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OSCILLATORY PROPERTIES OF SOLUTIONS OF THREE-DIMENSIONAL DIFFERENTIAL SYSTEMS OF NEUTRAL TYPE

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Abstract. The purpose of this paper is to obtain oscillation criteria for the differential system

$$\begin{aligned} [y_1(t) - a(t)y_1(g(t))]' &= p_1(t)f_1(y_2(h_2(t))) \\ y_2'(t) &= p_2(t)f_2(y_3(h_3(t))) \\ y_3'(t) &= -p_3(t)f_3(y_1(h_1(t))), \quad t \in \mathbb{R}_+ = [0, \infty). \end{aligned}$$

Keywords: differential system of neutral type, oscillatory (nonoscillatory) solution

MSC 2000: 34K15, 34K40

1. Introduction

In this paper we consider the neutral differential system of the form

$$\text{(S)}\qquad \begin{aligned} [y_1(t) - a(t)y_1(g(t))]' &= p_1(t)f_1(y_2(h_2(t))) \\ y_2'(t) &= p_2(t)f_2(y_3(h_3(t))) \\ y_3'(t) &= -p_3(t)f_3(y_1(h_1(t))), \quad t \in \mathbb{R}_+ = [0, \infty). \end{aligned}$$

The following conditions are assumed to hold throughout the paper:

(a) $p_i\colon \mathbb{R}_+ \to \mathbb{R}_+$, $i = 1, 2, 3$ are continuous functions not identically equal to zero in every neighbourhood of infinity,

$$\int^{\infty} p_j(t)\,\mathrm{d}t = \infty, \quad j = 1, 2;$$

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(b) $a\colon \mathbb{R}_+ \to \mathbb{R}$ is a continuous function satisfying $|a(t)| \leqslant \lambda < 1$, where λ is a constant and $a(t)a(g(t)) \geqslant 0$ on $\mathbb{R}_+$;

(c) $g\colon \mathbb{R}_+ \to \mathbb{R}$ is a continuous and increasing function, $g(t) < t$ on $\mathbb{R}_+$ and $\lim\limits_{t\to\infty} g(t) = \infty$;

(d) $h_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ are continuous functions and $\lim\limits_{t\to\infty} h_i(t) = \infty$, $i = 1, 2, 3$;

(e) $f_i\colon \mathbb{R} \to \mathbb{R}$ are continuous and nondecreasing functions, $uf_i(u) > 0$ for $u \neq 0$, $i = 1, 2, 3$.

The asymptotic properties of solutions of systems with deviating arguments or systems of neutral type are studied for example in the papers [1–12].

The purpose of this paper is to obtain oscillation criteria for the system (S). The paper is a generalization of the results obtained in the paper [12].

Let $t_0 \geqslant 0$. Denote

$$\tilde{t}_0 = \min\Big\{g(t_0), \inf_{t\geqslant t_0} h_i(t), i = 1, 2, 3\Big\}.$$

A function $y = (y_1, y_2, y_3)$ is a solution of the system (S) if there exists a $t_0 \geqslant 0$ such that y is continuous on $[\tilde{t}_0, \infty)$, $y_1(t) - a(t)y_1(g(t))$, $y_i(t)$, $i = 2, 3$, are continuously differentiable on $[t_0, \infty)$ and y satisfies (S) on $[t_0, \infty)$.

Denote by W the set of all solutions $y = (y_1, y_2, y_3)$ of the system (S) which exist on some ray $[T_y, \infty) \subset \mathbb{R}_+$ and satisfy

$$\sup\Big\{\sum_{i=1}^{3} |y_i(t)|\colon\ t \geqslant T\Big\} > 0 \quad \text{for any } T \geqslant T_y.$$

A solution $y \in W$ is nonoscillatory if there exists a $T_y \geqslant 0$ such that its every component is different from zero for all $t \geqslant T_y$. Otherwise a solution $y \in W$ is said to be oscillatory.

Denote

$$\begin{aligned} h_i^\star(t) &= \min\{t, h_i(t)\}, \quad i = 1, 2, 3;\\ \gamma_i(t) &= \sup\{s \geqslant 0, h_i^\star(s) \leqslant t\}, \quad t \geqslant 0,\ i = 1, 2, 3;\\ \beta(t) &= \sup\{s \geqslant 0, g(s) \leqslant t\}, \quad t \geqslant 0;\\ \gamma(t) &= \max\{\gamma_1(t), \gamma_2(t), \gamma_3(t), \beta(t)\}. \end{aligned}$$

For any $y_1(t)$ we define $z_1(t)$ by

$$z_1(t) = y_1(t) - a(t)y_1(g(t)). \tag{1}$$

2. Some basic lemmas

Lemma 1. ([6, Lemma 1]) *Let* $y \in W$ *be a solution of the system* (S) *with* $y_1(t) \neq 0$ *on* $[t_0, \infty)$, $t_0 \geqslant 0$. *Then* y *is nonoscillatory and* $z_1(t)$, $y_2(t)$, $y_3(t)$ *are monotone on some ray* $[T, \infty)$, $T \geqslant t_0$.

Lemma 2. ([6, Lemma 2]) *Let* $y = (y_1, y_2, y_3) \in W$ *be a nonoscillatory solution of the system* (S) *and let* $\lim\limits_{t\to\infty} |z_1(t)| = L_1$, $\lim\limits_{t\to\infty} |y_i(t)| = L_i$, $i = 2, 3$. *Then*

$$L_1 < \infty \quad \textit{implies} \quad L_2 = L_3 = 0. \tag{2}$$

Lemma 3. ([6, Lemma 4]) *Let* $y = (y_1, y_2, y_3) \in W$ *be a nonoscillatory solution of the system* (S) *on* $[t_0, \infty)$, $t_0 \geqslant 0$. *Then there exist an integer* $l \in \{1, 3\}$ *and a* $t_1 \geqslant t_0$ *such that for* $t \geqslant t_1$ *either*

$$\begin{aligned} z_1(t)y_1(t) &> 0 \\ y_2(t)y_1(t) &< 0 \\ y_3(t)y_1(t) &> 0 \end{aligned} \tag{3_1}$$

or

$$\begin{aligned} z_1(t)y_1(t) &> 0 \\ y_i(t)y_1(t) &> 0, \quad i = 2, 3. \end{aligned} \tag{3_3}$$

Remark. The case $z_1(t)y_1(t) < 0$ on $[t_1, \infty)$ cannot occur (see [6, Lemma 4]).

We denote by N_1^+ or N_3^+ the set of all nonoscillatory solutions of (S) which satisfy (3_1) or (3_3), respectively. Denote by N the set of all nonoscillatory solutions of (S). Then by Lemma 3 we have

$$N = N_1^+ \cup N_3^+.$$

Lemma 4. ([6, Lemma 5])

I) *Let* $y \in N_3^+$ *on* $[t_1, \infty)$. *Then*

$$|y_1(t)| \geqslant (1 - \lambda)|z_1(t)| \quad \textit{for large } t. \tag{4}$$

II) *Let* $y \in N_1^+$ *on* $[t_1, \infty)$.

i) *If* $\lim\limits_{t\to\infty} |z_1(t)| = L_1 > 0$, *then there exists an* $a_0\colon\ 0 < a_0 < 1$ *such that*

$$|y_1(t)| \geqslant a_0|z_1(t)| \quad \textit{for large } t; \tag{5}$$

ii) *if* $\lim\limits_{t\to\infty} z_1(t) = 0$ *then* $\liminf\limits_{t\to\infty} |y_1(t)| = 0$, $\lim\limits_{t\to\infty} y_i(t) = 0$, $i = 2, 3$.

3. Oscillation theorems

Theorem 1. *Let the following conditions be satisfied:*

$$xyf_i(xy) \geqslant Kxyf_i(x)f_i(y) \quad (0 < K = \text{const.}),\ i = 1,2,3; \tag{6}$$

$$h_j(t),\ j = 2,3 \quad \text{are nondecreasing functions}; \tag{7}$$

$$h_3(h_2(h_1(t))) \leqslant t; \tag{8}$$

$$\int_{\gamma(0)}^{\infty} p_2(t) f_2\Big(\int_{h_3(t)}^{\infty} p_3(s)\,\mathrm{d}s\Big)\,\mathrm{d}t = \infty; \tag{9}$$

$$\int_{\gamma(\gamma(0))}^{\infty} p_3(t) f_3\Big(\int_{\gamma(0)}^{h_1(t)} p_1(s) f_1\Big(\int_0^{h_2(s)} p_2(x)\,\mathrm{d}x\Big)\,\mathrm{d}s\Big)\,\mathrm{d}t = \infty; \tag{10}$$

$$\int_0^{\alpha} \frac{\mathrm{d}t}{f_3(f_1(f_2(t)))} < \infty, \quad \int_0^{-\alpha} \frac{\mathrm{d}t}{f_3(f_1(f_2(t)))} < \infty, \tag{11}$$
for every constant $\alpha > 0$.

Then every solution $y \in W$ *is either oscillatory or* $\liminf\limits_{t\to\infty} |y_1(t)| = 0$ *and* $\lim\limits_{t\to\infty} y_i(t) = 0, i = 2,3.$

P r o o f. Let $y \in W$ be a nonoscillatory solution of (S). Then $y \in N_1^+ \cup N_3^+$ on $[t_1, \infty)$.

A) Let $y \in N_1^+$ on $[t_1,\infty)$. Without loss of generality we suppose that $y_1(t) > 0$ for $t \geqslant t_1$. Then the function $z_1(t)$ is nonincreasing on $[\gamma(t_1), \infty)$ and $\lim\limits_{t\to\infty} z_1(t) = L_1 < \infty$. From (2) we obtain

$$\lim_{t\to\infty} y_2(t) = \lim_{t\to\infty} y_3(t) = 0. \tag{12}$$

We shall prove that $\lim\limits_{t\to\infty} z_1(t) = 0$. Let $\lim\limits_{t\to\infty} z_1(t) = L_1 > 0$. Lemma 4 implies that there exist a $t_2 \geqslant \gamma(t_1)$ and a constant $C_1 = a_0 L_1$ such that $y_1(t) \geqslant C_1$ for $t \geqslant t_2$. From (e) we get

$$f_3(y_1(h_1(t))) \geqslant C_2, \quad t \geqslant t_3 = \gamma(t_2), \quad \text{where} \quad C_2 = f_3(C_1) > 0. \tag{13}$$

Integrating the third equation of (S) from t to ∞ and then using (13) we have

$$y_3(t) \geqslant C_2 \int_t^{\infty} p_3(s)\,\mathrm{d}s, \quad t \geqslant t_3.$$

Then in view of (e), (6) and the last inequality we get

$$f_2(y_2(h_3(t))) \geqslant K f_2(C_2) f_2\Big(\int_{h_3(t)}^{\infty} p_3(s)\,\mathrm{d}s\Big), \quad t \geqslant t_4 = \gamma(t_3). \tag{14}$$

Integrating the second equation of (S) from t_4 to t and then using (14) we get

$$y_2(t) \geqslant y_2(t_4) + Kf_2(C_2)\int_{t_4}^{t} p_2(z) f_2\Big(\int_{h_3(z)}^{\infty} p_3(s)\,\mathrm{d}s\Big)\,\mathrm{d}z, \quad t \geqslant t_4.$$

By virtue of (9), the last inequality implies for $t \to \infty$ that $\lim\limits_{t\to\infty} y_2(t) = \infty$, which contradicts (12). Therefore $\lim\limits_{t\to\infty} z_1(t) = 0$ and from Lemma 4 we have $\liminf\limits_{t\to\infty} |y_1(t)| = 0$.

B) Let $y \in N_3^+$ on $[t_1, \infty)$. Without loss of generality we suppose that $y_1(t) > 0$ on $[t_1, \infty)$. Integrating the second equation of (S) from t_5 to t we get

$$y_2(t) - y_5(t_5) = \int_{t_5}^{t} p_2(s) f_2(y_3(h_3(s)))\,\mathrm{d}s, \quad t \geqslant t_5 = \gamma(t_1)$$

and

$$y_2(h_2(t)) \geqslant \int_{t_5}^{h_2(t)} p_2(s) f_2(y_3(h_3(s)))\,\mathrm{d}s, \quad t \geqslant t_6 = \gamma(t_5). \tag{15}$$

Using (e), (6), (15) and the monotonicity of $f_2(y_3(h_3(s)))$ we get

$$f_1(y_2(h_2(t))) \geqslant Kf_1(f_2(y_3(h_3(h_2(t)))))f_1\Big(\int_{t_5}^{h_2(t)} p_2(s)\,\mathrm{d}s\Big), \quad t \geqslant t_6.$$

Integrating the first equation of (S) from t_6 to t and then using the last inequality, we have

$$z_1(t) \geqslant K\int_{t_6}^{t} p_1(s) f_1(f_2(y_3(h_3(h_2(s)))))f_1\Big(\int_{t_5}^{h_2(s)} p_2(x)\,\mathrm{d}x\Big)\,\mathrm{d}s, \quad t \geqslant t_6. \tag{16}$$

Using (8), (16) and the monotonicity of $f_1(f_2(y_3(t)))$ we get

$$z_1(h_1(t)) \geqslant Kf_1(f_2(y_3(t)))\int_{t_6}^{h_1(t)} p_1(s) f_1\Big(\int_{t_5}^{h_2(s)} p_2(x)\,\mathrm{d}x\Big)\,\mathrm{d}s, \quad t \geqslant t_7 = \gamma(t_6). \tag{17}$$

In view of Lemma 4 there exists a $t_8 \geqslant t_7$ such that

$$y_1(h_1(t)) \geqslant (1-\lambda) z_1(h_1(t)), \quad t \geqslant t_9 = \gamma(t_8). \tag{18}$$

In view of (e), (6), (17) and (18) we have

$$f_3(y_1(h_1(t))) \geqslant C_3 f_3(f_1(f_2(y_3(t))))f_3\Big(\int_{t_6}^{h_1(t)} p_1(s) f_1\Big(\int_{t_5}^{h_2(s)} p_2(x)\,\mathrm{d}x\Big)\,\mathrm{d}s\Big), \tag{19}$$

$t \geqslant t_9$ where $C_3 = K^2 f_3\big((1-\lambda)K\big) > 0$.

Multiplying (19) by $\frac{p_3(t)}{f_3(f_1(f_2(y_3(t))))}$, using the third equation of (S) and then integrating from t_9 to t, we get

$$\int_t^{t_9} \frac{y_3'(z)\,\mathrm{d}z}{f_3(f_1(f_2(y_3(z))))} \geqslant C_3 \int_{t_9}^t p_3(z) f_3\biggl(\int_{t_6}^{h_1(z)} p_1(s) f_1\biggl(\int_{t_5}^{h_2(s)} p_2(x)\,\mathrm{d}x\biggr)\,\mathrm{d}s\biggr)\,\mathrm{d}z,$$

$t \geqslant t_9$. The last inequality for $t \to \infty$ gives a contradiction to (10) with (11). This case cannot occur. The proof of Theorem 1 is complete. □

Theorem 2. *Suppose that* (6)–(9) *hold and in addition*

$$f_3(f_1(f_2(t)))) = t; \tag{20}$$

$$\int_{\gamma(\gamma(0))}^{\infty} p_3(t)\biggl[f_3\biggl(\int_{\gamma(0)}^{h_1(t)} p_1(s)\biggl(\int_0^{h_2(s)} p_2(x)\,\mathrm{d}x\biggr)\,\mathrm{d}s\biggr)\biggr]^{(1-\varepsilon)}\,\mathrm{d}t = \infty, \tag{21}$$
$$\text{where} \quad o < \varepsilon < 1.$$

Then the conclusion of Theorem 1 *holds.*

P r o o f. Let $y \in W$ be a nonoscillatory solution of (S). Then $y \in N_1^+ \cup N_3^+$ on $[t_1, \infty)$. As in the proof of Theorem 1, we get two cases: A) and B). In the case A) we proceed in the same way as in the proof of Theorem 1. Consider now the case B). In this case the inequality (19) holds. Using (20), (19) implies

$$f_3(y_1(h_1(t))) \geqslant C_3 y_3(t) f_3\biggl(\int_{t_6}^{h_1(t)} p_1(s) f_1\biggl(\int_{t_5}^{h_2(s)} p_2(x)\,\mathrm{d}x\biggr)\,\mathrm{d}s\biggr), \quad t \geqslant t_9. \tag{22}$$

Raising (22) to $(1-\varepsilon)$th power we obtain

$$\bigl[C_3 y_3(t)\bigr]^{(1-\varepsilon)}\biggl[f_3\biggl(\int_{t_6}^{h_1(t)} p_1(s) f_1\biggl(\int_{t_5}^{h_2(s)} p_2(x)\,\mathrm{d}x\biggr)\,\mathrm{d}s\biggr)\biggr]^{(1-\varepsilon)} \tag{23}$$
$$\leqslant \bigl[f_3(y_1(h_1(t)))\bigr]^{(1-\varepsilon)}, \quad t \geqslant t_9.$$

Lemma 4 together with (6) implies that there exist a $t_{10} \geqslant t_9$ and a constant $C_4 > 0$ such that

$$f_3(y_1(h_1(t))) \geqslant C_4, \quad t \geqslant t_{10}. \tag{24}$$

Now (24) implies

$$\bigl[f_3(y_1(h_1(t)))\bigr]^{(1-\varepsilon)} \leqslant C_5 f_3(y_1(h_1(t))), \quad t \geqslant t_{10}, \tag{25}$$
$$\text{where} \quad C_5 = C_4^{-\varepsilon} > 0.$$

Combining (23) with (25), we get

$$\big[C_3y_3(t)\big]^{(1-\varepsilon)}\bigg[f_3\bigg(\int_{t_6}^{h_1(t)}p_1(s)f_1\bigg(\int_{t_5}^{h_2(s)}p_2(x)\,\mathrm{d}x\bigg)\,\mathrm{d}s\bigg)\bigg]^{(1-\varepsilon)} \leqslant C_5f_3(y_1(h_1(t))),\quad t\geqslant t_{10}. \tag{26}$$

Multiplying (26) by $p_3(t)\big[C_3y_3(t)\big]^{(\varepsilon-1)}$, using the third equation of (S), integrating from t_{10} to t and then using the fact that $y_3(t)$ is positive and decreasing, we have

$$\int_{t_{10}}^{t}p_3(z)\bigg[f_3\bigg(\int_{t_6}^{h_1(t)}p_1(s)f_1\bigg(\int_{t_5}^{h_2(s)}p_2(x)\,\mathrm{d}x\bigg)\,\mathrm{d}s\bigg)\bigg]^{(1-\varepsilon)}\,\mathrm{d}z \leqslant C_5(C_3)^{(\varepsilon-1)}\big(\varepsilon^{-1}\big)\big[y_3(t_{10})\big]^{\varepsilon}<\infty,\quad t\geqslant t_{10},$$

which contradicts (21). Therefore the case B) cannot occur.

The proof of Theorem 2 is complete. □

Theorem 3. *Suppose that* (6), (9), (11) *hold and in addition*

$$h_2(t)\geqslant t,\quad h_3(t)\leqslant t; \tag{27}$$

$$\int_{\gamma(\gamma(0))}^{\infty}p_3(t)f_3\bigg(\int_{\gamma(0)}^{h(t)}p_1(s)f_1\bigg(\int_0^s p_2(x)\,\mathrm{d}x\bigg)\,\mathrm{d}s\bigg)\,\mathrm{d}t=\infty, \quad \textit{where}\quad h(t)=h_1^{\star}(t)=\min\{t,h_1(t)\}. \tag{28}$$

Then the conclusion of Theorem 1 *holds.*

P r o o f. Let $y\in W$ be a nonoscillatory solution of (S) on $[t_1,\infty)$. Further, proceeding in the same way as in the proof of Theorem 2 we consider only the case B). Using (27) and the monotonicity of $f_1(y_2(t))$ on $[t_1,\infty)$ the first equation of system (S) implies

$$z_1'(t)\geqslant p_1(t)f_1(y_2(t)),\quad t\geqslant t_1. \tag{29}$$

Analogously to (29) we have

$$y_2'(t)\geqslant p_2(t)f_2(y_3(t)),\quad t\geqslant\gamma(t_1)\geqslant t_1. \tag{30}$$

Lemma 4 together with (*e*) and (6) implies that there exists a $t_2^{\star}\geqslant\gamma(t_1)$ such that

$$f_3(y_1(h_1(t)))\geqslant C_6f_3(z_1(h_1(t))),\quad t\geqslant t_2^{\star}, \quad \text{where}\quad C_6=Kf_3(1-\lambda)>0. \tag{31}$$

Using (31) and the monotonicity of $f_3(z_1(t))$ on $[t_2^\star,\infty)$ the third equation of (S) implies

$$y_3'(t) \leqslant -C_6 p_3(t) f_3(z_1(h(t))), \quad t \geqslant t_2^\star. \tag{32}$$

In view of (29), (30), (32) we modify the system (S) to the form

$$\begin{aligned} &z_1'(t) \geqslant p_1(t) f_1(y_2(t)) \\ &y_2'(t) \geqslant p_2(t) f_2(y_3(t)) \\ &y_3'(t) \leqslant -C_6 p_3(t) f_3(z_1(h(t))), \quad t \geqslant t_2^\star. \end{aligned} \tag{S*}$$

System ($S^\star$) yields

$$z_1(t) \geqslant \int_{t_2^\star}^t p_1(s) f_1(y_2(s))\,\mathrm{d}s, \quad t \geqslant t_2^\star \tag{33}$$

and

$$y_2(s) \geqslant \int_{t_2^\star}^s p_2(x) f_2(y_3(x))\,\mathrm{d}x, \quad s \geqslant t_2^\star. \tag{34}$$

In view of (e), (6) and the monotonicity of $f_2(y_3(x))$ on $[t_2^\star,\infty)$, from (34) we have

$$f_1(y_2(s)) \geqslant K f_1(f_2(y_2(s))) f_1\Big(\int_{t_2^\star}^s p_2(x)\,\mathrm{d}x\Big), \quad s \geqslant t_2^\star. \tag{35}$$

Combining (33) with (35) we get

$$z_1(t) \geqslant K \int_{t_2^\star}^t p_1(s) f_1(f_2(y_3(s))) f_1\Big(\int_{t_2^\star}^s p_2(x)\,\mathrm{d}x\Big)\,\mathrm{d}s, \quad t \geqslant t_2^\star. \tag{36}$$

Using (e), (6) and the monotonicity of $f_1(f_2(y_3(s)))$ on $[t_2^\star,\infty)$ we obtain

$$f_3(z_1(h(t))) \geqslant C_7 f_3(f_1(f_2(y_3(t)))) f_3\Big(\int_{t_2^\star}^{h(t)} p_1(s) f_1\Big(\int_{t_2^\star}^s p_2(x)\,\mathrm{d}x\Big)\,\mathrm{d}s\Big), \tag{37}$$
$$t \geqslant t_3^\star = \gamma(t_2^\star), \qquad \text{where} \quad C_7 = K^2 f_3(K) > 0.$$

Multiplying (37) by $\frac{C_6 p_3(t)}{f_3(f_1(f_2(y_3(t))))}$, integrating from $t_3^\star$ to t, using the third inequality of $(S^\star)$ and (11) we get

$$\begin{aligned} C_6 C_7 \int_{t_3^\star}^t p_3(z) f_3\Big(\int_{t_2^\star}^{h(z)} p_1(s)\Big(\int_{t_2^\star}^s p_2(x)\,\mathrm{d}x\Big)\,\mathrm{d}s\Big)\,\mathrm{d}z \\ \leqslant \int_{y_3(t)}^{y_3(t_3^\star)} \frac{\mathrm{d}z}{f_3(f_1(f_2(z)))} < \infty, \quad t \geqslant t_3^\star, \end{aligned}$$

which contradicts (28) and therefore the case B) cannot occur. The proof of Theorem 3 is complete. □

Theorem 4. *Suppose that* (6), (9), (20), (27) *hold and in addition*

$$(38)\int_{\gamma(\gamma(0))}^{\infty} p_3(t)\left[f_3\left(\int_{\gamma(0)}^{h(t)} p_1(s) f_1\left(\int_0^s p_2(x)\,\mathrm{d}x\right)\mathrm{d}s\right)\right]^{(1-\varepsilon)} \mathrm{d}t = \infty,\ 0 < \varepsilon < 1,$$
$$\text{where} \quad h(t) = h_1^\star(t).$$

Then the conclusion of Theorem 1 *holds.*

We can prove Theorem 4 analogously to Theorem 2 and Theorem 3.

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