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On the Synthesis of a Class of Interpolating Circuits

MIROSLAV PŘEUČIL

The paper explains the synthesis principles of a class of interpolating circuits which transform the discretely defined signal into the continuous one.

1. INTRODUCTION

The interpolating circuit transforms a signal from the discretly defined form into a continous analogue one. Each interpolator from the class in question consists of several basic blocks. They are as follows: the differential element, the sampler and data hold (S&DH), the forward transfer block $F(z, \varepsilon)$ and the backward transfer block $B(z)$. The fundamental circuit diagram of an interpolator is in Fig. 1. The principle

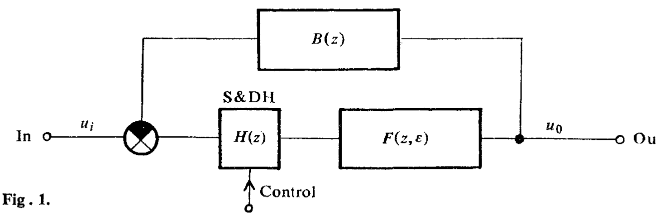

Fig. 1.

of the used sampler and data hold is schematically indicated in Fig. 2. It is the sampler and data hold of the zero order, it means of the simplest possible performance based on the switched invertor-integrator and is governed by controlling impulses coming from outside. Each controlling impulse causes a switching of the contact k. The time duration of these impulses is negligible with regard to their repetition period. Thus, except for negligibly short instant, the data hold works practically during the whole sampling interval (i.e. between two controlling impulses) as an analogue memory element. At first sight it is obvious that the circuit diagram in Fig. 1 is practically identical with a circuit diagram of a sampled control system. Also in discrete time instants the interpolator follows by its output the discrete values inserted into its input. In addition, the output between these discrete values is – according to given law – tracing a curve selected in such a way that it may connect the discrete values

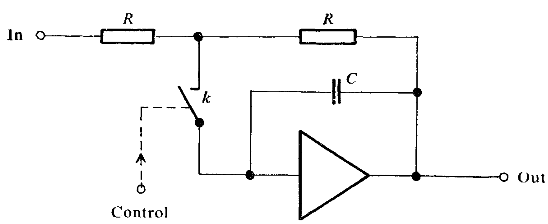

Fig. 2.

of the input function. Thus during the sampling interval the input function interpolated according to the given law. This interpolation law should be selected in such a way, that the result may correspond with our intention and the purpose it serves. For example, the interpolation of function of simple shapes by means of complicated interpolation law is unsuitable owing to a great distortion of the input signal. On the other hand, a simple interpolation law may considerably neglect the details of the more complicated shape of the input function and leads to a distortion again.

An ideal interpolating set would be able to identify exactly the character of the shape of the input function between two discrete instants and according to the need to generate this shape at its output. This interpolation would actually be an interpolation of a function by itself, which means that a parabola would be interpolated by a parabola again and similarly, e.g. a goniometric function by a goniometric function, etc. To execute such an interpolator with a switched generator of the interpolation function according to the arisen need is, of course, impossible. In practice, we are, therefore, mostly limited to a parabolic interpolation, where the output of the interpolator moves between two discrete values according to the polynomial law, which describes the parabola of the m-th order, going through $(m + 1)$ discrete values. The selected interpolation parabola is for the sake of simplicity of the third order at the most. Higher orders of interpolation bring about more complicated circuit diagrams and difficult adjustment as it will be seen later. Let us now neglect the problems connected with the choice of the interpolation law and let us concentrate on the synthesis of a practically utilizable circuit diagram of an interpolator with a parabolic interpolation of the m-th order.

2. THE DERIVING OF THE SHAPE OF THE INTERPOLATION PARABOLA

First we have to determine the form of a polynomial of the m-th order, which is to describe a parabola of the same order going trough $(m + 1)$ given points. These points – as it has been said above – are the discrete values of the input

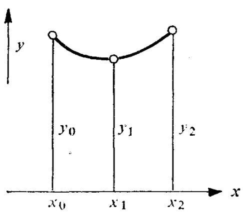

Fig. 3.

function. The situation for the parabola of the second order is illustrated in Fig. 3. We may write

$$(1)\qquad P(x) = \sum_{q=0}^{m} y_q \frac{(x - x_0)(x - x_1)\ldots(x - x_{q-1})(x - x_{q+1})\ldots(x - x_m)}{(x_q - x_0)(x_q - x_1)\ldots(x_q - x_{q-1})(x_q - x_{q+1})\ldots(x_q - x_m)}$$

which represents an expanded form of the Lagrange interpolation polynomial of the m-th order.

The polynomial $P(x)$ can be also written in a closed form

$$(2)\qquad P(x) = \sum_{q=0}^{m} y_q P_q(x) .$$

We have determined the expression describing that the interpolation parabola of the m-th order is going through $(m + 1)$ points. This parabola cannot appear at the output in all sampling intervals bounded by $(m + 1)$ points, but in the one only. This is obvious for the simple reason, that if we shift in time from one discrete point to the following one, the discrete values for creating the interpolation parabola change and it is neccesary to create a new parabola, which passes through those new $(m + 1)$ discrete values. Let us make the following consideration: m is the order of the interpolation (i.e. the order of the interpolation parabola), $(m + 1)$ is the number of the points, which the interpolation parabola is passing through. Let us have a parameter l, which describes the location of the used interval among the $(m + 1)$ given points. Let:

$$(3)\qquad k = q - l .$$

When q is changed now from 0 to m (through $(m + 1)$ values), the k is changed from the value $-l$ through 0 to the value r. The equation

$$r = m - l \tag{4}$$

must apply. The function of the parameter l can be clearly characterized in the following way: it states at every discrete instant how many discrete values before this instant (l) and how many of them after this (r) are determining the shape of the parabola which will be created in the just beginning sampling interval. This interval will be called the working interval. As the device in question is the sampling one, the modified Z-transformation can be used in the following considerations with advantage. For the description of the continuous character of the function between the discrete instants let us use a nondimensional time parameter ε, for which

$$\varepsilon = \frac{t}{\vartheta} \tag{5}$$

applies, ϑ being the time duration of the interval. The transform of the sequence of the functions $u_{in}(\varepsilon)$ may then be written in the Z-transformation as

$$Z[u_{in}(\varepsilon)] = U_i(z, \varepsilon)\,, \tag{6}$$

where the transformation of the actual sequence u_{in} is defined by the relation

$$U_i(z) = \sum_{n=0}^{\infty} f_n z^{-n}\,. \tag{7}$$

In the common use of the Z-transformation the parameter ε lies usually within the interval $0 \leqq \varepsilon < 1$. In our case the modification consists in the validity of the parameter being extended to the interval $-l \leqq \varepsilon < r$ (i.e. through all discrete values which determine the interpolation parabola). From the whole interval of the parameter only the part given by the inequality $0 \leqq \varepsilon < 1$ is actually utilized at the output of the interpolator, as in the original form of the transformation. Hence the independent variable x will be in the following considerations interchanged with the time parameter ε, and the interpolation Lagrange formula can be rewritten into the form

$$\begin{aligned} P(\varepsilon) &= \sum_{q=0}^{m} y_q \frac{(\varepsilon - \varepsilon_0)(\varepsilon - \varepsilon_1)\dots(\varepsilon - \varepsilon_{q-1})(\varepsilon - \varepsilon_{q+1})\dots(\varepsilon - \varepsilon_m)}{(\varepsilon_q - \varepsilon_0)(\varepsilon_q - \varepsilon_1)\dots(\varepsilon_q - \varepsilon_{q-1})(\varepsilon_q - \varepsilon_{q+1})\dots(\varepsilon_q - \varepsilon_m)} = \\ &= \sum_{k=-l}^{m-l} y_k \frac{(\varepsilon + l)(\varepsilon + l - 1)\dots(\varepsilon - k + 1)(\varepsilon - k - 1)\dots(\varepsilon - m + l)}{(k + l)(k + l - 1)\dots(+1)(-1)\dots(k - m + l)}\,. \end{aligned} \tag{8}$$

3. THE RESULTING DISCRETE TRANSFER FUNCTION OF THE INTERPOLATOR

Now let us estimate the form of the resulting discrete overall transfer function of the interpolator on the basis of the considerations we have made so far. The circuit is creating a parabola passing through $(m + 1)$ points. Let us consider the discrete value at a given instant. Since this instant the circuit must wait at least for the next r discrete values in order that the shape of the parabola starting from this discrete value may be determined. Hence it is clear that the interpolator will work with a given fixed time-lag dependent on the order of the interpolation and on the location of the working interval. This time-lag will be formed by r intervals and in the time measure

(9) $$d = r\vartheta = (m - l)\,\vartheta$$

and hence it will appear also as z^{-r} coefficient in discrete overall transfer functions. According to the rule of analytical description of a feedback system it is possible to write for the output of an interpolator

(10) $$U_0(z, \varepsilon) = K(z, \varepsilon)\, U_i(z) = \frac{H(z)\, F(z, \varepsilon)}{1 + F(z)\, B(z)\, H(z)}\, U_i(z) = \\ = E(z)\, F(z, \varepsilon)\, U_i(z)\,,$$

$U_i(z)$ being the transform of the input signal and $E(z)$ the discrete transfer function from the input to the output of the difference element

(11) $$E(z) = \frac{H(z)}{1 + F(z)\, B(z)\, H(z)}\,.$$

Assuming for the sake of simplicity that the transfer function of the used sampler and data hold is negative and equals one then:

$$H(z) = -1\,.$$

Thus

(12) $$E(z) = -\frac{1}{1 - F(z)\, B(z)}\,.$$

Considering the time-lag, the over-all transfer function must have the form

(13) $$K(z, \varepsilon) = -z^{-r} A\; L(z, \varepsilon)\,,$$

where

(14) $$L(z, \varepsilon) = \frac{P(z, \varepsilon)}{U_i(z)}$$

is the actual interpolation transfer function and $P(z, \varepsilon)$ the transform of the Lagrange interpolation formula (8). A is the over-all amplification factor. Comparing equation (13) and (10) we obtain the transfer function of the forward block

$$F(z, \varepsilon) = -z^{-r} A \frac{L(z, \varepsilon)}{E(z)} . \tag{15}$$

4. AN EXAMPLE FOR CUBIC INTERPOLATION

As an example of the use of these statements let us determine a transfer function of an interpolator of the third order (cubic) with the third interval as the working one.

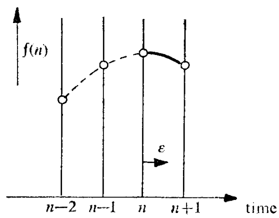

Fig.4.

The creation of the interpolation parabola is clearly illustrated in Fig. 4. At first let us determine the actual interpolation transfer function $L(z, \varepsilon)$ in the relation (14). For the above interpolation $m = 3$, $l = 2$, $r = 1$ applies. Hence

$$\begin{aligned} L(z, \varepsilon) = & -z^{-2} \frac{(\varepsilon + 1)\,\varepsilon(\varepsilon - 1)}{6} + z^{-1} \frac{(\varepsilon + 2)\,\varepsilon(\varepsilon - 1)}{2} - \\ & - z^{0} \frac{(\varepsilon + 2)(\varepsilon + 1)(\varepsilon - 1)}{2} + z^{1} \frac{(\varepsilon + 2)(\varepsilon + 1)\,\varepsilon}{6} = \\ & = z\left[\varepsilon^3\left(-\frac{1}{6z^3} + \frac{1}{2z^2} - \frac{1}{2z} + \frac{1}{6}\right) + \varepsilon^2\left(\frac{1}{2z^2} - \frac{1}{z} + \frac{1}{2}\right) + \right. \\ & \left. + \varepsilon\left(\frac{1}{6z^3} - \frac{1}{z^2} + \frac{1}{2z} + \frac{1}{3}\right) + \frac{1}{z}\right]. \end{aligned} \tag{16}$$

The coefficients in the round brackets can be arranged to give relations consisting of the transforms of the differences up to the m-th order. The we can write

$$\begin{aligned} & L(z, \varepsilon) = \\ = z\left[\varepsilon^3 \tfrac{1}{6}\Delta_3(z) + \varepsilon^2 \left(\tfrac{1}{2}\Delta_3(z) + \tfrac{1}{2}z^{-1}\,\Delta_2(z)\right) + \varepsilon\left(\tfrac{1}{3}\Delta_3(z) + \tfrac{3}{2}z^{-1}\,\Delta_2(z) + z^{-2}\,\Delta_1(z)\right) + \frac{1}{z}\right] \end{aligned} \tag{17}$$

where $\Delta_i(z) = [(z - 1)/z]^i$ is the transform of the i-th difference ($i = 1, 2, 3$) in the Z-transformation. Let us point out that the relation (17) is not the only possible variant of the arrangement of the relation (16). The coefficient at the first power of ε can be for example arranged as follows:

$$\frac{1}{6z^3} - \frac{1}{z^2} + \frac{1}{2z} + \frac{1}{3} = \tfrac{1}{3}\Delta_3(z) + \tfrac{1}{2}z^{-1}\,\Delta_2(z) + z^{-1}\,\Delta_1(z) =$$

$$= -\tfrac{1}{6}\Delta_3(z) + \tfrac{1}{2}\Delta_2(z) + z^{-1}\,\Delta_1(z) = -\tfrac{1}{6}\Delta_3(z) - \tfrac{1}{2}\Delta_2(z) + \Delta_1(z) =$$

$$= \tfrac{1}{3}\Delta_3(z) + \tfrac{3}{2}z^{-1}\,\Delta_2(z) + z^{-2}\,\Delta_1(z)\,.$$

From the next procedure of the synthesis of the over-all transfer function it follows that the only utilizable relation is the last one. The utilizable relation is always characterized by the feature that in every member the sum of the index of the difference (which determines its order) and the negatively taken exponent above z (which determines the value of the shift in every member) is constant and equals the order of the interpolation m.

After a new rearrangement we obtain the final form of the actual interpolation transfer function $L(z, \varepsilon)$:

$$L(z, \varepsilon) = z\,\Delta_3(z)\left[-\varepsilon^3\,\frac{1}{6} - \varepsilon^2\left(\frac{1}{2} + \frac{1}{2}\,\frac{1}{z-1}\right) - \varepsilon\left(\frac{1}{3} + \frac{3}{2}\,\frac{1}{z-1} + \frac{1}{(z-1)^2}\right) - \frac{z^2}{(z-1)^3}\right]. \tag{18}$$

The over-all transfer function of the interpolator will be

$$K(z, \varepsilon) = A\,\Delta_3(z)\left[-\varepsilon^3\,\frac{1}{6} - \varepsilon^2\left(\frac{1}{2} + \frac{1}{2}\,\frac{1}{z-1}\right) - \varepsilon\left(\frac{1}{3} + \frac{3}{2}\,\frac{1}{z-1} + \frac{1}{(z-1)^2}\right) - \frac{z^2}{(z-1)^3}\right]. \tag{19}$$

Using (10) and (15) it is possible to write:

$$E(z) = \Delta_3(z) = \left(\frac{z-1}{z}\right)^3 \tag{20}$$

and

$$F(z, \varepsilon) = -\varepsilon^3\,\frac{1}{6} - \varepsilon^2\left(\frac{1}{2} + \frac{1}{2}\,\frac{1}{z-1}\right) - \varepsilon\left(\frac{1}{3} + \frac{3}{2}\,\frac{1}{z-1} + \frac{1}{(z-1)^2}\right) - \frac{z^2}{(z-1)^3} \tag{21}$$

assuming that $A = 1$.

The main conclusion from the above equations is: The required interpolated output signal is obtained by inserting the m-th difference (in our case $m = 3$) of the discrete input signal into the input of the forward block with the transfer function according to (21). A convenient technique for the derivation of the m-th difference signal will be discussed in Chapter 6. 27

5. THE REALIZATION OF THE FORWARD TRANSFER FUNCTION

One of may many possible ways of realization of the forward transfer function is to determine the discrete transfer function of the circuit from Fig. 5 with undeter-

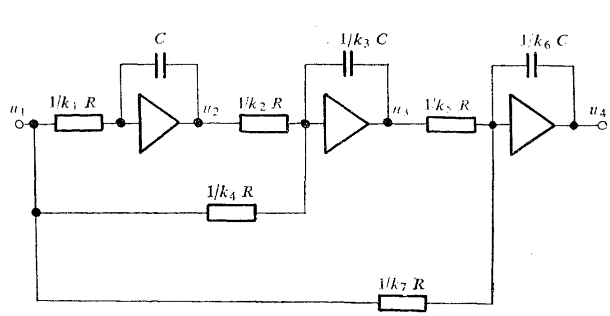

Fig. 5.

mined coefficients k_i. The structure of this circuit is based on the knowledge that the forward block must create a polynomial (parabolic) output signal. The derived transfer function is compared with the required transfer function (21). When comparing the coefficients it is possible to determine the time constants, or better, directly the gain factors of individual integrators. From the discrete point of view the gain factor of an integrator is given by the relation

$$k = \frac{\vartheta}{RC}. \tag{22}$$

This formulation of the gain factor proceeds from the idea that the gain factor equals one, when the time constant equals the sampling interval. In other words, if we bring into the input of this integrator a step-function, then in the time-duration of the interval ϑ the output signal will reach the inverted value of this input step (see Fig. 6). The function of a simple integrator attached behind a forming element of the zero order, which has a contant level during the sampling interval, can be described at discrete instants by the equation

$$u_{2n} = u_{2(n-1)} - k u_{1(n-1)} \tag{23}$$

 or written in the Z-transforms

(24) $$U_2(z) = z^{-1}\, U_2(z) - k\, z^{-1}\, U_1(z)$$

and hence

(25) $$U_2(z) = -\frac{k}{z-1}\, U_1(z)\,.$$

In the similar way it is possible to describe the function the whole circuit in Fig. 5 of course with respect to the fact that for the integrators which are connected with

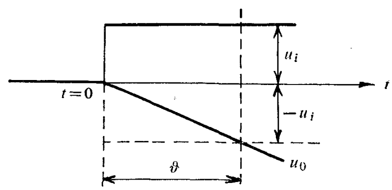

Fig. 6.

the output of the preceding integrators it is neccesary to express the mean value of the signal at this output. The whole procedure is time-consuming. Let us mention here the partial results and the final relation (mean values being marked with stripes). Supposing $RC = \vartheta$

(26) $$\bar{U}_1(z) = U_1(z)\,,$$

(27) $$U_2(z, \varepsilon) = U_2(z) - k_1\varepsilon\, U_1(z)\,,$$

(28) $$U_2(z) = -\frac{k_1}{z-1}\, U_1(z)\,,$$

(29) $$\bar{U}_2(z, \varepsilon) = -U_1(z)\, k_1 \left[\frac{1}{z-1} + \frac{\varepsilon}{2}\right],$$

(30) $$U_3(z, \varepsilon) = U_3(z) - k_1 k_2 \varepsilon\, \bar{U}_2(z, \varepsilon) - k_3 k_4 \varepsilon\, \bar{U}_1(z, \varepsilon) =$$
$$= U_3(z) + U_1(z)\, k_3 \varepsilon \left[k_1 k_2 \left(\frac{1}{z-1} + \frac{\varepsilon}{2}\right) - k_4\right],$$

(31) $$U_3(z) = U_1(z)\, k_3 \left[k_1 k_2 \left(\frac{1}{(z-1)^2} + \frac{1}{2}\frac{1}{z-1}\right) - \frac{k_4}{z-1}\right],$$

(32) $$\bar{U}_3(z, \varepsilon) = U_1(z)\left[a + \tfrac{1}{2}\varepsilon b + \tfrac{1}{6}\varepsilon^2 c\right],$$

where

(33) $$a = k_3 \left[k_1 k_2 \left(\frac{1}{(z-1)^2} + \frac{1}{2} \frac{1}{z-1} \right) - \frac{k_4}{z-1} \right],$$

(34) $$b = k_3 \left[k_1 k_2 \frac{1}{z-1} - k_4 \right],$$

(35) $$c = k_1 k_2 k_3 ,$$

and finally

(36) $$U_4(z, \varepsilon) = -U_1(z) (\alpha + \beta) = U_1(z) F(z, \varepsilon) ,$$

where

(37) $$\alpha = k_5 k_6 \frac{1}{z-1} \left[a + \tfrac{1}{2} b + \tfrac{1}{6} c + 1 \right]$$

and

(38) $$\beta = k_5 k_6 \, \varepsilon \left[a + \tfrac{1}{2} b \varepsilon + \tfrac{1}{6} c \varepsilon^2 + 1 \right] .$$

Assuming

(39) $$\begin{aligned} k_1 k_2 &= 1 , \\ k_3 k_5 &= 1 , \\ k_6 &= 1 \end{aligned}$$

then, after a comparison of the coefficients in (36) and (21), we obtain the values of the gain factors

(40) $$\begin{aligned} k_4 &= -1 , \\ k_7 &= \tfrac{1}{3} . \end{aligned}$$

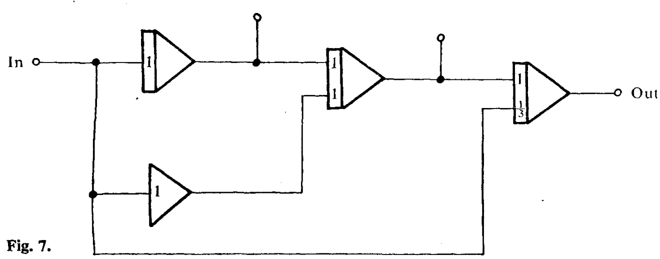

Fig. 7.

In the direct branch from the input to the second integrator an invertor is connected. The resulting circuit diagram is in Fig. 7. Neither this circuit diagram nor the method of obtaining it are the only possible ones. It is possible to select circuits with a smaller number of amplifiers or even with a single one (see [2]). Such circuits are convenient in the cases when a sufficient number of amplifiers is not available or for specialized use. The adjustment of the final exact and mostly unround values of time constants is then too complicated in practice because it is non-autonomous for individual elements, especially for interpolation of higher orders. For this reason we have selected the above method making possible the autonomous adjustment of individual time constants (i.e. gain factors), although the number of the amplifiers has been increased.

6. THE REALIZATION OF THE BACKWARD TRANSFER FUNCTION $B(z)$

Now let us closely examine the formation of the deviation signal, which is an input signal of the forward block. The corresponding transfer function, which forms it, is for our case taken from the relation (20), for other cases of interpolation it will be the m-th difference accordingly. The deviation transfer function $E(z)$ is according to the relation (12) given by the discrete forward transfer function $F(z)$ and the backward transfer function $B(z)$. The forward transfer function $F(z)$ is known, and hence it is possible to determine the backward transfer function $B(z)$.

This consideration, however, is theoretical and it is of importance only for the description of the principle of the circuit. The transfer function $B(z)$ obtained in this way would refer to the circuit diagram in Fig. 1, but could not possibly be executed. In the forward block a time-lag takes place and it follows that for the block $B(z)$ the transfer function would anticipate the present situation. An "anticipating" transfer block is naturally unfeasible. Instead, we shall use individual intermediate transfer functions of the forward block and mix the corresponding signal by means of a linear combinations of signals at the outputs of the individual integrators and of the input signal. We obtain the conditions at the outputs of the integrators of the circuit in Fig. 7 by inserting the corresponding gain factors for our case into the relations (28), (31) and (36). It is necessary to insert $\varepsilon = 0$ into the last equation in order to get the output signal at the discrete instants.

$$U_2(z) = -\frac{1}{z}\,\Delta_2(z)\,U(z)\,, \tag{41}$$

$$U_3(z) = \left[\frac{1}{z^2}\,\Delta_1(z) + \frac{3}{2}\,\frac{1}{z}\,\Delta_2(z)\right]U(z)\,, \tag{42}$$

$$U_4(z) = -\frac{1}{z}\,U(z)\,. \tag{43}$$

Thus for the synthesis of the third difference at the output of the forming element $(H(z) = -1)$ we can write

$$(44) \quad \frac{(z-1)^3}{z^3} = c_1 - c_2 \frac{1}{2} \frac{(z-1)^2}{z^2} + c_3 \frac{1}{z^2} \frac{z-1}{z} + c_3 \frac{3}{2} \frac{1}{z} \frac{(z-1)^2}{z^2} - c_4 \frac{1}{z} .$$

We assume the use of an amplifier as a differential element. The individual coefficients equal the gain factors of this amplifier from the corresponding inputs

$$(45) \qquad c_i = \frac{R}{R_i} .$$

After comparing the coefficients at indentical powers of z we shall obtain individual gain factors

$$(46) \qquad 1 = c_4 A \quad \text{where} \quad A = \frac{R_4}{R_1} .$$

If

$$A = 1 \quad \text{then} \quad c_4 = 1 .$$

Further:

$$c_3 = -1 \quad \text{and} \quad c_2 = \tfrac{1}{2} .$$

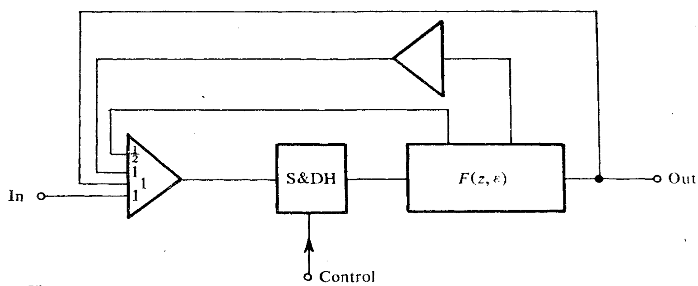

Fig. 8.

To the branch from the output of the second integrator an invertor is connected. The final circuit diagram of the whole interpolator together with the feedback branches is illustrated in Fig. 8. R and C are normalized values of resistance and capacity to which $RC = \vartheta$ applies.

By means of a similar procedure it is possible to build interpolating circuits with an arbitrary order of interpolation and location of the working interval. Let us illustrate this statement with the following circuit diagrams from the simplest to the most complicated ones (it is assumed that $H(z) = 1$ and $A = 1$):

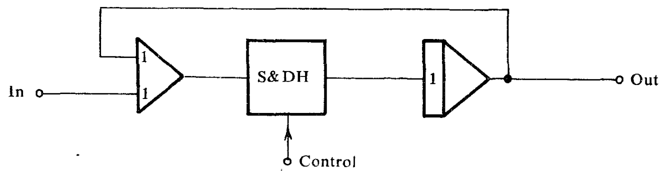

Fig. 9.

a) *Linear interpolator* (Fig. 9), $m = 1, l = 0$

$$K(z, \varepsilon) = -\Delta_1(z)\left[\varepsilon + \frac{1}{z-1}\right]. \tag{47}$$

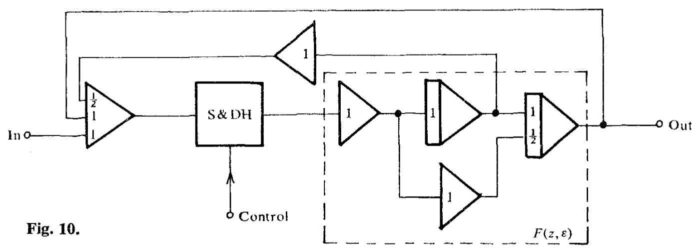

Fig. 10.

b) *Quadratic interpolator* (Fig. 10), $m = 2$ with the second interval as the working one, i.e. $l = 1$

$$K(z, \varepsilon) = -\Delta_2(z)\left[\varepsilon^2 \frac{1}{2} + \varepsilon\left(\frac{1}{2} + \frac{1}{z-1}\right) + \frac{z}{(z-1)^2}\right]. \tag{48}$$

c) *Quadratic interpolator* (Fig. 11), $m = 2$ with the first interval as the working one, i.e. $l = 0$.

$$K(z, \varepsilon) = -\Delta_2(z)\left[\varepsilon^2 \frac{1}{2} + \varepsilon\left(-\frac{1}{2} + \frac{1}{z-1}\right) + \frac{1}{(z-1)^2}\right]. \tag{49}$$

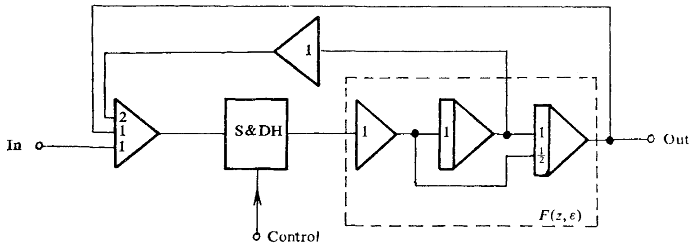

Fig. 11.

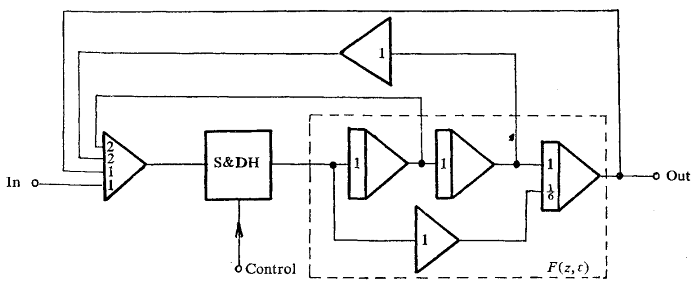

Fig. 12.

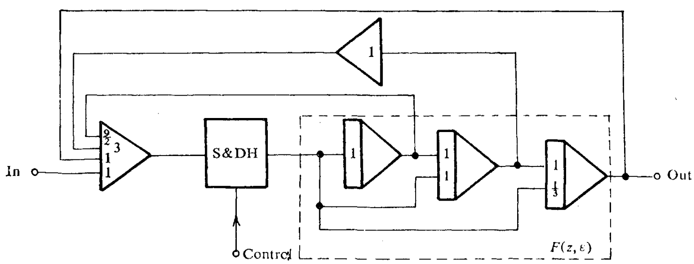

Fig. 13.

d) *Cubic interpolator* (Fig. 12), $m = 3$ with the second interval as the working one, i.e. $l = 1$

$$K(z, \varepsilon) = \tag{50}$$

$$= -\Delta_3(z)\left[\varepsilon^3 \frac{1}{6} + \varepsilon^2 \frac{1}{2}\frac{1}{z-1} + \varepsilon\left(-\frac{1}{6} + \frac{1}{2}\frac{1}{z-1} + \frac{1}{(z-1)^2}\right) - \frac{z}{(z-1)^3}\right].$$

e) *Cubic interpolator* (Fig. 13), $m = 3$ with the first interval as the working one, i.e. $l = 0$

$$K(z, \varepsilon) = -\Delta_3(z)\left[\varepsilon^3 \frac{1}{6} + \varepsilon^2\left(-\frac{1}{2} + \frac{1}{2}\frac{1}{z-1}\right) + \right. \tag{51}$$

$$\left. + \varepsilon\left(\frac{1}{3} - \frac{1}{2}\frac{1}{z-1} + \frac{1}{(z-1)^2}\right) + \frac{1}{(z-1)^3}\right].$$

All the circuit diagrams are rather simple and need not be commented. Let us mention only the fact that with every circuit a fixed time-lag is linked, given by the relation (9). This time-lag varies according to the order of the interpolation and to the location of the working interval.

8. SOME LIMITATIONS AND DISADVANTAGES OF THE METHOD

We should not omit to mention one serious obstacle in order to raise the order of the interpolation above a certain level. As it is clear from the relations (47) to (51) it is always necessary to bring into the input of the forward block the corresponding difference of the input signal, i.e. the difference of the order corresponding to the order of the interpolation. No obstacle arises in the case of interpolation of the input signal with gradual changes. Difficulties arise when interpolating the input signal subject to a great change in magnitute during the sampling interval (for example from one limit to the other). The very first difference of such a step-signal will exceed the magnitude of the permitted signal at the differential amplifier and at the amplifier of the forming element. The higher the order of the difference the greater the magnitute of the oscillations of the signal at these amplifiers (see Fig. 14). Hence, it is necessary to limit the magnitude of the input signal. After such a modification the circuit is suitable for a signal with great changes but some functional drawbacks of the data hold arise and accuracy falls accordingly. This situation is not improved even by an apparently more convenient distribution of the gain than that in our example (lowering the gain in the deviation transfer function and increasing the gain in the forward block). This main reason, besides others, compelled us to confine ourselves to the cubic interpolation as the highest one. Another disadvantage is the long duration of the transient at the output when step-function is brought into the input, etc. But it concerns rather the selection of a certain interpolation type according to many criteria and lies outside the scope of this paper which deals only with the synthesis of the circuits according to given conditions. A scope of some experience and practical results in modeling of interpolating circuits by means of an analogue computer is given in [1].

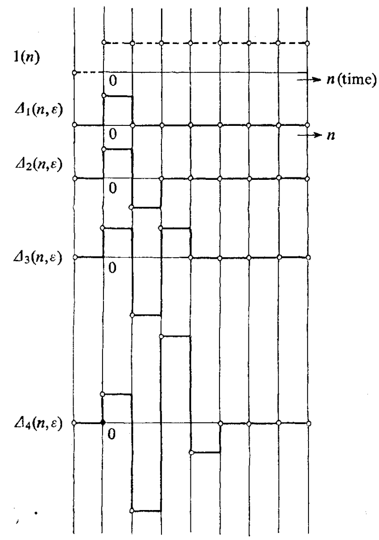

Fig. 14.

9. CONCLUSION

The paper explains the principle of the interpolating circuit which transforms the discretely defined signal into the continuous one. Conditions are derived for the creation of an interpolation parabola which is passing through $(m + 1)$ discrete points if m is the order of the interpolation. By means of the parameter l it is possible to select the location of the working interval which the parabole appears in. The example of the cubic interpolation with the third interval as the working one demonstrates the synthesis of the resulting transfer function, the forward block transfer function and the backward block transfer function. These transfer function can be modelled by an analogue computer. The conditions limiting the order of the interpolation in these interpolating circuits are given. Finally we find transfer functions and circuit diagrams for modeling the interpolators of the first, second and third order.

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VÝTAH

K syntéze jedné třídy interpolačních obvodů

Miroslav Přeučil

Práce vysvětluje činnost interpolačního zařízení, které přeměňuje diskrétně definovaný signál na spojitý. Jsou odvozeny podmínky pro vytváření interpolační paraboly, která je prokládána $(m + 1)$ diskrétními body, je-li m stupeň interpolace. Volbou parametru l lze zvolit umístění pracovního intervalu, ve kterém se prokládaná parabola uplatní. Na příkladu kubické interpolace s třetím intervalem pracovním je demonstrována syntéza výsledného přenosu, přenosu dopředného členu a zpětnovazebního přenosu. Tyto přenosy jsou realizovány poměrně jednoduchým zapojením, které je možno vytvořit na universálním analogovém počítači. Jsou vysvětleny okolnosti omezující zvyšování stupně interpolace v tomto zapojení. Na závěr jsou uvedeny přenosy a zapojení je modelující pro interpolaci prvního, druhého a třetího stupně.

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