Mathematica Slovaca

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Mathematica Slovaca, Vol. 27 (1977), No. 3, 257--265

Persistent URL: http://dml.cz/dmlcz/128859

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HORIZONTAL STRUCTURES ON FIBRE MANIFOLDS

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Libermann, [3], has defined a connection of the first order on a fibre space $E(B, F, \pi)$ as a global cross-section $\Gamma: E \to J^1E$. In this paper we find some properties of this structure. Our consideration are in the category C^∞. The standard terminology and notations of the theory of jets are used throughout the paper, see [2].

1. Let VTE denote the fibre bundle of vertical vectors on $E(B, F, \pi)$. A tensor field $\sigma: E \to VTE \otimes T^*E$ will be said to be a v-field. Let X be a vector field on E. Denote by $L_X(\sigma)$ the Lie derivative of σ by X. Locally, let (x^i, y^α), $i = 1, \ldots, n = \dim B$, $\alpha = 1, \ldots, \dim F$, be local coordinates on E. Direct evaluation yields for the v-field $\sigma: (x, y) \mapsto (a_k(x, y)dx^k + b^\alpha_\beta(x, y)dy^\beta) \otimes \partial y_\alpha$ and the vector field $X = a^i(x, y)\partial x_i + b^\alpha(x, y)\partial y_\alpha$:

$$
(1) \qquad L_X(\sigma) = -(a^\alpha_k dx^k + b^\alpha_\beta dy^\beta)\frac{\partial a^i}{\partial y^\alpha} \otimes \partial x_i + \left\{ \left(\frac{\partial a^\alpha_k}{\partial x^i} a^i + \right.\right.
$$

$$
\left. + \frac{\partial a^\alpha_k}{\partial y^\beta} b^\beta + a^\alpha_i \frac{\partial a^i}{\partial x^k} + b^\alpha_\beta \frac{\partial b^\beta}{\partial x^k} - \frac{\partial b^\alpha}{\partial y^\beta} a^\beta_k \right) dx^k +
$$

$$
+ \left(a^\alpha_k \frac{\partial a^k}{\partial y^\beta} + \frac{\partial b^\alpha_\beta}{\partial x^i} a^i + \frac{\partial b^\alpha_\beta}{\partial y^\gamma} b^\gamma + b^\alpha_\gamma \frac{\partial b^\gamma}{\partial y^\beta} - \right.
$$

$$
\left.\left. - \frac{\partial b^\alpha}{\partial y^\gamma} b^\gamma_\beta \right) dy^\beta \right\} \otimes \partial y_\alpha .
$$

This immediately gives

Lemma 1. *Let X be a vector field on E. Then the Lie derivative of every v-field on E by X is a v-field on E if and only if X is projectable.*

Let σ be a v-field, hence $\sigma(u) \in \mathrm{Hom}(T_uE, T_uE_x)$, $\pi u = x$. If $\sigma(u) | T_uE_x$ is regular for any $u \in E$, then σ determines a horizontal distribution of the kernels of $\sigma(u)$, i.e. a global cross-section $E \to J^1E$. Denote by $\varkappa(E)$ the set of all such v-fields on E that $\sigma(u) | T_uE_x = \mathrm{id} | T_uE_x$ for any $u \in E$. Let Γ_E be the set of all cross-sections $E \to J^1E$. There is a one to one correspondence $\delta: \varkappa(E) \to \Gamma_E$, where

$\delta(\sigma)$ is a cross-section $E \to J^1E$ determined by the horizontal distribution of the kernels of $\sigma(u)$, $u \in E$.

2. **Definition 1.** *Let $\Gamma: E \to J^1E$ be a cross-section. The pair (E, Γ) or the v-field $\delta^{-1}(\Gamma) \equiv {}^\Gamma\sigma$ will be called an H-structure or a tensor of the H-structure,* respectively.

Every 1 − jet $\Gamma(u)$ determines an element of $\mathrm{Hom}(T_xB, T_uE)$, $\pi u = x$. Thus we get a cross-section $\bar{\Gamma}: E \to TE \otimes T^*B$. Locally, let $(x^i, y^\alpha, y_i^\alpha)$ be local coordinates on J^1E. If $\Gamma: (x^i, y^\alpha) \to (x^i, y^\alpha, y_i^\alpha = -a_i^\alpha(x^k, y^\beta)$, then

$$
\begin{aligned}
{}^\Gamma\sigma &: (x, y) \mapsto (a_i^\alpha(x, y)dx^i + dy^\alpha) \otimes \partial y_\alpha, \\
\bar{\Gamma} &: (x, y) \mapsto dx^i \otimes \partial x_i - a_k^\alpha(x, y)dx^k \otimes \partial y_\alpha .
\end{aligned}
$$

By direct evaluation we get

Lemma 2. *Let X be a projectable vector field on E. Then $L_X(\bar{\Gamma})$ is a global cross-section $E \to VTE \otimes T^*M$ and*

$$(L_X{}^\Gamma\sigma)(u) = -(L_X\bar{\Gamma})(u)\pi_* .$$

Let X be a projectable vector field on E and ${}^1X$ be the first prolongation of X on J^1E. Let $\Gamma(E)$ be the set of all values of the cross-section $\Gamma: E \to J^1E$. By [1] a projectable field X on E is conjugate with Γ if $\Gamma_*(X)(h) = {}^1X(h)$. It is easy to prove

Proposition 1. *Let (E, Γ) be an H-structure. Let X be a projectable vector field on E. Then X is conjugate with Γ if and only if $L_X({}^\Gamma\sigma) = 0$.*

Denote by $\bar{Y}$ the Γ-lift of a vector field Y on B. Let $Z_1, Z_2 \in T_{x_0}B$. Let Y_1 or Y_2 be such a vector field on B that $Y_1(x_0) = Z_1$ or $Y_2(x_0) = Z_2$, respectively. Put

$$\Theta(u)(Z_1, Z_2) = {}^\Gamma\sigma(u)([\bar{Y}_1, \bar{Y}_2](u)).$$

It is easy to prove that $\Theta(u)(Z_1, Z_2)$ does not depend on the choice of the vector fields Y_1, Y_2 and that the mapping $u \mapsto \Theta(u)$ determines a global cross-section

$$\Theta: E \to VTE \otimes \wedge^2 T^*B,$$

which will be said to be the curvature field of the H-structure.

Let $\Gamma: E \to \bar{J}^2E$ denote the first prolongation of $\Gamma: E \to J^1E$, see [4]. In local coordinates, if

$$\Gamma: (x^i, y^\alpha) \mapsto (x^i, y^\alpha, y_i^\alpha = -a_i^\alpha(x^k, y^\beta)),$$

then

$$\Gamma' : (x^i, y^\alpha) \mapsto \left(x^i, y^\alpha, y_k^\alpha = -a_k^\alpha, y_{kj}^\alpha = \frac{\partial a_k^\alpha}{\partial y^\beta} a_i^\beta - \frac{\partial a_k^\alpha}{\partial x^i}\right). \tag{2}$$

Kolář, [4], introduced the difference tensor $\Delta(X)$ of an arbitrary semi-holonomic jet X. We recall that if $h \in \bar{J}^2_x E$, $\beta h = u \in E$, then $\Delta(h) \in T_u E_x \otimes \wedge^2 T^*_x B$. Locally, if $h = (x^i, y^\alpha, y^\alpha_i, y^\alpha_{ik})$, then $\Delta(h) = y^\alpha_{[i,k]} dx^i \wedge dx^k \otimes \partial y_\alpha$.

In the case of the H-structure (B, Γ) we obtain a global cross-section $\Delta(\Gamma'): E \to VTE \otimes \wedge^2 T^* B$. By the direct evaluation in local coordinates we get

Proposition 2. *Let* (E, Γ) *be an H-structure. Then*

$$\Theta(u) = -\Delta(\Gamma')(u) \tag{3}$$

for any $u \in E$.

By the relation (3) the curvature field Θ of the H-structure (E, Γ) is the curvature of the connection Γ by Libermann [3]. Relation (3) also gives in the comparison the curvature of the differential system Γ by Pradines [6].

Let $\bar{X} = a^i \partial x_i - a^\alpha_k a^k \partial y_\alpha$ be the Γ-lift of a vector field X on M. Using (1) we have

$$L_{\bar{X}}({}^\Gamma\sigma) = \left[\frac{\partial a^\alpha_k}{\partial x^i} - \frac{\partial a^\alpha_k}{\partial y^\beta} a^\beta_i + \frac{\partial a^\alpha_i}{\partial y^\beta} a^\beta_k - \frac{\partial a^\alpha_i}{\partial x^k}\right] a^i dx^k \otimes \partial y_\alpha \tag{4}$$

It immediately yields that the mapping

$$X \mapsto L_{\bar{X}}({}^\Gamma\sigma)$$

is a linear mapping of the modul $D(M)$ of all vector fields on M to the modul of all tensor fields $E \to VTE \otimes T^*M$. Moreover if the curvature field of (B, Γ) vanishes, then the Γ-lift $\bar{X}$ of X is conjugate with Γ.

Let $w \in J^1 E$, $\beta w = u$, $\pi u = x$. Denote by $L(w)$ the element of $T_u E \otimes T^*_x M$ determined by w. Then $L(w) - L(\Gamma(u)) \in T_u E_x \otimes T^*_x M$ and determines a 1-jet of $J^1_x(B, E_x)$, which we will denote by $w - \Gamma(u)$ and call the developement of w into E_x by means of Γ.

Let $v \in \bar{J}^2 E$, $\beta v = u$. Then the tensor $\bar{\tau}(v) = \Delta(v) - \Delta(\Gamma'(u))$ will be said to be the torsion of the 2-jet v. Let $\mathcal{S}: B \to \bar{J}^2 E$ be a global section of $\bar{J}^2 E$ over B. Let (E, Γ) be an H-structure. Then the threetuple $(E, \Gamma, \mathcal{S})$ will be called the SH-space. The tensor

$$\bar{\tau}(x) = \Delta(\mathcal{S}(x)) - \Delta(\Gamma'(\beta\mathcal{S}(x)))$$

will be said to be the torsion of the SH-space at $x \in B$.

Remark. The second prolongation of the section $S: B \to E$ gives a holonomic section $S^{(2)}: B \to J^2 E$ and determines the SH-space $(E, \Gamma, S^{(2)})$, the torsion of which has the property

$$\bar{\tau}(x) = \Theta(S(x)). \tag{5}$$

3. Let us compare our consideration with the theory of connections. Let Φ be a Lie grupoid of the operators on a fibre bundle $E(B, F, \pi)$. Let a, b be the projections of Φ and let $1_x \in \Phi$ denote the unit over $x \in B$. Let us recall (see [5])

that the connection (of the first order) on Φ is a global cross-section $C: B \rightarrow \bigcup_{x \in B} Q_x$, where Q_x denotes the set of all such elements $h \in J^1_x(a^{-1}(x), b, B)$ that $\beta h = 1_x$.

Let C be a connection on Φ, $C(x) = j^1_x \eta$. Let $v \in J^1_x E$, $v = j^1_x \xi$. We recall that

$$C^{-1}(x)(v) = j^1_x[\eta^{-1}(z)[\xi(z)]] \in J^1(B, E_x) \tag{6}$$

is the developement of v into E_x by means of C and analogously if $w \in \bar{J}^2_x E$, $w = j^1_x \xi$, then

$$C'^{-1}(x)(w) = C^{-1}(x)[j^1_x c^{-1}(z)(\xi(z))] \in \bar{J}^2(B, E_x) \tag{7}$$

is the developement of w into E_x by means of C.

Let $u \in E$, $\pi u = x$, $C(x) = j^1_x \eta$. Using the diffeomorphism $\eta(z): E_x \rightarrow E_z$ put

$$^C\Gamma(u) = j^1_x[z \mapsto \eta(z)(u)] \in J^1_x E. \tag{8}$$

It is easy to see that the mapping $u \mapsto {}^C\Gamma(u)$ determines a global cross-section ${}^C\Gamma: E \rightarrow J^1 E$. The H-structure $(E, {}^C\Gamma)$ will be said to be the representative of the connection C on E.

Denote by U the domain of the local cross-section η. We have a mapping f: $\pi^{-1}(U) \rightarrow E_x$ determined by $h \rightarrow \eta^{-1}(z)(h)$, $\pi h = z$. Let dC_u be the differential of f at $u \in E$, $\pi u = x$.

Proposition 3. *Let C be a connection on Φ. Then*

$$\mathrm{d}C_u = {}^C\sigma(u), \quad u \in E, \tag{9}$$

where ${}^C\sigma$ denotes the tensor of the H-structure $(E, {}^C\Gamma)$.

Proof. Since $\beta C(x) = 1_x$, $\mathrm{d}C_u | T_u(E_x) = \mathrm{id} | T_u(E_x)$. Let $Y \in H_u \subset T_u(E)$, where H_u is the subspace determined by ${}^C\Gamma(u)$. Then $\mathrm{d}C_u(Y) = O$. It proves our assertion.

Lemma 3. *Let $v \in J^1_x E$, $\beta v = u$. Then*

$$L(C^{-1}(x)(w)) = L(v) - L({}^C\Gamma(u)). \tag{10}$$

Proof. It is easy to see that $L(v) - L({}^C\Gamma(u)) = {}^C\sigma(u) L(v)$ and that dC_u $L(v) = L(C^{-1}(x)(v))$. Then the relation (9) completes our proof.

Using Lemma 3 the following assertion can be proved by direct evaluation in local coordinates.

Proposition 4. *Let $w \in \bar{J}^2 E$, $\beta w = u$, $\pi u = x$. Then*

$$\bar{\tau}(w) = \Delta C'^{-1}(x)(w). \tag{11}$$

Let $P(B, G, p)$ be a principal fibre bundle and let $E(B, F, \pi)$ be a fibre bundle associated with P. Let $\Phi = PP^{-1}$ be the grupoid associated with P. Let us recall that

$\Phi = (P \times P)|G$, $(h_1 g, h_2 g) \sim (h_1, h_2)$; if $\vartheta = (h_1, h_2) \in \Phi$, then $a\vartheta = ph_2$, $b\vartheta = ph_1$; if $\vartheta_1 = (h_1, h_2)$ and $\vartheta_2 = (h_3, h_4)$, then the composition $\vartheta_1 \vartheta_2$ is defined if and only if $h_3 = h_2$ and $\vartheta_1 \vartheta_2 = (h_1, h_4)$. Let us also recall that $\Phi = PP^{-1}$ is a grupoid of operators on $E(B, F, \pi)$. Let C be a connection on Φ and let $\Gamma: P \to J^1 P$ be the representative of C on P. It is known that $\Gamma(hg) = \Gamma(h)g$ (i.e. Γ is a connection on P). Hence the tensor ${}^\Gamma\sigma$ of the H-structure (P, Γ) is equivariant, i.e. if $\bar{Y} \in T_h P$ is generated by $Y \in \mathcal{G}$ ($\mathcal{G}$ denotes the Lie algebra of G) and ${}^\Gamma\sigma(X) = \bar{Y}$, then ${}^\Gamma\sigma((R_g)_* X) = \overline{Ad\, g^{-1}(Y)}$. Let $h \in P$, $p(h) = x$. Denote by $\bar{h}$ the map $P_x \to G$, $\bar{h}(q) = \bar{h}(hg) = g$. Let φ be the canonical form of the connection Γ. Then $\varphi(h) \in \mathcal{G} \otimes T_h^* P$ and

(12) $$\varphi(h) = \bar{h}_* {}^\Gamma\sigma(h).$$

Let Ω be the curvature form of the connection Γ on P, denote by $\Omega(h)$ the element of $\mathcal{G} \otimes \wedge^2 T_x^* M$ determined by Ω at $h \in P$, $ph = x$.

Proposition 5. *Let Θ be the curvature field of the H-structure (P, Γ) determined by the connection Γ on P. Let Ω be the curvature form of Γ. Then*

(13) $$\bar{h}_* \Theta(h) = -\Omega(h).$$

Proof. Let $\bar{X}$, $\bar{Y}$ be the Γ-lifts of vector fields X, Y on B. Using (12), the definitions of Ω and Θ yield

$$\Omega(h)(X, Y) = -\varphi([\bar{X}, \bar{Y}](h)) = -\bar{h}_* {}^\Gamma\sigma(h)[\bar{X}, \bar{Y}] = \\ = -\bar{h}_* \Theta(h)(X, Y) \cdot QED.$$

Denote by $(E, \tilde{\Gamma})$ the H-structure, which is the representative of the connection C on E. Every $h \in P$, $ph = x$, determines a mapping $\tilde{h}: P \to a^{-1}(x) \subset \phi$, $\tilde{h}(q) = (q, h)$. Analogously denote by $\tilde{u}$: $a^{-1}(x) \to E$ the map $\vartheta \to \vartheta(u)$, $u \in E_x$. Therefore $\tilde{u}\tilde{h}$: $P \to E$ is a fibre morphism from P to E. Let $(\tilde{u}\tilde{h})': J^1 P \to J^1 E$ denote the prolongation of the map $\tilde{u}\tilde{h}$. It is easy to see that the diagram

(14) $$\begin{array}{ccc} P & \xrightarrow{\tilde{u}\tilde{h}} & E \\ \Gamma\downarrow & & \downarrow\tilde{\Gamma} \\ J^1 P & \xrightarrow{(\tilde{u}\tilde{h})'} & J^1 E \end{array}$$

is commutative. Let $(\tilde{u}\tilde{h})_*$ denote the differential of $\tilde{u}\tilde{h}$ at $h \in P$. Using (14) we obtain

Proposition 6. *Let ${}^\Gamma\tilde{\sigma}$ or ${}^\Gamma\sigma$ be the tensor field of the $(E, \tilde{\Gamma})$, or (P, Γ), respectively. Then*

(15) $$(\tilde{u}\tilde{h})_* {}^\Gamma\sigma(h)(X) = {}^\Gamma\tilde{\sigma}(\tilde{u}\tilde{h})_*(X), \quad X \in T_h(P).$$

Proposition 7. *Let $h \in P_x$, $u \in E_x$. Let $\tilde{\Theta}$ be the curvature field of the H-structure $(E, \tilde{\Gamma})$. Then*

$$\tilde{\Theta}(u) = (\bar{u}\bar{h})_* \Theta(h). \tag{16}$$

Remark. Let G_x be the isotropy group of Φ over $x \in B$ and let $\mathscr{G}_x$ be its Lie algebra. Let $h \in P_x$. Denote by $\bar{h}_*$ the differential of the mapping $\bar{h}$: $G \to G_x$, $\bar{h}(g) = [hg, h] = \vartheta \in \Phi$, at $e \in G$, where e denotes the unit of G. Let Ω be the curvature form of the connection Γ on P which is the representative of the connection C. In [5] Kolář has introduced the curvature form of the connection C at $x \in B$ by

$$\Omega(x) = \bar{h}_* \cdot \Omega(h),$$

where the dot denotes the composition of mappings, and also introduced a generalized space with connection as a quadruple $\mathscr{S} = S(P(B, G), F, C, \eta)$, where $\eta: B \to E$ is a global cross-section. Let $u \in E_x$. Let $\bar{u}_*$ denote the differential of mapping $\bar{u}$: $G_x \to E_x$, $u(\vartheta) = \vartheta(u)$, at $1_x \in G_x$. Then the form

$$\tau(x) = (\overline{\eta(x)})_* \cdot \Omega(x)$$

is called by Kolář the torsion form of the generalized space $\mathscr{S}$ with connection at $x \in B$. The relations (13) and (16) give

$$\tilde{\Theta}(\eta(x)) = -\tau(x). \tag{17}$$

Moreover the generalized space $\mathscr{S}(P(B, G), F, C, \eta)$ with connection determines the SH-space $(E, {}^C\tilde{\Gamma}, \eta^{(2)})$. Let $\bar{\tau}(x)$ be the torsion of this SH-space. Then comparing (5) with (17) we get

$$\bar{\tau}(x) = -\tau(x).$$

4. Let us consider the special case of a vector bundle $E(B, \pi)$. Denote by V the Liouville field on E determined by the 1-parametric group of all homothetics on E. Locally, $V = y^\alpha \partial y_\alpha$. A v-field σ on E will be said to be k-homogeneous, if $L_V \sigma = k\sigma$.

Lemma 4. *Locally let $\sigma = (a_i(x^i, y^\beta)dx^i + b^\alpha_\gamma(c^i, y^\beta)dy^\gamma) \otimes \partial y_\alpha$. Then σ is k-homogeneous if and only if a^α_i or b^α_i are homogeneous functions of the degree $k+1$ or k with respect to variables y^β.*

Proof. Relation (1) gives

$$L_V\sigma = \left[\left(\frac{\partial a_k^\alpha}{\partial y^\beta} y^\beta - a_k^\alpha\right) dx^k + \frac{\partial b_\beta^\alpha}{\partial y^\gamma} y^\gamma dy^\beta\right] \otimes \partial y_\alpha . \tag{18}$$

This proves our assertion.

Proposition 8. *Let (E, Γ) be an H-structure. Then ${}^\Gamma\sigma$ is O-homogeneous if and only if the Liouville field V is conjugate with Γ.*

Proof. In the case of the tensor field ${}^\Gamma\sigma$ of the H-structure we have

$$L_V{}^\Gamma\sigma = \left[\left(\frac{\partial a_k^\alpha}{\partial y^\beta} y^\beta - a_k^\alpha\right) dx^k + \frac{\partial b_\beta^\alpha}{\partial y^\gamma} y^\gamma dy^\beta\right] \otimes \partial y_\alpha . \tag{19}$$

Using proposition 1, relation (19) and Lemma 4 complete our assertion.

Let $\bar{X}$ be the Γ-lift of a field X on B. Then

$$(L_V{}^\Gamma\sigma)(\bar{X}) = [V, \bar{X}].$$

This gives

Proposition 9. *The tensor field ${}^\Gamma\sigma$ of the H-structure (E, Γ) is O-homogeneous if and only if $[V, \bar{X}] = 0$ for every vector field X on B.*

Let (E, Γ) be an H-structure, Z be a vertical field on E. Then $\Gamma_*(Z)$ is a vector field on the submanifold $\Gamma(E)$. The values of $\Gamma_*(Z)$ are vertical tangent vectors on the vector bundle J^1E over B. Let i: $T_u(J_x^1E) \to J_x^1E$ be the canonical identification. Then $u \to i \cdot \Gamma_*(Z(u))$ determines a mapping $\zeta: E \to J^1E$. Locally, $Z = b^\alpha(x^i, y^\beta)\partial y_\alpha$ and

$$(x^i, y^\alpha) \overset{\zeta}{\mapsto} \left(x^i, b^\alpha(x^i, y^\beta), \frac{\partial a_i^\alpha}{\partial y^\beta} b^\beta\right).$$

therefore ζ is a global cross-section of J^1E over E if and only if $Z = V$. In this case denote by $(E, V(\Gamma))$ the H-structure determined by ζ. Locally

$${}^{V(\Gamma)}\sigma = \left(dy^\alpha + \frac{\partial a_i^\alpha}{\partial y^\beta} y^\beta dx^i\right) \otimes \partial y_\alpha . \tag{20}$$

Proposition 10. Let (E, Γ) be an H-structure. Then

$$(L_V({}^\Gamma\sigma))(u) = (\bar{\Gamma}(u) - \overline{V(\Gamma)}(u))\pi_* . \tag{21}$$

Proof. $\bar{\Gamma}$: $(x^i, y^\alpha) \mapsto dx^i \otimes \partial x_i - a_j^\alpha(x, y) dx^j \otimes \partial y_\alpha$,

$$\overline{V(\Gamma)}: (x^i, y^\alpha) \mapsto dx^i \otimes \partial x_i - \frac{\partial a_j^\alpha}{\partial y^\beta} y^\beta dx^j \otimes \partial y_\alpha .$$

Using (19) we get (21).

Corollary. *An H-structure (E, Γ) is O-homogeneous if and only if $\Gamma = V(\Gamma)$.*

Remark. As it is well known, the H-structure (E, Γ) is a connection on E if and only if the cross-section $\Gamma\colon E \to J^1 E$ is a vector bundle morphism over B. Locally, Γ is a connection on E if and only if $a_i^\alpha = \Gamma_{i\beta}^\alpha(x) y^\beta$. Hence the Liouville field V is conjugate with every connection on E.

Further, if (E, Γ) is an H-structure and $\varepsilon\colon B \to E$ is a global cross-section, then, using the identifications $j\colon E_x \to T_{\varepsilon(x)} E_x$, $i\colon T_{\Gamma(\varepsilon(x))} J_x^1 E \to J_x^1 E$, we get the mapping

$$\Gamma^{\varepsilon(x)} \equiv i \cdot \Gamma_* \cdot j$$

from E_x to $J_x^1 E$. It is easy to see that Γ^ε is a connection on E. Locally, if the functions $a_i^\alpha(x, y)$ determine the H-structure (E, Γ), then the functions

$$\frac{\partial a_i^\alpha(x^k, \varepsilon^\gamma(x^k))}{\partial y^\beta} y^\beta$$

determine the connection Γ^ε.

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Received December 8, 1975

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ГОРИЗОНТАЛЬНЫЕ СТРУКТУРЫ НА РАССЛОЕННЫХ ПРОСТРАНСТВАХ

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Резюме

Пусть E расслоенное пространство. Горизонтальная структура или обобщенная связаность это сечение Γ: $E \to J^1 E$ расслоения $J^1 E$. В статье определено поле и форма кривизны горизон тальной структуры. Пользуясь теорией струей наиден джет-вид формы кривизны. Обоснованы некоторые свойства производной Ли поля горизонтальной структуры. Специально иследованы горизонтальные структуры на векторных расслоенных пространствах. Результаты соединены с полем и формой кривизны горизонтальной структуры сравнены с теорией связности на главном расслоенном пространстве и пространствах ассоциированных с этим пространством.