3. *Assume that the conditions* (h1) *and* (h2) *of Lemma 2 hold and* $t_{k+1} - t_k = \tau$ *for all* $k = 1, 2, \dots$ *. Suppose that there exist a positive integer* k_0 *and a constant* $\alpha > 0$ *such that* $\frac{1}{c_k} \geq \left(\frac{t_{k+1}}{t_k}\right)^\alpha$ *for* $k \geq k_0$*, where*

$$c_k = \begin{cases} b_1 & \text{if } k = 1\,, \\ \frac{b_k}{\overline{a}_{k-1}} & \text{if } k = 2, 3, \dots\,. \end{cases}$$

If

$$\lim_{t\to+\infty} \int_{t_0}^{t} s^\alpha P(s)\, \mathrm{d}s = +\infty\,,$$

then every solution of (1) *is oscillatory.*

Corollary 3 can be deduced from Theorem 2. Its proof is similar to that of Corollary 2. Here we omit it.

EXAMPLE 1. Consider

$$\begin{aligned} &x''(t) + \frac{1}{t \ln t} x(t-1) = 0\,, \qquad t \geq \frac{3}{2}\,, \quad t \neq 2^k\,,\ k = 1, 2, \dots\,, \\ &x\big((2^k)^+\big) = \frac{2(k+1)}{k} x(2^k)\,, \quad x'\big((2^k)^+\big) = x'(2^k)\,, \qquad k = 1, 2, \dots\,, \end{aligned} \tag{18}$$

where $a_k = \overline{a}_k = \frac{2(k+1)}{k}$, $b_k = \overline{b}_k = 1$, $P(t) = \frac{1}{t \ln t}$, $t_0 = \frac{3}{2}$, $t_k = 2^k$, $k = 1, 2, \dots$. Obviously, the condition (h1) of Lemma 2 is satisfied and

$$\lim_{t\to+\infty}\int_{t_0}^{t}\prod_{t_0<t_k<s}\frac{\overline{b}_k}{a_k}\,\mathrm{d}s=\int_{\frac{3}{2}}^{+\infty}\prod_{\frac{3}{2}<t_k<s}\frac{k}{2(k+1)}\,\mathrm{d}s$$

$$=\int_{\frac{3}{2}}^{t_1}\prod_{\frac{3}{2}<t_k<s}\frac{k}{2(k+1)}\,\mathrm{d}s+\int_{t_1^+}^{t_2}\prod_{\frac{3}{2}<t_k<s}\frac{k}{2(k+1)}\,\mathrm{d}s$$

$$+\int_{t_2^+}^{t_3}\prod_{\frac{3}{2}<t_k<s}\frac{k}{2(k+1)}\,\mathrm{d}s+\int_{t_3^+}^{t_4}\prod_{\frac{3}{2}<t_k<s}\frac{k}{2(k+1)}\,\mathrm{d}s+\dots$$

$$=\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times 2+\left(\frac{1}{2}\right)^2\times\frac{1}{2}\times\frac{2}{3}\times 2^2$$

$$+\left(\frac{1}{2}\right)^3\times\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times 2^3+\dots=+\infty\,.$$

Let $k_0=1$. Then

$$\overline{a}_k\geq 1\,,\qquad k\geq k_0\,,$$

and

$$\int_{\frac{3}{2}}^{+\infty}P(t)\,\mathrm{d}t=\int_{\frac{3}{2}}^{+\infty}\frac{1}{t\ln t}\,\mathrm{d}t=+\infty\,.$$

By Corollary 1, every solution of (18) is oscillatory.

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