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Mathematica Slovaca, Vol. 33 (1983), No. 1, 87--97

Persistent URL: http://dml.cz/dmlcz/128675

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LEVELS IN L-SYSTEMS

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1. Introduction

L-systems, as a kind of parallel rewriting systems were created on the basis of the formal model suggested by A. Lindenmayer in [6] for the development of simple biological organisms.

Nowadays we have an intensively developing theory of L-systems, which originated the study of such interesting mathematical subjects as iterated homomorphisms and substitutions [9] and also the study of formal power series from the point of view of computer science [10]. The theory has considerable influence in the research activity in classical formal language theory. An exhaustive information on the theory of L-systems can be obtained on the basis of monographs [2, 9, 10, 13] and proceedings [7, 8].

Roughly speaking, an L-system is a rewriting system determined by the initial word (i.e. by a finite string of symbols over a fixed set, called alphabet) and by a finite set of production rules (which are prescriptions for replacing the letters in a derivation process). The derivation of words in an L-system proceeds in discrete time instants in a parallel manner, and starting with the initial word it produces a word in every time instant. (All symbols of a given word are simultaneously rewritten into the words according to the production rules.)

A growth function and a letter occurrence function are examples of such notions of L-systems which have biological origin. The growth function of a deterministic L-system (i.e. a system with a uniquely determined step of derivation) is a function defined for nonnegative integers. Its value for t is given by the length of the t-th word produced by L-system. A detailed survey of results concerning the growth functions can be find, e.g., in [11].

Some symbols in words can be more significant or important than others (e.g. in the biological original they can be more accessible or easier measurable than others. This, for example, is the case in the model of cell cycle [4]). This motivation leads to the study of a letter occurrence function, i.e. a function, which associates with the natural number t the number of occurrences of a given letter in the t-th word produced by an L-system.

In the present paper we wish to stress connections between the type of a growth function or a letter occurrence function of an L-system and the structural properties of an alphabet given by its production rules. For this purpose structural properties will be characterized by levels of an L-system, which are equivalence classes on the alphabet of the L-system defined by production rules. (In the context-free grammar a grammatical level is an equivalent for the level of L-system. Grammatical levels are used as the basis for the study of the structural complexity of context-free languages [1, 5].)

Throughout the paper we shall deal only with deterministic L-systems without interactions (abbreviated as $D0L$-systems).

After the present Introduction, Section 2 contains definitions and preliminary results. In Section 3 various types of levels of an L-system are investigated. Section 4 contains definitions of the structural complexity measures of L-systems, which are later used for the reformulation of results from [11] and [3] to obtain a proper characterization of the type of growth functions or letter occurrence functions.

Analogical characterizations of the deterministic table L-systems ($DT0L$-systems) are studied in [12].

2. Definitions and preliminary results

We shall briefly review definitions and notations as well as propositions used in the following parts of the paper. The readers wishing more detailed information on the topic are referred to, e.g., [11] or [9].

We shall use the following notations:

W^* for the set of all words (finite strings) over the set W, i.e. $W^* = \varepsilon \cup \{a_1 a_2 \ldots a_n\colon a_i \in W, 1 \leqslant i \leqslant n, n$ is a natural number$\}$, where ε is the empty word (i.e. the string, which does not contain any symbol); a_i is also called a letter;

Z^+ for the set of all nonnegative integers;

$|w|$ for the length of the word w;

$\#_a(w)$ for the number of occurrences of the letter a in the word w.

Definition 2.1. *A $D0L$-system (a deterministic Lindenmayer system without interaction) is a triple $G = (W, h, w)$, where W is a finite nonempty set called an alphabet, $w \in W^*$ is an initial word and $h\colon W \to W^*$ determines the production rules.*

We extend the domain of the function h to W^* and define in natural manner

$$h(\varepsilon) = \varepsilon$$
$$h(aw) = h(a)h(w) \quad \text{for} \quad a \in W, \; w \in W^*.$$

For an integer t, $t \geq 2$ and for $w \in W^*$ let $h^t(w) \overset{\text{def}}{=} h(h^{t-1}(w))$.

In the paper we shall consider only reduced $D0L$-systems, i.e. such systems, in which all letters in W are accessible from w, i.e. for every $a \in W$ there is an integer t such that $h^t(w) = xay$, $xy \in W^*$.

Definition 2.2. *Let $G = (W, h, w)$ be a $D0L$-system. Then the function f_G: $Z^+ \to Z^+$ defined by $f_G(t) = |h^t(w)|$ is said to be the growth function of G.*

Definition 2.3. *Let $G = (W, h, w)$ be a $D0L$-system and let $a \in W$. Then the function $o_{G,a}$: $Z^+ \to Z^+$ defined by $o_{G,a}(t) = \#_a(h^t(w))$ is said to be an occurrence function of the letter a in G.*

A function g: $Z^+ \to Z^+$ is said to be of the type

i) exponential or type 3 if there is a real number $x > 1$ such that

$$\limsup_{t \to \infty} \frac{g(t)}{x^t} > 0;$$

ii) polynomial or type 2 if $g(t)$ is unbounded, i.e. $\limsup_{t \to \infty} g(t) > c$ for all constants c and there exist polynomials p, q such that $p(t) \leq g(t) \leq q(t)$ for all t;

iii) limited or type 1 if there exists an integer m such that $g(t) \leq m$ for $t \in Z^+$ and $\{t\colon g(t) \neq 0\}$ is infinite;

iv) terminating or type 0 if there is an integer t_0 such that $g(t) = 0$ for all $t \geq t_0$.

Now we shall list some structural properties of letters in a $D0L$-system $G = (W, h, w)$.

A letter $a \in W$ is mortal, $a \in M$, if $h^i(a) = \varepsilon$ for some $i \in Z^+$; a letter $a \in W$ is recursive, $a \in R$, if $h^i(a) \in W^*aW^*$ for some $i \geq 1$; a letter $a \in W$ is monorecursive, $a \in MR$, if $h^i(a) \in M^*aM^*$ for some $i \geq 1$; a letter $a \in W$ is expanding, $a \in E$, if $h^i(a) \in W^*aW^*aW^*$ for some $i \geq 1$. For $z \in W$ a letter $a \in W$ is z-mortal, $a \in z\text{-}M$, if $\#_z(h^i(a)) = 0$ for all $i \geq i_0$; a letter $a \in W$ is z-monorecursive, $a \in z\text{-}MR$, if $h^i(a) \in z\text{-}M^*az\text{-}M^*$; a letter $a \in W$ is accessible from $v \in W^*$, $a \in U(v)$, if $h^i(v) \in W^*aW^*$ for some $i \geq 1$.

Results in [11, pp. 140—144] and in [3] can be reformulated as the following propositions:

Proposition 2.1 [11]. *The only possible types of growth functions for* D0*L-systems are the types* 0, 1, 2 *and* 3.

Proposition 2.2 [11]. *Let $G = (W, h, w)$ be a reduced $D0L$-system. The growth function f_G is of the type* 0 *iff all letters in w are mortal; f_G is of the type* 1 *iff all recursive letters accessible from w are monorecursive; f_G is of the type* 2 *iff G does not contain an expanding letter and it contains a recursive letter, which is not monorecursive; f_G is of the type* 3 *iff G contains an expanding letter.*

Proposition 2.3 [3]. *The only possible types of letter occurrence functions for D0L-systems are the types* 0, 1, 2 *and* 3.

Proposition 2.4 [3]. *Let* $G=(W, h, w)$ *be a reduced D0L-system and let* $a \in W$. *The letter occurrence function* $o_{G,a}$ *is of the type* 0 *iff all letters in* w *are* a*-mortal;* $o_{G,a}$ *is of the type* 1 *iff all recursive letters producing the letter* a *and accessible from* w *are* a*-monorecursive;* $o_{G,a}$ *is of the type* 2 *iff* G *does not contain an expanding letter and it contains a recursive letter, which is not* a*-monorecursive;* $o_{G,a}$ *is of the type* 3 *iff* G *contains an expanding letter* b *and* $a \in U(b)$.

3. Levels of L-system

Binary relations $\triangleright$, $\triangleright^{+}$, $\triangleright^{*}$ and $\equiv$, defined below are known from the descriptional complexity of formal languages.

Definition 3.1. *Let* $G=(W, h, w)$ *be a D0L-system and let* $a, b \in W$.

$a \triangleright_G b$ iff $h(a)=xby$ for some $x, y \in W^*$;

$\triangleright_G^{+}$ is transitive closure of $\triangleright_G$;

$\triangleright_G^{*}$ is the reflexive and transitive closure of $\triangleright_G$;

$a \equiv_G b$ iff $a \triangleright_G^{*} b$ and $b \triangleright_G^{*} a$.

Definition 3.2. *Let* $G=(W, h, w)$ *be a D0L-system and let* $a \in W$. *The equivalence class* $[a]_G=\{b\colon b \in W, b \equiv_G a\} \in W/{\equiv_G}$ *is called the level of the L-system* G *generated by* a.

The subscript G in $[a]_G$ will be omitted if it is clear, which G is under consideration.

We shall use the following notations:

Let $[a], [b] \in W/{\equiv_G}$ and let t be a nonnegative integer.

Then

a) $[a]<[b]$ iff $b \triangleright^{*} a$;

b) $[a]^t=\{b_1 b_2 \dots b_t\colon b_i \in [a], 1 \leqslant i \leqslant t\}$;

c) $[a]^*=\varepsilon \cup \{b_1 b_2 \dots b_k\colon k$ is a positive integer, $b_i \in [a]$ for $1 \leqslant i \leqslant k\}$.

Lemma 3.1. *Let* $a, b, z \in W$, $b \in [a]$ *and* P *is one from the sets* M, R, E, MR, z*-*M, z*-*MR. *Then* $a \in P$ *iff* $b \in P$.

Proof. Follows easily from the definitions.

Definition 3.3. *The level* $[a]$ *is said to be mortal, recursive, monorecursive, expanding,* z*-mortal,* z*-monorecursive if the letter* a *is mortal, recursive, expanding,* z*-mortal,* z*-monorecursive, respectively.*

Definition 3.4. *Let* $G=(W, h, w)$ *be a D0L-system. A graph of levels of* G *is a digraph* $GL(G)=(W/{\equiv_G}, E_G)$, *the nodes of which are levels of* G *and* $([a], [b]) \in E_G$ *iff* $c \triangleright_G d$ *for some* $c \in [a]$ *and* $d \in [b]$.

Lemma 3.2. *A level $[a]$ is recursive iff $[a]$ has at least two elements or if $[a]=\{a\}$ and $h(a)=xay$ for some $x, y \in W^*$.*

Proof. The case $h(a)=xay$ is trivial. Suppose that $[a]$ contains two different elements a and b. Then $a \trianglerighteq^+ b$, $b \trianglerighteq^+ a$, i.e. there are integers i, j such that $h^i(a)=x_1by_1$ and $h^j(b)=x_2ay_2$ for x_1, y_1, x_2, $y_2 \in W^*$. Hence $h^{i+j}(a)=h^j(x_1)x_2ay_2h^j(y_1)$ and $[a]$ is recursive.

Lemma 3.3. *A level $[a]$ is mortal iff every $[b]$ satisfying the condition $[b]\leqslant[a]$ is nonrecursive.*

Proof. If $[a]$ is mortal, then $h^i(a)=\varepsilon$ for some i and obviously $[a]$ is nonrecursive.

Let us suppose by contradiction that there is a recursive level $[b]$ in G and $[b]<[a]$. Then $h^j(a)=x_1by_1$ and $h^k(b)=x_2by_2$ for some integers j, k and for x_1, x_2, y_1, $y_2 \in W^*$. This implies that $h^{j+sk}(a)$ contains at least a letter b for every natural number s and therefore $[a]$ is not mortal.

Because of the finiteness of W the assumption that all $[b]$-s, satisfying condition $[b]<[a]$, are nonrecursive implies the mortality of $[a]$.

Corollary 3.1. *If $[a]$ is mortal, then $[b]$, satisfying condition $[b]<[a]$, is mortal.*

By e_a we shall denote a mapping erasing from words in W^* all letters x, $x \notin [a]$, i.e. e_a: $W^* \to [a]^*$ such that

$$\begin{aligned} e_a(x)&=x \text{ for } x\in[a], \\ e_a(x)&=\varepsilon \text{ otherwise for } x\in W \text{ and} \\ e_a(xy)&=e_a(x)e_a(y) \text{ for } x,\ y\in W^*. \end{aligned}$$

Lemma 3.4 [11, p. 141]. *A level $[a]$ is expanding iff there is $x\in[a]$ such that $e_a(h(x))\in[a]^2[a]^*$.*

Lemma 3.5. *A recursive level $[a]$ is monorecursive iff it is not expanding and all levels $[b]$, $[b]<[a]$ are not recursive.*

Proof. a) Monorecursivity of the level obviously implies that the level is not expanding (according to the Corollary 3.1).

Suppose that $[b]<[a]$. Then $b\notin[a]$ and from the monorecursivity of $[a]$ it follows that $[b]$ is mortal, so $[b]$ is nonrecursive by Lemma 3.3.

b) Suppose that $[a]$ is recursive and nonexpanding and that $[b]$ is nonrecursive for every $[b]$, $[b]<[a]$. Then $h^j(a)=xay$ for some j and x, y do not contain any letter from $[a]$ because of Lemma 3.4, i.e. for z being a letter of xy, we have $[z]<[a]$. Since $[z]$ is nonrecursive, by Lemma 3.3 we get that all letters of xy are mortal, i.e. $[a]$ is monorecursive.

Lemma 3.6. a) *A level $[a]$ is z-mortal for $z\in[a]$ iff $[a]$ is not recursive.*

b) *A level $[a]$ is z-mortal for $z\notin[a]$ iff $[z]<[a]$ does not hold or if $[z]<[a]$ and each $[b]$, $[z]\leqslant[b]\leqslant[a]$ is nonrecursive.*

Proof. a) If $[a]$ is recursive and $z \in [a]$, then obviously $h^i(a)$ contains a letter z for infinitely many indexes i.

If $[a]$ is not recursive, then according to the Lemma 3.2 $[a]=\{a\}$ and $h(a)$ does not contain a letter a, therefore $[a]$ is a-mortal.

b) If $[z]<[a]$ does not hold, then trivialy $[a]$ is z-mortal. Suppose that $[z]<[a]$.

The assumptions that 1°$[a]$ is recursive or 2° that for some $[b]$, $[z]<[b]<[a]$, $[b]$ is recursive lead immediately to the conclusion that $[a]$ is not z-mortal.

If $[z]<[a]$, then $h^i(a)=x_1zy_1$. Since all levels $[b]$ such that $[z]\leqslant[b]\leqslant[a]$ are not recursive, there is a finite number of indices such that $h^i(a)$ contains a letter z, i.e. $[a]$ is z-mortal.

Lemma 3.7. a) *A level $[a]$ is z-monorecursive for $z \in [a]$ iff $[a]$ is recursive and nonexpanding.*

b) *A level $[a]$ is z-monorecursive for $z \notin [a]$ iff $[a]$ is recursive and $[z]<[a]$ does not hold or if $[z]<[a]$, $[a]$ is recursive and nonexpanding and all levels $[b]$, such that $[z]\leqslant[b]<[a]$ are nonrecursive.*

Proof. The z-monorecursive level is obviously recursive. a) Suppose that $z \in [a]$.

Let moreover $[a]$ be expanding. Then $h^i(a)=xayaz$ for some index i and obviously $[a]$ is not z-monorecursive.

Let $[a]$ be a nonexpanding level. Then by Lemma 6.3 $e_a(h(x)) \in [a]^2[a]^*$ for all $x \in [a]$, i.e. $e_a(h^i(a)) \in [a]$ for all indices $i=0, 1, \ldots$, i.e. $h^i(a)=xay$ and a is not a letter of the word $h^i(xy)$, $i=0, 1, \ldots$. Since $z \in [a]$, then z is not a letter of $h^i(xy)$, $i=0, 1, \ldots$ and therefore $[a]$ is z-monorecursive.

b) Suppose that $z \notin [a]$.

1st case: if $[z]<[a]$ does not hold, it is trivial.

2nd case: $[z]<[a]$.

i) Let $[a]$ be z-monorecursive. Using the same method as in the proof of part a) of the present lemma one can prove that $[a]$ is nonexpanding.

Suppose for a moment that for some level $[b]$, $[z]\leqslant[b]<[a]$, $[b]$ is recursive. Then there are indices i, j, k such that $h^i(a)=xay$, $h^j(a)=x_1by_1$, $h^k(b)=x_2by_2$ for $x, y, x_1, y_1, x_2, y_2 \in W^*$. We shall discuss two cases:

α) $i\leqslant j$. Since $[b]\neq[a]$ there can be chosen a letter $c \in W$ in such a way that $h^i(a)=x_3cy_3ay$ or $h^i(a)=xax_3cy_3$ and $h^{j-i}(c)=x_4by_4$. The level $[b]$ is recursive and $[z]\leqslant[b]$, therefore there is an index s such that $h^s(b)=x_5zy_5$. Since $h^{j-i+kr+s}(c)$ contains a letter z for $r=0, 1, \ldots$, we have a contradiction with the assumption that $[a]$ is z-monorecursive.

β) $i>j$. A letter $c \in W$ can be chosen in such a way that for some $x_3y_3 \in W^*$, $h^i(a)=x_3cy_3ay$ or $h^i(a)=xax_3cy_3$ and for

$$s=\frac{i-j}{k}-\left[\frac{i-j}{k}\right]$$

(i.e. s is the rest of the integer division of $i-j$ by k), $h^s(b)=x_6cy_6$ and c produces a letter z infinitely many times. Therefore c is not z-mortal. This is the contradiction with the assumption that $[a]$ is z-monorecursive.

ii) Let $[a]$ be nonexpanding and all levels $[b]$, $[z]\leqslant[b]<[a]$, are nonrecursive.

Suppose for a moment that $[a]$ is not z-monorecursive. Then for some integer i and for some $x, y, z \in W^*$, $h^i(a)=xaycz$ or $h^i(a)=xcyaz$ and c is not z-mortal. According to Lemma 3.6 there is a recursive level $[d]$, $[z]\leqslant[d]\leqslant[c]$, which is a contradiction with the assumption that all levels $[b]$, $[z]\leqslant[b]<[a]$ are nonrecursive.

4. Characterizations of growth functions and letter occurrence functions

For a $D0L$-system $G=(W, h, w)$ and for $z \in W$ we shall denote by Lev G the number of levels in G, by RLev G the number of recursive levels in G, by MRLev G the number of monorecursive levels in G, by ELev G the number of expanding levels in G, by z-MRLev G the number of z-monorecursive levels in G.

Remark 4.1. Trivially for a given $D0L$-system G

$$\mathrm{Lev}\, G \geqslant \mathrm{RLev}\, G \geqslant \mathrm{MRLev}\, G + \mathrm{ELev}\, G \geqslant 0$$

and moreover at least one of the inequalities above is strong.

Theorem 4.1. *There is an effective procedure which, for $D0L$-system $G=(W, h, w)$ and for $z \in W$, produces the values of* Lev G, RLev G, MRLev G, ELev G and z-MRLev G.

Proof. Let G be a given $D0L$-system. Construct the graph $GL(G)$ of the levels of system G. According to Lemmas 3.2, 3.3, 3.4, 3.5, 3.6 and 3.7 test whether a given level is recursive, mortal, expanding, monorecursive, z-mortal or z-monorecursive, respectively. The total number of levels with property P gives a value PLev.

Theorem 4.2. *Let $G=(W, h, w)$ be a reduced $D0L$-system and let f_G be the growth function of G.*

Then a) *f_G is of the type* 0 *iff* RLev $G=0$;

b) *f_G is of the type* 1 *iff* RLev $G=$ MRLev $G\neq 0$ *and* Elev $G=0$;

c) *f_G is of the type* 2 *iff* RLev $G>$ MRLev G *and* ELev $G=0$;

d) *f_G is of the type* 3 *iff* ELev $G>0$.

Proof. Following the Proposition 2.2 we have:

a) f_G is of the type 0 iff all letters in w are mortal. If RLev $G=0$, then trivially all letters of G are mortal (see Lemmas 3.2 and 3.3). For $a \in W$, a being a letter of w, $[a]$ is mortal and so $[a]$ is not recursive. For $a \in W$, a being not a letter of w, there is a letter b in w such that $[b]>[a]$. According to Lemma 3.3 $[a]$ is not recursive, therefore RLev $G=0$;

b) f_G is of the type 1 iff only recursive letters accessible from w are monorecursive. This together with part a) of this theorem gives immediately equivalent conditions $\mathrm{RLev}\, G = \mathrm{MRLev}\, G \neq 0$ and $\mathrm{ELev}\, G = 0$;

c) f_G is of the type 2 iff G does not contain an expanding letter and it contains a recursive letter which is not monorecursive, i.e. iff $\mathrm{ELev}\, G = 0$ and $\mathrm{RLev}\, G > \mathrm{MRLev}\, G$;

d) f_G is of the type 3 iff G contains an expanding letter, i.e. iff $\mathrm{ELev}\, G > 0$.

With a $D0L$-system $= (W, h, w)$ and with $a \in W$ we shall associate a $D0L$-system $G_a = (W, h_a, w)$, where

$$h_a(x) = h(x) \text{ for } x \triangleright^* a$$
$$h_a(x) = \varepsilon \text{ otherwise.}$$

Remark 4.2. Let $G_a = (W, h_a, w)$ and let M_a be the set of all mortal levels in G_a.

Then a) $\{[c] \in W/\equiv_G : [c] > [a] \text{ does not hold}\} \subseteq M_a$,

b) $[c] >_G [a]$ iff $[c] >_{G_a} [a]$.

Lemma 4.1. *Let $G = (W, h, w)$ be a $D0L$-system and let $a \in W$. An occurrence function $o_{G,a}$ and a growth function f_{G_a} are of the same type.*

Proof. Following Propositions 2.2 and 2.4 it is sufficient to prove that

i) $[b]$ is a-mortal in G iff $[b]$ is mortal in G_a;

ii) G contains a recursive letter which is not a-monorecursive iff G_a contains a recursive letter which is not monorecursive;

iii) G contains an expanding letter b and $b \triangleright^* a$ iff G_a contains an expanding letter b.

Using Remark 4.2 the property i) follows immediately from Lemma 3.3 and Lemma 3.6; the property ii) follows from Lemma 3.5 and Lemma 3.7 and the property iii) is trivial.

Theorem 4.3. *Let $G = (W, h, w)$ be a reduced $D0L$-system, $a \in W$ and $G_a = (W, h_a, w)$ be a $D0L$-system associated with G and a.*

Then $o_{G,a}$ is of the type 0 iff $\mathrm{RLev}\, G_a = 0$;

$o_{G,a}$ is of the type 1 iff $\mathrm{RLev}\, G_a = \mathrm{MRLev}\, G_a > 0$ and $\mathrm{ELev}\, G_a = 0$;

$o_{G,a}$ is of the type 2 iff $\mathrm{RLev}\, G_a > \mathrm{MRLev}\, G_a$ and $\mathrm{ELev}\, G_a = 0$;

$o_{G,a}$ is of the type 3 iff $\mathrm{ELev}\, G_a > 0$.

Proof. Follows immediately from Lemma 4.1 and Theorem 4.2.

Example: Let $G = (\{a, b, c, d, e, f\}, h, a)$ be a $D0L$-system and

$$\begin{array}{ll} h(a) = bc & h(d) = df \\ h(b) = bc & h(e) = eed \\ h(c) = ecdf & h(f) = d. \end{array}$$

We shall describe the graph $GL(G)$ of levels of G and for every x in $\{a, b, c, d, e, f\}$ the graph $GL(G_x)$ of levels of G_x.

G_a: The levels $\{a\}$, $\{b\}$, $\{c\}$ are mortal. $\mathrm{Lev}\, G_a = 3$, $\mathrm{RLev}\, G_a = 0$. $o_{G,a}$ is of the type 0 (fig. 1).

G_b: The level $\{a\}$ is nonrecursive, $\{b\}$ is monorecursive, $\{c\}$ is mortal. $\mathrm{RLev}\, G_b = \mathrm{MRLev}\, G_b = 1$, $\mathrm{ELev}\, G_b = 0$. $o_{G,b}$ is of the type 1 (fig. 2).

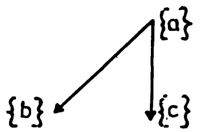

Fig. 1: $GL(G_a)$

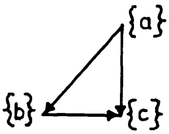

Fig. 2: $GL(G_b)$

G_c: The level $\{a\}$ is nonrecursive, $\{b\}$ is recursive and not monorecursive, $\{c\}$ is monorecursive, $\{d\}$, $\{e\}$, $\{f\}$ are mortal. $\mathrm{ELev}\, G_c = 0$, $\mathrm{RLev}\, G_c = 2$, $\mathrm{MRLev}\, G_c = 1$. $o_{G,c}$ is of the type 2 (fig. 3).

G_e: The level $\{a\}$ is nonrecursive, $\{b\}$, $\{c\}$ are recursive and not monorecursive, $\{d\}$, $\{f\}$ are mortal, $\{e\}$ is expanding. $\mathrm{ELev}\, G_e = 1$. $o_{G,e}$ is of the type 3 (fig. 4).

G_d: The level $\{a\}$ is nonrecursive, $\{b\}$, $\{c\}$ are recursive and not monorecursive, $\{d\}$ is monorecursive, $\{e\}$ is expanding and $\{f\}$ is mortal. $\mathrm{ELev}\, G_d = 1 \cdot o_{G,d}$ is of the type 3 (fig. 5).

$G_f = G$: The level $\{a\}$ is not recursive, $\{b\}$ and $\{c\}$ are recursive, $\{e\}$, $\{d, f\}$ are expanding. $\mathrm{ELev}\, G_f = \mathrm{ELev}\, G = 2$. $o_{G,f}$ is of the type 3 and f_G is of the type 3 (fig. 6).

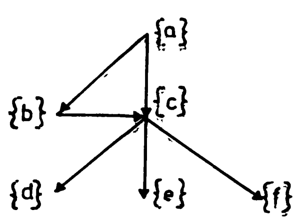

Fig. 3: $GL(G_c)$

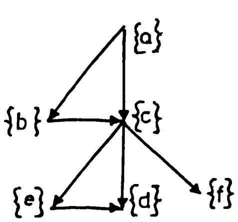

Fig. 4: $GL(G_e)$

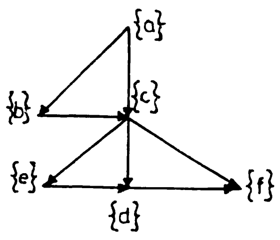

Fig. 5: $GL(G_a)$

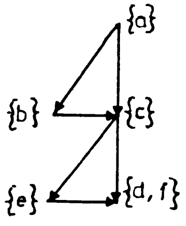

Fig. 6: $GL(G_f) = GL(G)$

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Received July 7, 1981

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УРОВНИ В L-СИСТЕМАХ

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Резюме

Для $D0L$-систем (т.е. детерминистических систем Линденмаиера без взаимодействий) в статье

1° определены погибающие, рекурсивные, монорекурсивные, расширяющиеся, z-погибающие и z-монорекурсивные уровни;

2° даны простые необходимые и достаточные условия для того, чтобы уровень имел некоторое из свойств, перечисленных в 1°;

3° характеризован тип функции роста в $D0L$-системе G при помощи числа рекурсивных, монорекурсывных и расширяющихся уровней системы G и тип функции появления буквы а в $D0L$-системе G при помощи числа рекурсивных, монорекурсивных и расширяющихся уровней $D0L$-системы G_a, присоединеной к системе G и букве a.