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SEQUENTIAL CONVERGENCES IN DISTRIBUTIVE LATTICES

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Summary. In this paper we investigate the system $\operatorname{Conv} L$ of all sequential convergences on a distributive lattice L.

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Filter convergences on distributive lattices were studied in [1]. In the paper [8] there were investigated FLUSH sequential convergences (cf., e.g., [9], [10]) on lattices such that certain natural relations between the convergence and the order were fulfilled.

In the present paper the definitions and notations from [8] will be used. Throughout the paper, L denotes a distributive lattice.

The following results (A), (B) and (C) will be established.

(A) The partially ordered set $\operatorname{Conv} L$ of all sequential convergences in L is a complete lattice if and only if the following condition (i) and its dual are satisfied:

 (i) Whenever $((x_n), x), ((x'_n), x)$ are regular elements of $L^{\mathbb{N}} \times L$ and y is an element of L such that $x \leqslant x_n, x \leqslant x'_n$ and $x \leqslant y \leqslant x_n \vee x'_n$ for each $n \in \mathbb{N}$, then $x = y$.

(B) If L is $(\aleph_0, 2)$-distributive, then $\operatorname{Conv} L$ is a complete lattice.

For lattice ordered groups and for Boolean algebras the results analogous to (B) were proved in [7]. A related result (concerning filter convergences in completely distributive lattice ordered groups) was established in [2], Propos. 1.15.

If L is represented as a direct product $L = \prod_{i \in I} L_i$ and if $\alpha_i \in \operatorname{Conv} L_i$ for each $i \in I$, then $\alpha = \prod_{i \in I} \alpha_i$ is defined in a natural way.

(C) Under the above notations, α is a maximal element of $\operatorname{Conv} L$ if and only if, for each $i \in I$, α_i is a maximal element of $\operatorname{Conv} L_i$.

For the case of lattice ordered groups the result analogous to (C) was shown to be valid in [6]. On the other hand, the analogous result does not hold for groups (cf. [3], [5]); neither does the analogous result hold for topological groups (cf. [4]).

The questions whether the above theorems (A) and (C) remain valid without assuming the distributivity of L remain open.

1. Regular sets

For the notion of convergence in L cf. [8], Definition 1.1. We recall that a set $A \subseteq L$ is said to be regular if there exists $\alpha \in \operatorname{Conv} L$ such that $A \subseteq \alpha$.

In this section the assertion (A) mentioned above will be proved.

Let A be a nonempty subset of $L^{\mathbb{N}} \times L$. Denote

$$A^+ = \{(x_n \vee x), x): ((x_n), x) \in A\},$$

and let A^- be defined dually.

1.1. Lemma. *Let $\emptyset \neq A \subseteq L^{\mathbb{N}} \times L$. Then the following conditions are equivalent:*

(i) *A is regular.*

(ii) *Both A^+ and A^- are regular.*

Proof. Let A be regular. Thus there is $\alpha \in \operatorname{Conv} L$ with $A \subseteq \alpha$. Then both A^+ and A^- are subsets of α, hence they are regular. Conversely, let (ii) hold. There are $\alpha_1, \alpha_2 \in \operatorname{Conv} L$ with $A^+ \subseteq \alpha_1$ and $A^- \subseteq \alpha_2$. Hence $A^+ \subseteq \alpha_1^+$ and $A^- \subseteq \alpha_2^-$. In view of 3.7, [8], there is $\alpha \in \operatorname{Conv} L$ such that $\alpha = \alpha_1^+ \vee \alpha_2^-$. Thus $A^+ \subseteq \alpha$ and $A^- \subseteq \alpha$. According to 3.2, [8], we obtain $A \subseteq \alpha$. Therefore A is regular. □

If $\emptyset \neq A \subseteq L^{\mathbb{N}} \times L$, then we denote by $T(A)$ the m-convergence in L generated by the set A (cf. [8], Section 2).

We recall the following notation from [8].

Let $\emptyset \subseteq A \subseteq L^{\mathbb{N}} \times L$. We denote by A^1 the set of all $((x_n), x) \in L^{\mathbb{N}} \times L$ such that either

(i) there exists $((y_n), y) \in A$ such that $x = y$ and (y_n) is a subsequence of (x_n), or

(ii) there is $m \in \mathbb{N}$ such that $x_n = x$ for each $n \geqslant m$.

1.2. Lemma. *Let $\emptyset \neq A \subseteq L^{\mathbb{N}} \times L$, $A^+ = A$. Let $((x_n), x) \in L^{\mathbb{N}} \times L$ be such that $x_n \geqslant x$ for each $n \in \mathbb{N}$. Then the following conditions are equivalent:*

(i) $((x_n), x) \in T(A)$.

(ii) *There are $k \in \mathbb{N}$, a k-ary lattice polynomial f, elements $z^1, z^2, \dots, z^k$ in L and $(z_n^j) \in L^{\mathbb{N}}$ $(j = 1, 2, \dots, k)$ such that $((z_n^j), z^j) \in A^1$ for each $j \in \{1, 2, \dots, k\}$ and*

$$f(z^1, z^2, \dots, z^k) = x \leqslant x_n \leqslant f(z_n^1, z_n^2, \dots, z_n^k) \text{ for each } n \in \mathbb{N}.$$

Proof. The implication (ii)⇒(i) is obvious. The converse implication is a consequence of [8] (2.3 and 2.4). □

1.3. Lemma. *Let A and $((x_n), x)$ be as in 1.2. Then the following conditions are equivalent:*

(i) $((x_n), x) \in T(A)$.

(ii) *There are $l \in \mathbb{N}$ and $(t_n^j) \in L^{\mathbb{N}}$ $(j = 1, 2, \dots, k)$ such that $((t_n^j), x) \in A^1$ for each $j \in \{1, 2, \dots, k\}$, $t_n^j \geqslant x$ for each $n \in \mathbb{N}$ and each $j \in \{1, 2, \dots, k\}$, and*

$$x_n \leqslant t_n^1 \vee t_n^2 \vee \dots \vee t_n^k \text{ for each } n \in \mathbb{N}.$$

Proof. The assertion follows from 1.2 by applying the same steps as in the proof of 5.1, [8]. □

1.4. Lemma. *Let A be as in 1.2. Then the following conditions are equivalent:*

(i) *A is regular.*

(ii) *If $((x_n), x)$ and $((x_n), y)$ belong to $T(A)$, then $x = y$.*

Proof. This is an immediate consequence of the definition of regularity. □

1.5. Lemma. *Let A be as in 1.2. Then the following conditions are equivalent:*

(i) *A is not regular.*

(ii) *There are $x, z \in L$ and $(x_n) \in L^{\mathbb{N}}$ such that $((x_n), x) \in T(A)$ and $x < z \leqslant x_n$ for each $n \in \mathbb{N}$.*

Proof. Let (i) be valid. Hence $T(A) \notin \operatorname{Conv} L$. Thus in view of 1.4 there exist $((x_n), x)$ and $((x_n), y)$ in $T(A)$ such that $x \neq y$. Denote $x \vee y = z$. Then either $z > x$ or $z > y$; without loss of generality we can assume that $z > x$. According to 1.2 we have $x_n \geqslant x$ and $x_n \geqslant y$ for each $n \in \mathbb{N}$, hence $x_n \geqslant z$ for each $n \in \mathbb{N}$. Therefore (ii) is valid.

Conversely, assume that (ii) holds. By way of contradiction, assume that A is regular. Hence there is $\alpha \in \operatorname{Conv} L$ with $A \subseteq \alpha$. Then $T(A) \subseteq \alpha$, thus $((x_n), x) \in \alpha$. Now in view of (ii) we would have $x < z$ and $(\operatorname{const} z, x) \in \alpha$, which is impossible. □

From 1.5 and 1.3 we obtain

1.6. Theorem. *Let $\emptyset \neq A \subseteq L^{\mathbb{N}} \times L$, $A^+ = A$. Then the following conditions are equivalent:*

(i) *A is not regular.*

(ii) *There exist $(t_n^j) \in L^{\mathbb{N}}$ $(j = 1, 2, \dots, k)$, $x \in L$ and $z \in L$ with $((t_n^j), x) \in A^1$ for each $j \in \{1, 2, \dots, k\}$, $t_n^j \geqslant x$ for each $j \in \{1, 2, \dots, k\}$ and each $n \in \mathbb{N}$ such that*

$$x < z \leqslant t_n^1 \vee t_n^2 \vee \dots \vee t_n^k \quad \text{for each } n \in \mathbb{N}.$$

In view of 1.6, 1.1 and by using duality we infer:

1.7. Corollary. *Let A be a nonempty subset of $L^{\mathbb{N}} \times L$. Then the following conditions are equivalent:*

(i) *A is not regular.*

(ii) *Either A^+ satisfies the condition* (ii) *from* 1.6 *or A^- satisfies the condition dual to the condition* (ii) *from* 1.6.

1.8. Lemma. *The following conditions are equivalent:*

(i) $\operatorname{Conv} L$ *possesses the greatest element.*

(ii) *The condition* (i) *from* (A) *and its dual are satisfied.*

Proof. Let (i) be valid and let $((x_n), x), ((x'_n), x)$ be as in the condition (i) from (A). Then there exist $\alpha_1, \alpha_2 \in \operatorname{Conv} L$ such that $((x_n), x) \in \alpha_1$ and $((x'_n), x) \in \alpha_2$. In view of (i) there exists the greatest element α in $\operatorname{Conv} L$. Hence both $((x_n), x)$ and $((x'_n), x)$ belong to α. Thus $((x_n \vee x'_n), x)$ belongs to α as well. If $y \in L$ such that $x \leqslant y \leqslant x_n \vee x'_n$ for each $n \in \mathbb{N}$, then $((\operatorname{const} y), x) \in \alpha$, hence $y = x$, Therefore the condition (i) from (A) is satisfied. Analogously we can verify that the dual of this condition is satisfied as well.

Assume that (ii) is valid. By way of contradiction, suppose that $\operatorname{Conv} L$ has no greatest element. Hence there exists a subset $\{\alpha_i\}_{i \in I}$ of $\operatorname{Conv} L$ which is not upper bounded in $\operatorname{Conv} L$. Thus $T\big(\bigcup_{i \in I} \alpha_i\big)$ fails to belong to $\operatorname{Conv} L$. In other words, $\bigcup_{i \in I} \alpha_i$ fails to be regular. Thus in view of 1.1, at least one of the sets $\big(\bigcup_{i \in I} \alpha_i\big)^+$, $\big(\bigcup_{i \in I} \alpha_i\big)^-$ is not regular.

Suppose $\big(\bigcup_{i \in I} \alpha_i\big)^+$ is not regular (if this assumption fails to be valid, then we proceed by applying a dual method). Let us use Theorem 1.6 with $A = \big(\bigcup_{i \in I} \alpha_i\big)^+$. Then $A = A^+$ and the condition (ii) from 1.6 holds for some $k \in \mathbb{N}$.

Hence there are regular elements $((t_n^j), x) \in L^{\mathbb{N}} \times L$ ($j = 1, 2, \ldots, k$) such that $t_n^j \geqslant x$ for each $j \in \{1, 2, \ldots, k\}$ and each $n \in \mathbb{N}$, and in view of 1.6, the set

$$\{((t_n^1), x), \ldots, ((t_n^k), x)\} \tag{1}$$

fails to be regular. Let k be the least positive integer having this property.

The case $k = 1$ is impossible. Suppose that $k > 2$. Then

$$\{((t_n^1), x), \ldots, ((t_n^{k-1}), x)\}$$

is regular. Hence the element $((t_n^1 \vee t_n^2 \vee \ldots \vee t_n^{k-1}), x)$ of $L^{\mathbb{N}} \times L$ is regular. In view of the induction assumption, the set

$$\{(t_n^1 \vee t_n^2 \vee \ldots \vee t_n^{k-1}), x), \ ((t_n^k), x)\}$$

is regular and then

$$((t_n^1 \vee t_n^2 \vee \ldots \vee t_n^k), x)$$

is regular as well. This implies that the set (1) is regular, which is a contradiction. Therefore we would have $k = 2$; let z be as in 1.6. We arrived at a contradiction with the condition (ii). This completes the proof. □

Lemma 1.8 and [8], Corollary 1.4, yield that Theorem (A) above is valid.

2. The $(\aleph_0, 2)$-distributivity

In this section Theorem (A) above will be applied for proving that Theorem (B) is valid.

2.1. Lemma. *Let $((x_n), x) \in L^{\mathbb{N}} \times L$. Assume that $((x_n), x)$ is regular and that $x_n \geqslant x$ for each $n \in \mathbb{N}$. Let $1(1), 2(1), 3(1), \ldots$ be a subsequence of the sequence $1, 2, \ldots$. Then $\bigwedge_{n \in \mathbb{N}} x_{n(1)} = x$.*

Proof. In view of the condition (i) of 1.1 in [8], $((x_{n(1)}), x)$ is regular. Thus there is $\alpha \in \operatorname{Conv} L$ with $((x_{n(1)}), x) \in \alpha$. By way of contradiction, suppose that the relation $\bigwedge_{n \in \mathbb{N}} x_{n(1)} = x$ does not hold. Hence there is $y \in L$ such that

$$x < y \leqslant x_{n(1)} \quad \text{for each } n \in \mathbb{N}.$$

Then in view of the condition (vi) of 1.1 in [8] we have

$$((\operatorname{const} y), x) \in \alpha.$$

According to the condition (iv) of 1.1 in [8] we arrived at a contradiction. □

The assertion dual to 2.1 can be verified analogously.

Proof of Theorem (B).

Assume that L is $(\aleph_0, 2)$-distributive. By way of contradiction, suppose that $\operatorname{Conv} L$ fails to be a complete lattice. Then in view of (A), either the condition (i) from (A) or the corresponding dual condition does not hold.

Suppose that the condition (i) in (A) is not satisfied (in the opposite case we apply a dual method). Hence there are regular elements $((x_n), x)$ and $((x'_n), x)$ of $L^{\mathbb{N}} \times L$; and $y \in L$ such that $x \leqslant x_n, x \leqslant x'_n$ and $x < y \leqslant x_n \vee x'_n$ for each $n \in \mathbb{N}$.

Denote $y_n = x_n \wedge y$ and $y'_n = x'_n \wedge y$ for each $n \in \mathbb{N}$. Then $((y_n), x)$ and $((y'_n), x)$ are regular elements of $L^{\mathbb{N}} \times L$. Moreover, $y_n \geqslant x$ and $y'_n \geqslant x$ for each $n \in \mathbb{N}$. Hence in view of 2.1 we have

$$\bigwedge_{n \in \mathbb{N}} y_{n(1)} = x = \bigwedge_{n \in \mathbb{N}} y'_{n(1)} \tag{1}$$

for each subsequence $(y_{n(1)})$ of (y_n) and each subsequence $(y'_{n(1)})$ of (y_n). Next

$$y_n \vee y'_n = (x_n \wedge y) \vee (x'_n \wedge y) = (x_n \vee x'_n) \wedge y = y \quad \text{for each } n \in \mathbb{N}. \tag{2}$$

According to (2) we have

$$(y_1 \vee y'_1) \wedge (y_2 \vee y'_n) \wedge \ldots \wedge (y_n \vee y'_n) \wedge \ldots = y. \tag{3}$$

Let Φ be the set of all mappings φ of the set $\mathbb{N}$ into the set $\{1, 2\}$. We put $z_{n,\varphi(n)} = x_n$ if $\varphi(n) = 1$, and $x_{n,\varphi(n)} = y_n$ otherwise.

Let φ be fixed; consider the set

$$\{z_{n,\varphi(n)}\}_{n \in \mathbb{N}}. \tag{4}$$

We have $z_{n,\varphi(n)} \geqslant x$ for each $n \in \mathbb{N}$. At least one of the sets $\{n \in \mathbb{N} \colon \varphi(n) = 1\}$, $\{n \in \mathbb{N} \colon \varphi(n) = 2\}$ is infinite. Hence according to (2),

$$\bigwedge_{n \in \mathbb{N}} z_{n,\varphi(n)} = x \quad \text{for each } \varphi \in \Phi.$$

Therefore

$$\bigvee_{\varphi \in \Phi} \bigwedge_{n \in \mathbb{N}} z_{n,\varphi(n)} = x. \tag{5}$$

Since $x < y$, the relations (3) and (5) show that L is not $(\aleph_0, 2)$-distributive, which is a contradiction. Hence $\operatorname{Conv} L$ must be a complete lattice. □

Let I be a nonempty set and for each $i \in I$ let L_i be a distributive lattice. Consider the direct product $L = \prod_{i \in I} L_i$. For $x \in L$ and $i \in I$ we denote by $x(i)$ the i-th component of x.

For each $i \in I$ let $\alpha_i \in \operatorname{Conv} L_i$. The set of all $((x_n), x) \in L^{\mathbb{N}} \times L$ such that $((x_n(i)), x(i)) \in \alpha_i$ for each $i \in I$ will be denoted by $\prod_{i \in I} \alpha_i$.

From the definition of convergence in L (cf. [8], 1.1) we obtain immediately:

3.1. Lemma. *Let $L = \prod_{i \in I} L_i$, $\alpha_i \in \operatorname{Conv} L_i$ for each $i \in I$. Then $\prod_{i \in I} \alpha_i \in \operatorname{Conv} L$.*

Let $\beta \subseteq L^{\mathbb{N}} \times L$ and $i \in I$. We put

$$\beta(i) = \{(x_n(i)), x(i) \colon ((x_n), x) \in \beta\}.$$

3.2. Lemma. *Let L be as in 3.1. Let $\emptyset \neq \beta \subseteq L^{\mathbb{N}} \times L$ such that $\beta^+ = \beta$. Then the following conditions are equivalent:*

(i) *The set β is regular.*

(ii) *For each $i \in I$, the set $\beta(i)$ is regular.*

Proof. In view of $\beta^+ = \beta$ we have $\beta(i)^+ = \beta(i)$ for each $i \in I$.

Let (i) be valid. By way of contradiction, suppose that (ii) does not hold. Hence there is $i \in I$ such that $\beta(i)$ fails to be regular. Thus either the condition (ii) from 1.6 or the dual of this condition is valid, where A and L are replaced by $\beta(i)$ and L_i. Let the first case be true. This means that there are $((t_n^j), x) \in \beta(i)^1$ $(j = 1, 2, \ldots, k)$ with $t_n^j \geqslant x$ for each $(n, j) \in \mathbb{N} \times \{1, 2, \ldots, k\}$ such that

$$x < z \leqslant t_n^1 \vee t_n^2 \vee \ldots \vee t_n^k$$

for each $n \in \mathbb{N}$ and for some $z \in L_i$.

Let $j \in \{1, 2, \ldots, k\}$. There exists $((u_n^j), u^j) \in \beta^1$ such that $u_n^j(i) = t_n^j$ for each $n \in \mathbb{N}$, and $u^j(i) = x$. Since β is regular, there is $\alpha \in \operatorname{Conv} L$ with $\beta \subseteq \alpha$; thus $\beta^1 \subseteq \alpha$. Denote

$$u = u^1 \vee u^2 \vee \ldots \vee u^k, \quad u_n = u_n^1 \vee u_n^2 \vee \ldots \vee u_n^k \quad \text{for each } n \in \mathbb{N}.$$

Then we have $((u_n), u) \in \alpha$ and $u \leqslant u_n$ for each $n \in \mathbb{N}$. Next, there is $v \in L$ such that $v(i) = z$ and $v(i(1)) = u(i(1))$ for each $i(1) \in I \setminus \{i\}$. Hence $u < v$. Also, $u_n \geqslant v$ for each $n \in \mathbb{N}$. Thus according to 1.6 the set α is not regular, which is a contradiction.

Conversely, let (ii) be valid. By way of contradiction, assume that (i) fails to hold. Thus the condition (ii) from 1.6 (or the dual of this condition) is satisfied for $A = \beta$. Let the first mentioned case hold. There is $i \in I$ such that $x(i) < z(i)$. Now it is easy to verify that the condition (ii) from 1.6 holds if L and A are replaced by L_i and $\beta(i)$; therefore in view of 1.6 the set $\beta(i)$ is not regular, which is a contradiction. □

3.3. Lemma. *Let L be as in 3.1 and let $i \in I$, $((x_n), x) \in L^{\mathbb{N}} \times L$, $x_n \geqslant x$ for each $n \in I$, $x_n(i(1)) = x(i(1))$ whenever $n \in I$ and $i(1) \neq i$. Then the following conditions are equivalent:*

(i) *$((x_n), x)$ is regular.*

(ii) *$((x_{n(i)}), x(i))$ is regular.*

Proof. This is a consequence of 3.2 (in view of the condition (iii) from 1.1, [8], and 1.2, [8]). □

The assertions dual to 3.2 and 3.3 can be verified analogously. Hence in view of 1.1, the assumption $\beta^+ = \beta$ in 3.2 can be omitted. Similarly, the assumption $x_n \geqslant x$ for each $n \in \mathbb{N}$ in 3.3. can be omitted.

The following assertion is easy to verify.

3.4. Lemma. *Let L be as in 3.1. Let $\alpha \in \operatorname{Conv} L$. Then $\alpha \leqslant \prod_{i \in I} \alpha(i)$.*

Let us remark that there are a distributive lattice L and $\alpha \in \operatorname{Conv} L$ such that $\alpha < \prod_{i \in I} \alpha(i)$.

3.5. Corollary. *Let L be as in 3.1. Let α be a maximal element of* Conv L. *Then $\alpha = \prod_{i \in I} \alpha(i)$.*

3.6. Lemma. *Let L be as in 3.1 and let α be a maximal element of* Conv L. *Then for each $i \in I$, $\alpha(i)$ is a maximal element of* Conv L_i.

Proof. By way of contradiction, suppose that there exists $i \in I$ such that $\alpha(i)$ is not maximal in Conv L_i. Hence there is β^i in Conv L_i with $\alpha(i) < \beta^i$. Thus there exists $((x_n), x) \in \beta^i \setminus \alpha(i)$ such that either $x_n \geqslant x$ or $x_n \leqslant x$ for each $n \in \mathbb{N}$; without loss of generality we can suppose that the first case occurs. Choose $y^j \in L_j$ for each $j \in I$ and let $x^0, x^0_n \in L$ $(n = 1, 2, \ldots)$ such that

$$x^0(i) = x, \quad x^0(j) = y^j \quad \text{for each } j \in I \setminus \{i\},$$
$$x^0_n(i) = x_n, \quad x^0_n(j) = y^j \quad \text{for each } j \in I \setminus \{i\}.$$

Next, put $\beta_i = \beta^i$ and $\beta_j = \alpha(j)$ for each $j \in I \setminus \{i\}$, $\beta = \prod_{i \in I} \beta_i$. Then $\alpha \leqslant \beta$. Moreover, $((x_n^0), x^0) \in \beta \setminus \alpha$, whence $\alpha < \beta$ and $\beta \in \operatorname{Conv} L$, which is a contradiction. □

3.7. Lemma. *Let L be a distributive lattice and let $\alpha \in \operatorname{Conv} L$. Then there is $\beta \in \operatorname{Conv} L$ such that* (i) *$\alpha \leqslant \beta$, and* (ii) *β is a maximal element of* $\operatorname{Conv} L$.

Proof. This is an immediate consequence of the definition of $\operatorname{Conv} L$. □

3.8. Lemma. *Let L be as in 3.1. Assume that for each $i \in I$, α_i is a maximal element of* $\operatorname{Conv} L_i$. *Then $\prod_{i \in I} \alpha_i = \alpha$ is a maximal element of* $\operatorname{Conv} L$.

Proof. By way of contradiction, assume that α is not maximal. Then there is a maximal element β of $\operatorname{Conv} L$ such that $\alpha < \beta$. In view of 3.5 we have $\alpha_i < \beta(i)$ for some $i \in I$. Hence α_i is not maximal in $\operatorname{Conv} L_i$, which is a contradiction. □

By summarizing, from 3.5, 3.6 and 3.8 we obtain:

3.9. Theorem. *Let $L = \prod_{i \in I} L_i$ and $\alpha \in \operatorname{Conv} L$. Then the following conditions are equivalent:*

(i) *α is a maximal element of* $\operatorname{Conv} L$.

(ii) *For each $i \in I$, $\alpha(i)$ is a maximal element of* $\operatorname{Conv} L_i$, *and $\alpha = \prod_{i \in I} \alpha(i)$.*

Next, 3.9 yields:

3.10. Corollary. *Let $L = \prod_{i \in I} L_i$ and $\alpha \in \operatorname{Conv} L$. Then the following conditions are equivalent:*

(i) *α is the greatest element of* $\operatorname{Conv} L$.

(ii) *For each $i \in I, \alpha(i)$ is the greatest element of* $\operatorname{Conv} L_i$, *and $\alpha = \prod_{i \in I} \alpha(i)$.*

3.11. Corollary. *Let $L = \prod_{i \in I} L_i$. Then the following conditions are equivalent:*

(i) $\operatorname{Conv} L$ *has a greatest element.*

(ii) *For each $i \in I$,* $\operatorname{Conv} L_i$ *has a greatest element.*

References

[1] *R. Ball*: Distributive Cauchy lattices. Algebra Universalis *18* (1984), 134–174.
[2] *R. Ball, G. Davis*: The α-completion of a lattice ordered group. Czechoslovak Math. J. *33* (1983), 111–118.
[3] *D. Dikranjan, R. Frič, F. Zanolin*: On convergence groups with dense coarse subgroups. Czechoslovak Math. J. *37* (1987), 471–479.
[4] *D. Doitchinov*: Produits de groupes topologiques min imaux. Bull. Sci. Math. *97* (1972), 59–64.
[5] *R. Frič*: Products of coarse convergence groups. Czechoslovak Math. J. *38* (1988), 285–290.
[6] *M. Harminc, J. Jakubík*: Maximal convergences and minimal proper convergences in ℓ-groups. Czechoslovak Math. J. *39* (1989), 631–640.
[7] *J. Jakubík*: Convergences and higher degrees of distributivity in lattice ordered groups and in Boolean algebras. Czechoslovak Math. J. *40* (1990), 453–458.
[8] *J. Jakubík*: Sequential convergences in lattices. Math. Bohemica *117* (1992), 239–250.
[9] *P: Mikusiński*: Problems posed at the conference. Proc. Conf. on Convergence, Szczyrk 1979; Katowice 1980, 110–112.
[10] *E. Pap*: Funkcionalna analiza, nizovne konvergenciji, neki principi funkcionalne analize. Novi Sad, 1982.

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