J. Gałuszka

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0. Introduction. An algebra $\mathfrak{A} = (A, +, \cdot)$ of type (2,2) is said to be a *bisemilattice* if it satisfies the following identities:

$$\begin{array}{ll} x + x = x, & x \cdot x = x, \\ x + y = y + x, & x \cdot y = y \cdot x, \\ (x + y) + z = x + (y + z), & (x \cdot y) \cdot z = x \cdot (y \cdot z) \end{array}$$

(cf. [8], [9]). In the sequel we shall write xy instead $x \cdot y$.

Let $\mathfrak{A}$ be a bisemilattice. Similarly as in [1] and [2] we define two partial orders on A:

$$x \leqslant_{+} y \overset{\text{df}}{\Leftrightarrow} x + y = y,$$

$$x \leqslant_{.} y \overset{\text{df}}{\Leftrightarrow} xy = x.$$

Each finite and nonempty subset of A has a least upper bound with respect to the first order and a greatest lower bound with respect to the second. And conversely on each set A with two partial orders satisfying the last conditions we can define two binary operations $+$ and, such that $\mathfrak{A} = (A, +, \cdot)$ is a bisemilattice andd the orders $\leqslant_{+}$ and $\leqslant_{.}$ are identical with the initial orders respectively.

An *essentially binary* polynomial of a bisemilattice is a binary polynomial depending on both its variables (see p. 38 of [6]). In the sequel the cardinality of the set of all esentially binary polynomials of a bisemilattice $\mathfrak{A}$ is denoted by $p_2(\mathfrak{A})$, the variety of all bisemilattices is denoted by $\mathscr{B}$. If I is a set of identities for bisemilattices, then $\mathscr{B}(I)$ denotes the subvariety of $\mathscr{B}$ defined by I. We say that a bisemilattice is *trivial* iff its underlying set has one element only, it is a *semilattice* iff its fundamental polynomials are equal, and it is *proper* iff its fundamental polynomials are distinct. Evidently a proper bisemilattice is nontrivial. We say that a binary polynomial $f(x, y)$ is *commutative* iff $f(x, y) = f(y, x)$. We can easily see that in a nontrivial bisemilattice every binary and commutative polynomial is essentially binary.

Essentially binary polynomials play an important rôle in a research of the variety $\mathcal{B}$ (see e.g. [3] or [4]). The main purpose of this paper is to extend the main theorem of [4] (we quote this theorem in Section 1 — Theorem 1.1) and to characterize bisemilattices with five essentially binary polynomials.

1. In this section we state without proof a few results concerning bisemilattices, used in this paper.

The following identities and their duals play an important rôle in the sequel:

$$(x+y)y = y, \tag{1}$$
$$(x+y)y = xy + y, \tag{2}$$
$$(x+y)y = x+y, \tag{3}$$
$$xy + x + y = x + y, \tag{4}$$
$$(xy+y)(yx+x) = xy, \tag{5}$$
$$(xy+y)(yx+x) = x+y. \tag{6}$$

Dual identities will be denoted by (1d), (2d), etc.

Theorem 1.1 ([4]). *If $\mathfrak{A}$ is a bisemilattice, then*:
(*i*) $p_2(\mathfrak{A}) = 0$ *iff $\mathfrak{A}$ is trivial,*
(*ii*) $p_2(\mathfrak{A}) = 1$ *iff $\mathfrak{A}$ is a nontrivial semilattice,*
(*iii*) $p_2(\mathfrak{A}) = 2$ *iff $\mathfrak{A}$ is a nontrivial lattice,*
(*iv*) *there are no bisemilattices for which* $p_2(\mathfrak{A}) = 3$,
(*v*) $p_2(\mathfrak{A}) = 4$ *iff $\mathfrak{A}$ is a nontrivial, non-semilattice, non-lattice member of one of the varieties* $\mathcal{B}(1,4,5)$, $\mathcal{B}(1\mathrm{d},4\mathrm{d},5\mathrm{d})$, $\mathcal{B}(3,4,6)$, $\mathcal{B}(3\mathrm{d},4\mathrm{d},6\mathrm{d})$, $\mathcal{B}(2)$.

Lemma 1.2 ([3]). *Let $\mathfrak{A}$ be a nontrivial bisemilattice. The following conditions are equivalent*:
(*i*) *$\mathfrak{A}$ is a lattice,*
(*ii*) $(xy+y)(x+y)$ *is not essentially binary.*

Lemma 1.3 ([3]). *In a nontrivial bisemilattice* $(x+y)y \neq x$ *and* $xy+y \neq x$.

Lemma 1.4 ([3]). *Let $\mathfrak{A}$ be a bisemilattice. Then the following statements hold*:
(*i*) *The following identities are equivalent in $\mathfrak{A}$*: (3), $(x+y)y = (y+x)x$, $(x+y)xy = x+y$.
(*ii*) *If $\mathfrak{A}$ is a proper bisemilattice and $(x+y)y$ is commutative, then $xy+y$ is essentially binary and noncommutative.*

Lemma 1.5 ([5]). *Let $f(x,y)$ be a polynomial over a nontrivial bisemilattice. Then both the polynomials $f(x,y)+y$ and $f(x,y)y$ depend on y.*

Lemma 1.6 ([4], [5]). *If $\mathfrak{A}$ is a nontrivial member from $\mathcal{B}(3)$, then every binary polynomial in $\mathfrak{A}$ is essentially binary.*

Evidently the dual versions of Lemmas 1.2, 1.4, 1.6 are also true.

2. In this section we formulate the main theorem of this paper (Theorem 2.1). Besides the identities (1)—(6) we need the following identities:

$$(xy+y)(x+y)=xy+y, \tag{7}$$
$$(xy+y)(yx+x)=xy+x+y, \tag{8}$$
$$(xy+y)(x+y)=xy+x+y, \tag{9}$$
$$(xy+y)(yx+x)+x=x+y, \tag{10}$$
$$(xy+y)(yx+x)+xy=x+y, \tag{11}$$
$$(xy+y)(yx+x)+xy=(xy+y)(yx+x). \tag{12}$$

Theorem 2.1. *Let* $\mathfrak{A}$ *be a bisemilattice. Then* $p_2(\mathfrak{A})=5$ *iff* $\mathfrak{A}$ *is a member of one of the classes* $\mathscr{B}(1,5,7)\setminus\mathscr{B}(4)$, $\mathscr{B}(1d,5d,7d)\setminus\mathscr{B}(4d)$, $\mathscr{B}(6)\setminus\mathscr{B}(4)$, $\mathscr{B}(6d)\setminus\mathscr{B}(4d)$, $\mathscr{B}(3,8,9)\setminus\mathscr{B}(4)$, $\mathscr{B}(3d,8d,9d)\setminus\mathscr{B}(4d)$, $\mathscr{B}(3,10,11)\setminus\mathscr{B}(6)$, $\mathscr{B}(3d,10d,11d)\setminus\mathscr{B}(6d)$, $\mathscr{B}(3,10,12)\setminus\mathscr{B}(6)$, $\mathscr{B}(3d,10d,12d)\setminus\mathscr{B}(6d)$.

Before proving this theorem we formulate and prove needed lemmas.

Lemma 2.2. *If the identity* $(x+y)y=yx+x$ *holds in a bisemilattice* $\mathfrak{A}$, *then* $\mathfrak{A}$ *is a semilattice.*

Proof. If the identity $(x+y)y=yx+x$ holds in $\mathfrak{A}$, then we have $(x+y)y=((x+y)+y)y=y(x+y)+(x+y)=yx+x+y$. Hence, the polynomials $(x+y)y$ and $xy+y$ are both commutative. By (ii) of Lemma 1.4 we obtain that $\mathfrak{A}$ is a semilattice.

Lemma 2.3. *If in a bisemilattice* $\mathfrak{A}$ *the polynomials* $(x+y)y$ *and* $xy+y$ *are both essentially binary, noncommutative and distinct, then* $p_2(\mathfrak{A})\geqslant 6$.

Proof. Evidently $\mathfrak{A}$ is proper. By the assumption and Lemma 2.2 the polynomials $x+y$, xy, $(y+x)x$, $(x+y)y$, $xy+y$, $yx+x$ are all essentially binary and distinct. Hence we get $p_2(\mathfrak{A})\geqslant 6$.

Lemma 2.4. *If* $\mathfrak{A}$ *is a bisemilattice such that* $p_2(\mathfrak{A})=5$, *then* $\mathfrak{A}$ *is in one of the classes* $\mathscr{B}(1)$, $\mathscr{B}(1d)$, $\mathscr{B}(3)$, $\mathscr{B}(3d)$.

Proof. If $p_2(\mathfrak{A})=5$, then by Lemma 2.3 and Lemma 1.3 we get that one of the identities (1), (1d), (3), (3d), (2) holds in $\mathfrak{A}$. By Theorem 1.1(v) the identity (2) does not hold in $\mathfrak{A}$.

Lemma 2.5. *If* $\mathfrak{A}$ *is a nontrivial bisemilattice from* $\mathscr{B}(1)$, *then the following statements are true*:

(*i*) *the polynomial* $(xy+y)x$ *is essentially binary,*
(*ii*) *the polynomials* $xy+y$, $yx+x$, $(xy+y)x$ *are all distinct,*
(*iii*) *the polynomial* $(xy+y)(x+y)$ *is noncommutative,*
(*iv*) *the polynomials* $xy+y$ *and* $(yx+x)(y+x)$ *are distinct.*

Proof. (i) By Lemma 1.5 the polynomial $(xy+y)x$ depends on x. Assume that the polynomial $(xy+y)x$ does not depend on y. Then the identity $x=(xy+y)x$ holds in $\mathfrak{A}$. Hence, putting $x+y$ for x we get $x+y=$

$= ((x + y)y + y)(x + y) = y(x + y) = y$, a contradiction with the assumption that $\mathfrak{A}$ is nontrivial.

(ii) Evidently $\mathfrak{A}$ is proper. Then by Lemma 1.4(ii) we infer that the polynomials $xy + y$ and $yx + x$ are distinct. Assume that the identity $xy + y = (xy + y)x$ holds in $\mathfrak{A}$. Then $x + y = x + (x + y) = x(x + y) + (x + y) = (x(x + y) + (x + y))x = x$, a contradiction. The case $yx + x = (xy + y)x$ can be treated analogously.

(iii) Assume that the identity $(xy + y)(x + y) = (yx + x)(y + x)$ holds in $\mathfrak{A}$. Then $y + x = (y + (y + x))(y + (y + x)) = (y(y + x) + (y + x))(y + (y + x)) = ((y + x)y + y)((y + x) + y) = y(y + x) = y$, a contradiction.

(iv) If the identity $xy + y = (yx + x)(y + x)$ holds in $\mathfrak{A}$, then $x + y = (y(x + y) + (x + y))(y + (x + y)) = (x + y)y + y = y$, a contradiction.

Lemma 2.6. *If* $\mathfrak{A}$ *is a bisemilattice from* $\mathscr{B}(1)$ *and the polynomial* $(xy + y)x$ *is commutative in* $\mathfrak{A}$, *then*

$$(xy + y)x = xy \tag{5'}$$

and (5) *are satisfied in* $\mathfrak{A}$.

Proof. Under the above assumption we have $xy = (x + xy)xy = ((yx + x)y)x = (xy + y)x$. Thus the identity (5′) holds in $\mathfrak{A}$. By (5′) $xy = (xy + y)x = ((xy + y)x + x)(xy + y) = (xy + x)(xy + y)$. Thus the identity (5) holds in $\mathfrak{A}$.

Lemma 2.7. *Let* $\mathfrak{A}$ *be a bisemilattice from* $\mathscr{B}(1)$. *Then* $p_2(\mathfrak{A}) = 5$ *iff* $\mathfrak{A}$ *is a member of the class* $\mathscr{B}(5,7)\setminus\mathscr{B}(4)$.

Proof. Let $\mathfrak{A}\in\mathscr{B}(1)$ and $p_2(\mathfrak{A}) = 5$. By Theorem 1.1, Lemma 1.3 and the dual version of Lemma 1.4(ii) the polynomials $x + y$, xy, $xy + y$, $yx + x$ are all essentially binary and distinct. Thus by (i) and (ii) of Lemma 2.5 the polynomial $(xy + y)x$ is commutative in $\mathfrak{A}$. Hence, by Lemma 2.6, the identity (5) holds in $\mathfrak{A}$. By Lemma 2.5(iii), Lemma 1.2 and Theorem 1.1(iii) the polynomials $(xy + y)(x + y)$ and $(yx + x)(y + x)$ are both essentially binary and distinct. Hence, by Lemma 2.5(iv), the identity (7) holds in $\mathfrak{A}$. Thus $\mathfrak{A}\in\mathscr{B}(1,5,7)$. By Theorem 1.1(v) $\mathfrak{A}\notin\mathscr{B}(4)$.

Assume that $\mathfrak{A}\in\mathscr{B}(1,5,7)\setminus\mathscr{B}(4)$. Thus $\mathfrak{A}\notin\mathscr{B}(4)$. Hence the polynomials $x + y$ and $xy + x + y$ are distinct. Evidently $\mathfrak{A}$ is not a lattice and $\mathfrak{A}$ is nontrivial. Hence and by Lemma 1.3 the polynomials $x + y$, xy, $xy + y$, $yx + x$, $xy + x + y$ are all essentially binary. By the dual version of Lemma 1.4 the polynomials $xy + y$ and $yx + x$ are noncommutative and the polynomials xy and $xy + x + y$ are distinct. Hence we get that the polynomials $x + y$, xy, $xy + y$, $yx + x$, $xy + x + y$ are all essentially binary and distinct. Recall that $P^{(2)}(\mathfrak{A}) = \bigcup_{i=0}^{\infty} P_i^{(2)}$, where $P_0^{(2)} = \{x, y\}$, $P_{n+1}^{(2)} = P_n^{(2)} \cup \{p + q,\ pq \mid p, q \in P_n^{(2)}\}$ (see

[6], [7]). Hence $P_2^{(2)} = \{x, y, x + y, xy, xy + y, yx + x, xy + x + y\}$. Observe that using (1) and (5) we obtain $(xy + y)x = (xy + y)(yx + x)x = xy = (yx + x)y$. By (5′) and (5) $P^{(2)}(\mathfrak{A}) = P_2^{(2)}$ and $p_2(\mathfrak{A}) = 5$.

Lemma 2.8. *If $\mathfrak{A}$ is a proper bisemilattice from $\mathscr{B}(3)$, then the following statements are true:*

(*i*) *the polynomials $xy + y$, $yx + x$, $(xy + y)(x + y)$ are all essentially binary and distinct,*

(*ii*) *the polynomials $xy + y$, $yx + x$, $(xy + y)(yx + x) + x$ are all essentially binary and distinct,*

(*iii*) *the polynomials xy and $(xy + y)(yx + x)$ are both essentially binary and distinct,*

(*iv*) *the polynomials xy and $(xy + y)(yx + x) + xy$ are both essentially binary and distinct.*

Proof. To prove this lemma we use the analogous methods as in the proof of Lemma 2.5. For example we prove (i).

(i) By Lemma 1.6 the polynomials $xy + y$, $yx + x$, $(xy + y)(x + y)$ are all essentially binary. By Lemma 1.4(ii) the polynomial $xy + y$ is noncommutative. Suppose that the identity $xy + y = (xy + y)(x + y)$ holds in $\mathfrak{A}$. Then $xy = x(xy) + xy = (x(xy) + xy)(x + xy) = x + xy$, a contradiction. Assume that the identity $yx + x = (xy + y)(x + y)$ holds in $\mathfrak{A}$. Then $xy = y(xy) + xy = ((xy)y + y)(xy + y) = xy + y$, a cotradiction. Hence the polynomials $xy + y$, $yx + x$, $(xy + y)(x + y)$ are all essentially binary and distinct.

We can easily see that the identity (6) implies the identity (3). Indeed, by (6) (putting xy for x) we get $xy + y = ((xy)y + y)(y(xy) + xy) = (xy + y)xy$. Hence $(x + y)y = (xy + y)(yx + x)xy$. Thus the polynomial $(x + y)y$ is commutative. Hence (and by Lemma 1.4(i)) we obtain the identity (3).

Lemma 2.9. *Let $\mathfrak{A}$ be a bisemilattice from $\mathscr{B}(3)\setminus\mathscr{B}(4)$. Then $p_2(\mathfrak{A}) = 5$ iff $\mathfrak{A}$ is a member of one of the classes $\mathscr{B}(6)$ and $\mathscr{B}(8, 9)$.*

Proof. Let $\mathfrak{A} \in \mathscr{B}(3)\setminus\mathscr{B}(4)$ and $p_2(\mathfrak{A}) = 5$. Thus $\mathfrak{A} \notin \mathscr{B}(4)$. Hence the polynomials $xy + x + y$ and $x + y$ are distinct. By Lemma 1.4 the polynomials $xy + y$ and $yx + x$ are noncommutative. By the dual version of (i) of Lemma 1.4 the polynomials xy and $xy + x + y$ are dictinct. Hence the polynomials $x + y$, xy, $xy + y$, $yx + x$, $xy + x + y$ are all essentially binary and distinct (recall that $\mathfrak{A}$ is nontrivial). By Lemma 2.8(iii) one of the identities (6) and (8) holds in $\mathfrak{A}$. Let $\mathfrak{A} \in \mathscr{B}(8)$. By Lemma 2.8(i) the polynomial $(xy + y)(x + y)$ is commutative. Hence and by (8) and (9) we obtain that $(xy + y)(x + y) = (xy + y)(x + y)\cdot(yx + x)(y + x) = (xy + x + y)(x + y) = xy + x + y$. Thus $\mathfrak{A} \in \mathscr{B}(8, 9)$.

Assume that $\mathfrak{A} \in \mathscr{B}(6)\setminus\mathscr{B}(4)$. Then as above we can prove that the polynomials $x + y$, xy, $xy + y$, $yx + x$, $xy + x + y$ are all essentially binary and distinct (recall that (6) implies (3)). By (6) we have $(xy + y)(y + y) = = (xy + y)(xy + y)(yx + x) = x + y$. Hence and by (3) we conclude that $P_2^{(2)} = \{x, y, x + y, xy, xy + y, yx + x, xy + x + y\}$ and $P^{(2)}(\mathfrak{A}) = P_2^{(2)}$. Thus $p_2(\mathfrak{A}) = 5$.

Now, assume that $\mathfrak{A} \in \mathscr{B}(3,8,9) \setminus \mathscr{B}(4)$. Then the polynomials $x + y$, xy, $xy + y$, $yx + x$, $xy + x + y$ are all essentially binary and distinct. By (8) and (9) we have that $P^{(2)}(\mathfrak{A}) = \{x, y, x + y, xy, xy + y, yx + x, xy + x + y\}$. Hence $p_2(\mathfrak{A}) = 5$.

Lemma 2.10. *Let $\mathfrak{A}$ be a bisemilattice from $\mathscr{B}(3,4) \setminus \mathscr{B}(6)$. Then $p_2(\mathfrak{A}) = 5$ iff $\mathfrak{A}$ is a member of one of the classes $\mathscr{B}(10,11)$ and $\mathscr{B}(10,12)$.*

Proof. We can easily see that $\mathscr{B}(3, 4, 10, 11) = \mathscr{B}(3, 10, 11)$ and $\mathscr{B}(3, 4, 10, 12) = \mathscr{B}(3, 10, 12)$. Indeed, if $\mathfrak{A} \in \mathscr{B}(11)$, then $x + y = ((xy + y)\cdot(yx + x) + xy) + xy = x + y + xy$.If $\mathfrak{A} \in \mathscr{B}(10,12)$, then $x + y = ((xy + y)\cdot(yx + x) + xy) + x = ((xy + y)(yx + x) + x) + xy = x + y + xy$.

Let $p_2(\mathfrak{A}) = 5$. By Lemma 1.4(ii) and Lemma 2.8(iii) the polynomials $x + y$, xy, $xy + y$, $yx + x$, $(xy + y)(yx + x)$ are all essentially binary and distinct. Hence $p^{(2)}(\mathfrak{A}) = \{x, y, x + y, xy, xy + y, yx + x, (yx + y)(yx + x)\}$. By Lemma 2.8(ii) the polynomial $(xy + y)(yx + x) + x$ is commutative in $\mathfrak{A}$. Hence and by (4) we get $(xy + y)(yx + x) + x = (xy + y)(yx + x) + (x + y) = = (xy + y)(yx + x) + xy + x + y = (xy + y)(yx + x) + (xy + y) + (yx + x) = = xy + x + y = x + y$. Thus the identity (10) holds in $\mathfrak{A}$. By Lemmas 1.4(ii) and 2.8(iv) we conclude that one of the identities (11) and (12) holds in $\mathfrak{A}$.

Assume that $\mathfrak{A} \in \mathscr{B}(3, 10, 11) \setminus \mathscr{B}(6)$. The polynomials $x + y$, xy, $xy + y$, $yx + x$, $(xy + y)(yx + x)$ are all essentially binary and distinct. By (3) $xy + y = = (xy + y)x$. Hence and by (3) and (4) we infer that $P_3^{(2)} = \{x, y, x + y, xy, xy + y, yx + x, (xy + y)(yx + x)\}$. By (10) and (11) $P_4^{(2)} = P_3^{(2)}$. Hence $P^{(2)}(\mathfrak{A}) = P_3^{(2)}$ and $p_2(\mathfrak{A}) = 5$.

Now assume that $\mathfrak{A} \in \mathscr{B}(3, 10, 12) \setminus \mathscr{B}(6)$. As above we obtain that $P_3^{(2)} = \{x, y, x + y, xy, xy + y, yx + x, (xy + y)(yx + x)\}$. By (3), (4), (10) and (12) we infer that $P_4^{(2)} = P_3^{(2)}$. Hence $P^{(2)}(\mathfrak{A}) = P_3^{(2)}$ and $p_2(\mathfrak{A}) = 5$.

Proof of the main theorem. We get Theorem 2.1 as a consequence of Lemma 2.4, Lemmas 2.7, 2.9, 2.10 (and their duals) and Theorem 1.1.

3. In this section we describe the free bisemilattices on two generators in the varieties $\mathscr{B}(1,5,7)$, $\mathscr{B}(6)$, $\mathscr{B}(3,8,9)$, $\mathscr{B}(3,10,12)$. By duality we obtain the free bisemilattices on two generators in the dual varieties.

1) The free bisemilattice on two generators in the variety $\mathscr{B}(1,5,7)$ has seven elements in the form presented in Figure 1 (cf. Figure 6 in [2]).

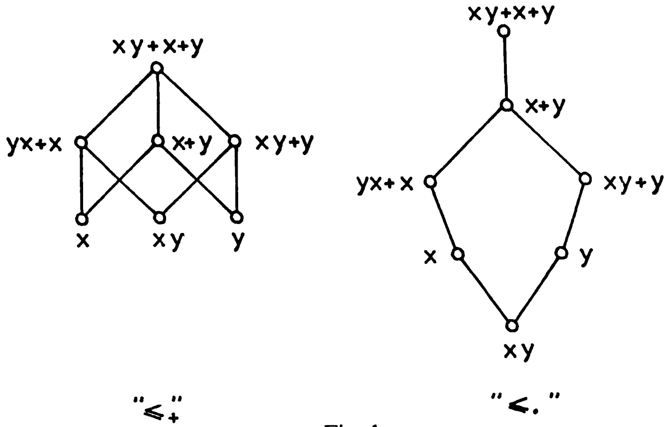

Fig. 1

2) The free bisemilattice on two generators in the variety $\mathscr{B}(6)$ has seven elements in the form presented in Figure 2.

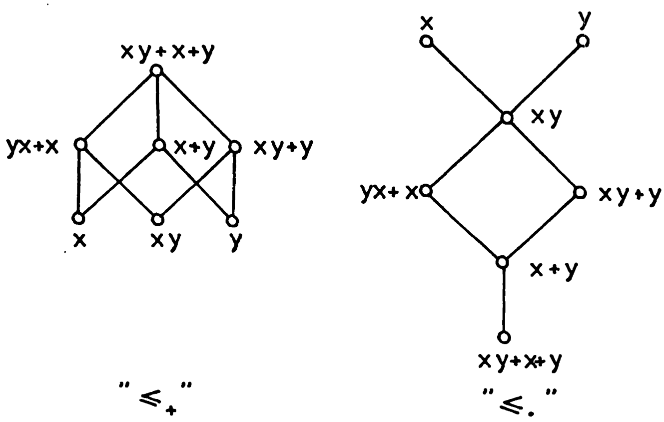

Fig. 2

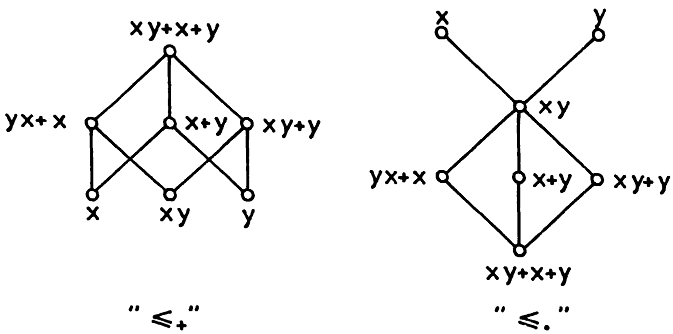

Fig. 3

3) The free bisemilattice on two generators in the variety $\mathscr{B}(3, 8, 9)$ has seven elements in the form presented in Figure 3.

4) The free bisemilattice on two generators in the variety $\mathscr{B}(3, 10, 11)$ has seven elements in the form presented in Figure 4.

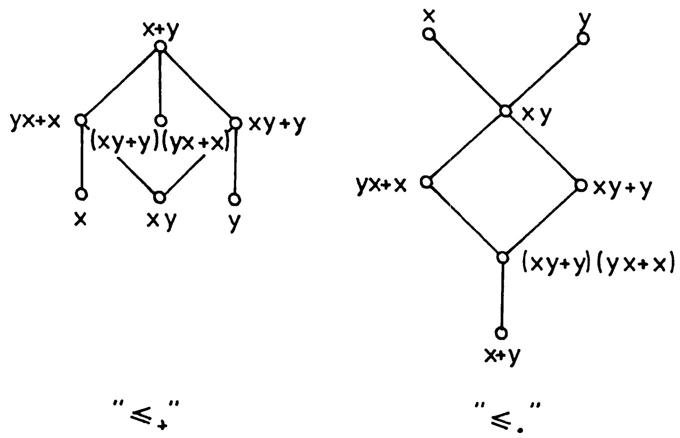

Fig. 4

5) The free bisemilattice on two generators in the variety $\mathscr{B}(3, 10, 12)$ has seven elements in the form presented in Figure 5.

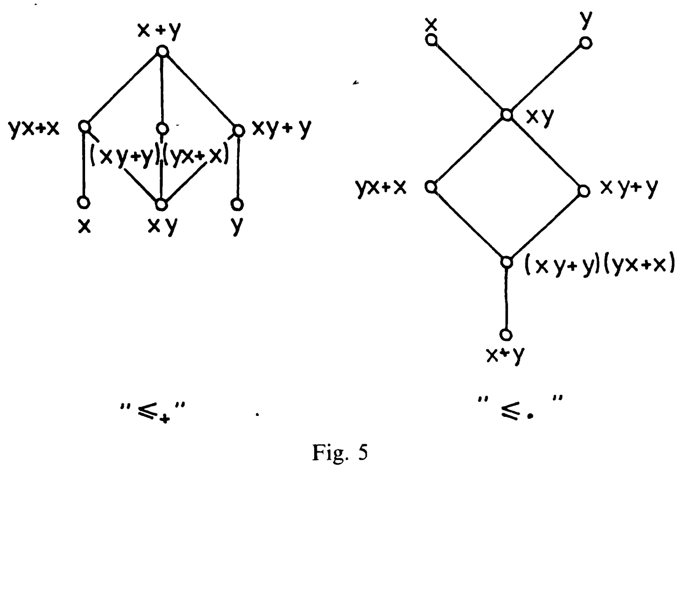

Fig. 5

Corollary 3.1. The classes $\mathscr{B}(1,5,7)\setminus\mathscr{B}(4)$, $\mathscr{B}(6)\setminus\mathscr{B}(4)$, $\mathscr{B}(3,8,9)\setminus\mathscr{B}(4)$, $\mathscr{B}(3,10,11)\setminus\mathscr{B}(6)$, $\mathscr{B}(3,10,12)\setminus\mathscr{B}(6)$ are all nonempty.

For a given class $\mathscr{K}$ of bisemilattices we denote by $N_2(\mathscr{K})$ the set of all k for which there exists a bisemilattice $\mathfrak{A}$ from $\mathscr{K}$ such that $p_2(\mathfrak{A}) = k$ (see [3]).

Corollary 3.2.

$$N_2(\mathscr{B}(1,5,7)) = \{0,2,4,5\},$$

$$N_2(\mathscr{B}(6)) = N_2(\mathscr{B}(3,8,9)) = N_2(\mathscr{B}(3,10,11)) =$$

$$N_2(\mathscr{B}(3,10,12)) = \{0,1,4,5\}.$$

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Jagiellonian University
Cracow, Poland

БИПОЛУРЕШЕТКИ, ИМЕЮЩИЕ ТОЧНО ПЯТЬ СУЩЕСТВЕННО БИНАРНЫХ ПОЛИНОМОВ

Jan Gałuszka

Резюме

В работе доказано, что класс всех биполурешеток, в которых находится точно дять существенно бинарных полиномов, разбивается на точно десять розличных субклассов.