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INTERVALS IN PARTIAL MONOUNARY ALGEBRAS

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Summary. In this paper the notion of an interval in a partial monounary algebra is introduced and pairs (A,f), (A,g) of partial monounary algebras are investigated such that each interval in (A,f) is also an interval in (A,g), and conversely.

Keywords: partial monounary algebra, convex set, interval.

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1. PRELIMINARIES AND RESULTS

G. Birkhoff and M. K. Bennett [4] dealt with pairs of partially ordered sets P_1 and P_2 which are defined on the same underlying set such that each convex subset of P_1 is, at the same time, a convex subset of P_2 and conversely. For further related results of the same authors cf. [1]–[3]. V. Slavík [8] studied an analogous question for lattices, replacing convex subsets by intervals. The system of all intervals of a lattice was studied by M. Kolibiar [7].

Pairs of partial monounary algebras defined on the same set and having the same convex subsets were investigated in [5].

In the present paper the notion of an interval in a partial monounary algebra will be defined (by applying the notion of a convex subset). Pairs of partial monounary algebras (A,f) and (A,g) possessing the same intervals will be dealt with.

For the basic definitions and notation concerning partial monounary algebras cf. [5]. Let $\mathcal{U}$ be the class of all partial monounary algebras. If $A \neq \emptyset$ is a set, then $F(A)$ is the system of all partial mappings of A into A. The set of all (positive) integers is denoted by $Z(N)$.

To each partial monounary algebra (A,f) there corresponds a directed graph $G(A,f) = (A, E)$ without loops and multiple edges which is defined as follows: an ordered pair (a, b) of distinct elements of A belongs to E iff $f(a) = b$. (The nonexistence of loops is a consequence of the fact that in our definition of an edge (a, b) it is assumed that $a \neq b$.)

A subset B of A is said to be *convex* (in (A,f)) if, whenever a, b_1 and b_2 are distinct elements of A such that $b_1, b_2 \in B$ and there is a path (in $G(A,f)$) going from b_1 to b_2 and containing a, then a belongs to B as well.

Let $a, b \in A$. Assume that there is $n \in N \cup \{0\}$ such that $b = f^n(a)$. The least convex set containing a and b will be called *an interval* in (A, f) and denoted by $[a, b]_f$.

We denote by $\text{Int}(A, f)$ the system of all intervals in (A, f) including $\emptyset$ and by $\text{Co}(A, f)$ the system of all convex subsets of (A, f). Both these systems are partially ordered by inclusion. Then $\text{Co}(A, f)$ is an atomic lattice (cf. [6]) and $\text{Int}(A, f)$ fails to be a lattice in general.

The notion of coherency for connected partially ordered sets was introduced by Birkhoff and Bennett [4]. In [5] an analogous notion for connected partial monounary algebras was defined; by this definition a connected partial monounary algebra (A, f) is non-coherent if either (i) $\text{card}\, A = 2$, or (ii) there are $a, c \in A$, $a \neq c$ with $f(a) = c$, $f^{-1}(a) = \emptyset$ and either $f(c) = c$ or $f(c)$ does not exist. A partial monounary algebra (A, f) will be called *coherent*, if no connected component of (A, f) is non-coherent.

In [5], Thm 6.3, all partial operations g on A with $\text{Co}(A, f) = \text{Co}(A, g)$ were described for a given partial monounary algebra (A, f).

By applying results of [5] the following theorems will be proved in Section 2 of the present paper.

1.1. Theorem. *Let* (A, f), $(A, g) \in \mathcal{U}$. *Then* $\text{Int}(A, f) = \text{Int}(A, g)$ *implies* $\text{Co}(A, f) = \text{Co}(A, g)$.

1.2. Theorem. *Let* $(A, f) \in \mathcal{U}$, *let* (A, f) *be not coherent. Then there exists* $(A, g) \in \mathcal{U}$ *such that* $\text{Co}(A, f) = \text{Co}(A, g)$ *and* $\text{Int}(A, f) \neq \text{Int}(A, g)$.

1.3. Theorem. *Let* $(A, f), (A, g) \in \mathcal{U}$ *be coherent. Then* $\text{Co}(A, f) = \text{Co}(A, g)$ *implies* $\text{Int}(A, f) = \text{Int}(A, g)$.

Let (A, f) be a given partial monounary algebra. In Section 3 we describe all partial monounary algebras (A, g) such that $\text{Int}(A, f) = \text{Int}(A, g)$.

2. PROOFS OF 1.1—1.3

Proof of Thm. 1.1. According to the definition of intervals and convex sets we have

(1) $$B \in \text{Co}(A, f) \Leftrightarrow (\forall b_1, b_2 \in B) \ (\text{if } [b_1, b_2]_f \text{ exists then } [b_1, b_2]_f \subseteq B).$$

If $\text{Int}(A, f) = \text{Int}(A, g)$, then (1) yields that the following conditions are equivalent

(i) $B \in \text{Co}(A, f)$

(ii) $(\forall b_1, b_2 \in B)$ $([b_1, b_2]_f \text{ exists} \Rightarrow [b_1, b_2]_f \subseteq B)$,

(iii) $(\forall b_1, b_2 \in B)$ $([b_1, b_2]_g \text{ exists} \Rightarrow [b_1, b_2]_g \subseteq B)$,

(iv) $B \in \text{Co}(A, g)$.

Proof of Thm. 1.2. Let $(A,f) \in \mathscr{U}$, let (A,f) be not coherent. Then there exists a connected component A_1 of (A,f) such that either (i) card $A_1 = 2$ or (ii) there are $a, c \in A_1$, $a \neq c$ with $f(a) = c$, $f^{-1}(a) = \emptyset$ and either $f(c) = c$ or $f(c)$ does not exist. First, let card $A_1 = 2$, $A_1 = \{u, v\}$. Put

$$g(x) = \begin{cases} f(x), & \text{if } x \in A - A_1, \\ x, & \text{if } x \in A_1. \end{cases}$$

It is obvious that $\mathrm{Co}(A,f) = \mathrm{Co}(A, g)$. Further, $[u, v]_f = \{u, v\} \notin \mathrm{Int}(A, g)$, hence $\mathrm{Int}(A,f) \neq \mathrm{Int}(A, g)$. Now let (ii) hold. Put

$$g(x) = \begin{cases} f(x), & \text{if } x \in A - \{a\}, \\ x, & \text{if } x = a. \end{cases}$$

Obviously, $\mathrm{Co}(A,f) = \mathrm{Co}(A, g)$, $[a, c]_f = \{a, c\} \notin \mathrm{Int}(A, g)$, thus $\mathrm{Int}(A,f) \neq \mathrm{Int}(A, g)$.

2.1. Lemma. *Let (N,f_i) (for $i = 0, 1, 2, 3$) be such that $f_0(n) = n + 1$, $f_1(n + 1) = f_2(n + 1) = f_3(n + 1) = n$ for each $n \in N$, $f_1(1) = 1$, $f_3(1) = 2$ and $f_2(1)$ does not exist. If $g \in F(N)$, $i \in \{0, 1, 2, 3\}$, then $\mathrm{Co}(N,f_i) = \mathrm{Co}(N, g)$ implies $\mathrm{Int}(N,f_i) = \mathrm{Int}(N, g)$.*

Proof. Let $i \in \{0, \ldots, 3\}$, $(N, g) \in \mathscr{U}$. Then $\mathrm{Co}(N, g) = \mathrm{Co}(N,f_i)$ if and only if $g \in \{f_0, f_1, f_2, f_3\}$ (cf. [5], Thm. 5.3.2). Further, if $g \in \{f_0, f_1, f_2, f_3\}$, then the relation $\mathrm{Int}(N, g) = \mathrm{Int}(N,f_i)$ follows immediately from the definition of intervals.

2.2. Lemma. *Let $N_1 = \{1, 2, \ldots, n\}$, $n > 2$ and assume $f_1(k) = f_2(k) = f_3(k) = k - 1$ for each $k \in N_1 - \{1\}$, $f_1(1) = 1$, $f_2(1)$ does not exist, $f_3(1) = 2$. If $g \in F(N_1)$, $i \in \{1, 2, 3\}$, then $\mathrm{Co}(N_1,f_i) = \mathrm{Co}(N_1, g)$ implies $\mathrm{Int}(N_1,f_i) = \mathrm{Int}(N_1, g)$.*

Proof. Let $i \in \{1, 2, 3\}$. Put $f_4(k) = f_5(k) = f_6(k) = k + 1$ for each $k \in N_1 - \{n\}$, $f_4(n) = n$, $f_5(n)$ does not exist, $f_6(n) = n - 1$. According to [5], Thm. 5.3.3,

$$\{g \in F(N_1)\colon \mathrm{Co}(N_1,f_i) = \mathrm{Co}(N_1, g)\} = \{f_1, f_2, \ldots, f_6\}.$$

It is clear that $\mathrm{Int}(N_1,f_i) = \mathrm{Int}(N_1,f_j)$ for each $j \in \{1, \ldots, 6\}$, thus $\mathrm{Co}(N_1,f_i) = \mathrm{Co}(N_1, g)$ implies $\mathrm{Int}(N_1,f_i) = \mathrm{Int}(N_1, g)$.

2.3. Lemma. *Let (Z,f) be such that $f(k) = k + 1$ for each $k \in Z$, $g \in F(Z)$. Then $\mathrm{Co}(Z,f) = \mathrm{Co}(Z, g)$ implies $\mathrm{Int}(Z,f) = \mathrm{Int}(Z, g)$.*

Proof. Put $h(k) = k - 1$ for each $k \in Z$. Then (in view of [5], 5.1), $\{g \in F(Z)\colon \mathrm{Co}(Z,f) = \mathrm{Co}(Z, g)\} = \{f, h\}$, and the assertion of the lemma is obvious.

2.4. Lemma. *Let (A,f) be a connected monounary algebra possessing no cycle and such that there are $a, b \in A$, $a \neq b$ with $f(a) = f(b)$. Further, let $g \in F(A)$. Then $\mathrm{Co}(A,f) = \mathrm{Co}(A, g)$ implies $\mathrm{Int}(A,f) = \mathrm{Int}(A, g)$.*

Proof. The assertion follows from the fact that $\{g \in F(A)\colon \mathrm{Co}(A,f) = \mathrm{Co}(A,g)\} = \{f\}$ (cf. [5], 5.1).

2.5. Lemma. *Let (A,f) be a coherent partial monounary algebra and let c be an element such that one of the following conditions is satisfied*: (a) $f(c) = c$, $\operatorname{card} f^{-1}(c) = 2$, (b) $f(c)$ *does not exist*, $\operatorname{card} f^{-1}(c) = 1$, (c) $f^{-1}(c) = \{f(c)\} \neq \{c\}$. *If $(A,g) \in \mathcal{U}$ is coherent, then* $\mathrm{Co}(A,f) = \mathrm{Co}(A,g)$ *implies* $\mathrm{Int}(A,f) = \mathrm{Int}(A,g)$.

Proof. Put $f_1(x) = f_2(x) = f_3(x) = f(x)$ for each $x \in A - \{c\}$, $f_1(c) = c$, $f_2(c)$ does not exist and $f_3(c) = a$, where $\{a\} = f^{-1}(c) - \{c\}$ (such a exists and is uniquely determined). We have

$$\{g \in F(A)\colon (A,g) \text{ is coherent, } \mathrm{Co}(A,f) = \mathrm{Co}(A,g)\} = \{f_1, f_2, f_3\},$$

in view of [5], 5.4.2. Since $\mathrm{Int}(A,f) = \mathrm{Int}(A,f_i)$ for $i = 1, 2, 3$, we obtain that $\mathrm{Co}(A,f) = \mathrm{Co}(A,g)$ implies $\mathrm{Int}(A,f) = \mathrm{Int}(A,g)$.

2.6. Lemma. *Let (A,f) be a coherent partial monounary algebra and let $c \in A$ be such that* $\operatorname{card}(f^{-1}(c) - \{c\}) \geqq 2$ *and either $f(c) = c$ or $f(c)$ does not exist. If $(A,g) \in \mathcal{U}$ is coherent then* $\mathrm{Co}(A,f) = \mathrm{Co}(A,g)$ *implies* $\mathrm{Int}(A,f) = \mathrm{Int}(A,g)$.

Proof. Let f_1, f_2 be defined analogously as in 2.5. Then [5], 5.5.2 implies

$$\{g \in F(A)\colon (A,g) \text{ is coherent, } \mathrm{Co}(A,f) = \mathrm{Co}(A,g)\} = \{f_1, f_2\}.$$

We obtain the assertion of the lemma similarly as above.

2.7. Lemma. *Let (A,f) be a coherent connected monounary algebra possessing a cycle with more than 1 element. Assume that there exists no $c \in A$ satisfying the condition $f^{-1}(c) = \{f(c)\} \neq \{c\}$. If $g \in F(A)$, then* $\mathrm{Co}(A,f) = \mathrm{Co}(A,g)$ *implies* $\mathrm{Int}(A,f) = \mathrm{Int}(A,g)$.

Proof. Let $C = \{c_1, \ldots, c_n\}$ be the cycle of (A,f) and let S_n be the set of all permutations of $1, 2, \ldots, n$. For $\alpha \in S_n$ put $f_\alpha(x) = f(x)$ for each $x \in A - C$, $f_\alpha(c_{\alpha(i)}) = c_{\alpha(i+1)}$ for each $i \in \{1, 2, \ldots, n-1\}$, $f_\alpha(c_{\alpha(n)}) = c_{\alpha(1)}$. According to [5], Thm. 5.6.2, we have

$$\{g \in F(A)\colon \mathrm{Co}(A,f) = \mathrm{Co}(A,g)\} = \{f_\alpha\colon \alpha \in S_n\}.$$

Further, $\mathrm{Int}(A,f_\alpha) = \mathrm{Int}(A,f)$, which proves the assertion of the lemma.

2.8. Corollary. *Let (A,f), (A,g) be coherent connected partial monounary algebras. Then* $\mathrm{Co}(A,f) = \mathrm{Co}(A,g)$ *implies* $\mathrm{Int}(A,f) = \mathrm{Int}(A,g)$.

Proof. Since (A,f) is connected and coherent, we have $\operatorname{card} A > 2$. If (A,f) is complete, contains no cycle and there are $a, b \in A$, $a \neq b$ with $f(a) = f(b)$, then the assertion follows from 2.4. If (A,f) is complete, contains no cycle and $f(a) = f(b)$

implies $a = b$ for each $a, b \in A$, then (A, f) is isomorphic to (N, f_0) from 2.1 or to (Z, f_0) from 2.3. If either (A, f) posesses a cycle with at least 3 elements or (A, f) possesses a cycle with 2 elements c_1, c_2 and there are $d_1, d_2 \in A - \{c_1, c_2\}$ with $f(d_1) = c_1$, $f(d_2) = c_2$, then (A, f) possesses a cycle with more than one element and there is no $c \in A$ with $f^{-1}(c) = \{f(c)\} \neq \{c\}$; in this case the required assertion is obtained by virtue of 2.7. Now assume that (A, f) possesses a cycle $\{c, d\}$, $c \neq d$ and that $f^{-1}(c) = \{d\}$. Then (A, f) satisfies the condition (c) of 2.5 and thus 2.5 implies the assertion. If there is $c \in A$ with $f(c) = c$ and $\operatorname{card}(f^{-1}(c) - \{c\}) \geqq 2$, then 2.6 yields that the assertion is valid. If $c \in A, f(c) = c$ and $\operatorname{card}(f^{-1}(c) - \{c\}) < < 2$, then $\operatorname{card}(f^{-1}(c) - \{c\}) = 1$, $\operatorname{card} f^{-1}(c) = 2$ and we can use 2.5 (a). Now assume that there is $c \in A$ such that $f(c)$ does not exist. Again, either $\operatorname{card}(f^{-1}(c) - - \{c\}) \geqq 2$ or $\operatorname{card}(f^{-1}(c) - \{c\}) < 2$, i.e., either $\operatorname{card}(f^{-1}(c) - \{c\}) \geqq 2$ or $\operatorname{card} f^{-1}(c) = 1$; hence 2.6 or 2.5(c) yield that the assertion of the corollary holds.

Proof of Thm. 1.3. Let $(A, f), (A, g)$ be coherent partial monounary algebras such that $\operatorname{Co}(A, f) = \operatorname{Co}(A, g)$. It follows from [5], Thm. 4.9, that (A, f) and (A, g) have the same connected components. Further, if A' is a connected component of (A, f) and of (A, g), then $\operatorname{Co}(A', f) = \operatorname{Co}(A', g)$. According to 2.8 we obtain

$$\operatorname{Co}(A', f) = \operatorname{Co}(A', g) \quad \text{implies} \quad \operatorname{Int}(A', f) = \operatorname{Int}(A', g),$$

and hence the assertion of Thm. 1.3 is valid.

3. PAIRS OF PARTIAL MONOUNARY ALGEBRAS WITH COMMON INTERVALS

Let (A, f) be a given partial monounary algebra. In this section we describe the method of construction of all partial monounary algebras (A, g) such that $\operatorname{Int}(A, f) = \operatorname{Int}(A, g)$.

3.1. Lemma. *Let $(A, f) \in \mathcal{U}$, $x, y \in A$. Then x and y belong to the same connected component of (A, f) if and only if there exist $z \in A$ and $B_1, B_2 \in \operatorname{Int}(A, f)$ such that $\{x, z\} \subseteq B_1$ and $\{y, z\} \subseteq B_2$.*

Proof. If x, y belong to the same connected component of (A, f), then there are $m, n \in N \cup \{0\}$ such that $f^n(x) = f^m(y)$. Put $z = f^n(x)$. Then $\{x, z\} \subseteq [x, z]_f \in \operatorname{Int}(A, f)$, $\{y, z\} \subseteq [y, z]_f \in \operatorname{Int}(A, f)$. Conversely, let $z \in A$, $\{x, z\} \subseteq B_1 \in \operatorname{Int}(A, f)$, $\{y, z\} \subseteq B_2 \in \operatorname{Int}(A, f)$. Then x and z (y and z) belong to the same connected component of (A, f), therefore x and y belong to the same connected component of (A, f).

3.2. Corollary. *Let $(A, f), (A, g) \in \mathcal{U}$, $\operatorname{Int}(A, f) = \operatorname{Int}(A, g)$, $\emptyset \neq B \subseteq A$. Then B is a connected component of (A, f) if and only if B is a connected component of (A, g).*

3.3. Lemma. *Let (A,f), $(A,g) \in \mathcal{U}$, $\mathrm{Int}(A,f) = \mathrm{Int}(A,g)$ and let $B \subseteq A$, $\mathrm{card}\, B > 1$. Then B is a coherent connected component of (A,f) if and only if B is a coherent connected component of (A,g).*

Proof. Let B be a coherent connected component of (A,f). According to 3.2, B is a connected component of (A,g). Further, the relation $\mathrm{Int}(A,f) = \mathrm{Int}(A,g)$ implies $\mathrm{Int}(B,f) = \mathrm{Int}(B,g)$, thus Thm. 1.1 yields $\mathrm{Co}(B,f) = \mathrm{Co}(B,g)$. In view of [5], 6.3 and 6.2.2 we obtain that (B,g) is coherent.

3.4. Lemma. *Let $(A,f) \in \mathcal{U}$, $a \in A$. Suppose that the connected component of (A,f) containing a has more than two elements and is non-coherent. Then the following conditions are equivalent:*

(i) *$f(a) = a$ or $f(a)$ does not exist;*
(ii) *if $X = \{x \in A - \{a\}: \{x, a\} \in \mathrm{Int}(A,f)\}$, then $\mathrm{card}\, X \geqq 2$ and $\{a, x_1, x_2\} \notin \mathrm{Int}(A,f)$ for each $x_1, x_2 \in X$, $x_1 \neq x_2$.*

Proof. Let (i) hold. According to the definition of intervals, X is the set of all $x \in A - \{a\}$ such that $f(x) = a$. Since the connected component containing the element a is non-coherent, there is $x \in X$ with $f^{-1}(x) = \emptyset$. Further, this component contains more than two elements, thus there is $y \in X$, $y \neq x$. It is obvious that $\{a, x_1, x_2\} \notin \mathrm{Int}(A,f)$ for $x_1, x_2 \in X$, $x_1 \neq x_2$. Now suppose that (ii) is valid and that (i) fails to hold. Put $f(a) = b$. Then $b \neq a$, $b \in X$. Since $\mathrm{card}\, X \geqq 2$, there is $x \in X - \{b\}$. Then $x \in f^{-1}(a)$, but $\{x, a, b\} \in \mathrm{Int}(A,f)$, which is a contradiction.
From 3.2, 3.3 and 3.4 we obtain

3.5. Corollary. *Let (A,f), $(A,g) \in \mathcal{U}$, $\mathrm{Int}(A,f) = \mathrm{Int}(A,g)$. Suppose that $a \in A$, the connected component of (A,f) containing a has more than two elements and is non-coherent. Then the following conditions are equivalent:*

(i) *$f(a) = a$ or $f(a)$ does not exist;*
(ii) *$g(a) = a$ or $g(a)$ does not exist.*

3.6. Lemma. *Let (A,f), $(A,g) \in \mathcal{U}$, $\mathrm{Int}(A,f) = \mathrm{Int}(A,g)$. Suppose that $a \in A$, the connected component B of (A,f) containing a has more then two elements and is non-coherent. Further, assume that either $f(a) = a$ or $f(a)$ does not exist. If $x \in B - \{a\}$, then $f(x) = g(x)$.*

Proof. According to 3.2 and 3.5 we obtain

(1) B is a connected component of (A,g),

(2) either $g(a) = a$ or $g(a)$ does not exist.

Thm. 1.1 implies that $\mathrm{Co}(B,f) = \mathrm{Co}(B,g)$. Put

$$B_1 = \{y \in A - \{a\}: f(y) = a,\ f^{-1}(y) = \emptyset\}.$$

Then (1), the relation $\operatorname{Co}(B,f) = \operatorname{Co}(B,g)$ and [5], 3.2 yield

$$B_1 = \{y \in A - \{a\} : g(y) = a,\ g^{-1}(y) = \emptyset\}\ .$$

Therefore we obtain

(3) $g(y) = f(y)$ for each $y \in B_1$.

If $B - B_1 = \emptyset$, then the assertion nnder consideration is valid. Let $B_2 = B - B_1 \neq \emptyset$. We get that B_2 is a coherent connected subalgebra of (A,f) (if B_2 is non-coherent, then the definition implies that there is $x \in B_2 - \{a\}$ with $f(x) = a$, $f^{-1}(x) = \emptyset$, i.e., $B_1 \cap B_2 \neq \emptyset$). Further, B_2 is a coherent connected subalgebra of (A, g) and $\operatorname{Co}(B_2,f) = \operatorname{Co}(B_2, g)$. Then (B_2,f) is isomorphic to some of the partial algebras considered in 2.1, 2.2, 2.5 or 2.6. In the proofs of these lemmas we have described (up to isomorphism) all partial algebras (B_2, h) with $\operatorname{Co}(B_2,f) = \operatorname{Co}(B_2, h)$. Hence in view of (2) the relation $\operatorname{Co}(B_2,f) = \operatorname{Co}(B_2, g)$ implies that $g(x) = f(x)$ for each $x \in B_2$. According to (3), $g(x) = f(x)$ for each $x \in B - \{a\}$.

3.7. Notation. *For $(A,f) \in \mathscr{U}$ let $C(f)$ be the set of all elements of A which belong to a coherent connected component of (A,f) possessing more than one element. The set of all $a \in A$ such that the connected component containing a has more than two elements, it is non-coherent and either $f(a) = a$ or $f(a)$ does not exist, will be denoted by the symbol $T(f)$. Further, let $Q(f)$ be the set of all $x \in A - T(f)$ which belong to the same connected component as some $a \in T(f)$.*

3.8. Theorem. *Let $(A,f), (A, g) \in \mathscr{U}$. The following conditions are equivalent:*

(i) $\operatorname{Int}(A,f) = \operatorname{Int}(A, g)$;

(ii) (a) *(A,f) and (A, g) have the same connected components*;
  (b) *$C(f) = C(g)$ and $\operatorname{Co}(C(f),f) = \operatorname{Co}(C(g), g)$*;
  (c) *$T(f) = T(g)$*;
  (d) *$Q(f) = Q(g)$ and $f(x) = g(x)$ for each $x \in Q(f)$.*

Proof. Let $\operatorname{Int}(A,f) = \operatorname{Int}(A, g)$. Then 3.2, 3.3 and 3.5 imply (a), (b) and (c). Using (a)–(c) we obtain that $Q(f) = Q(g)$ and then 3.6 yields that (d) holds. The converse implication follows from Thm. 1.3 and the definition of intervals.

Let us now describe how we will proceed by looking for all partial monounary algebras (A, g) such that $\operatorname{Int}(A,f) = \operatorname{Int}(A, g)$, if a partial monounary algebra (A,f) is given. (In this procedure we shall repeatedly apply Thm. 3.8 without mentioning it explicitly.)

The partial monounary algebras (A,f) and (A, g) must have the same connected components. Thus it suffices to investigate all $\emptyset \neq B \subseteq A$, where B runs over the set of all connected components of (A,f).

Let B be a connected component of (A,f). There are three possible cases:

1) card $B \leqq 2$. Then g on B can be defined in an arbitrary way, only B must be a connected component of (A, g).

2) $B \subseteq C(f)$, i.e., B is a coherent connected component of (A, f), card $B > 1$. Then we consider the condition $\mathrm{Co}(B, f) = \mathrm{Co}(B, g)$ and look for all g on B fulfilling this condition. Here we have to proceed as in [5], § 5. The partial algebra (B, f) is isomorphic to some of the partial algebras considered in 2.1 – 2.7. In the proofs of 2.1 – 2.7 we have described (up to isomorphism) all partial algebras (B, g) with $\mathrm{Co}(B, f) = \mathrm{Co}(B, g)$. E.g., if (B, f) is isomorphic to (N_1, f_2) from 2.2, then for g on B we have six possibilities.

3) $B \cap T(f) = \{a\}$, i.e., there is $a \in B$ such that either $f(a) = a$ or $f(a)$ does not exist, where card $B > 2$ and B is non-coherent. Then we have two possibilities for g on B: either $g(a) = a$ or $g(a)$ does not exist, and if $x \in B - \{a\}$, then $g(x) = f(x)$.

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Súhrn

INTERVALY V PARCIÁLNYCH MONOUNÁRNYCH ALGEBRÁCH

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V článku se zavádza pojem intervalu v parciálnej monounárnej algebre. Vyšetrujú sa dvojice (A, f), (A, g) parciálnych monounárnych algebier takých, že každý interval v (A, f) je tiež intervalom v (A, g) a obrátene.

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