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ON VARIOUS DYNAMIC COMPENSATIONS

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The aim of this modest note is to show that certain compensation schemes frequently used in control theory, namely regular output feedback, combined dynamic compensation, and dynamic precompensation, are input-output equivalent.

1. INTRODUCTION

Let $R^{p\times q}$ and $R^{p\times q}\{w\}$ be respectively the sets of scalar and proper rational $p \times q$ matrices in one indeterminate w over the real field R. The units of the rings $R^{n\times n}$ and $R^{n\times n}\{w\}$ are respectively the non-singular and bi-proper matrices. Let us recall that an $H(w)$ is a unit of $R^{n\times n}\{w\}$ if and only if $H(0)$ is a unit of $R^{n\times n}$. That is to say, a rational matrix is bi-proper if it is proper together with its inverse. As usual, I will denote the identity matrix.

Consider a system Σ

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{1}$$

described by the quadruple of matrices $A \in R^{n\times n}$, $B \in R^{n\times q}$, $C \in R^{p\times n}$ and $D \in R^{p\times q}$ which gives rise to the transfer matrix

$$T(w) = D + wC(I - wA)^{-1} B \in R^{p\times q}\{w\} .$$

Thus w is the inverse differential operator.

Further let

$$\dot{x}_d = u_d \tag{2}$$

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be a dynamic extension of Σ, a bunch of n_d integrators, and let v denote a command signal, say r-dimensional one.

Consider the problem od modifying $T(w)$ by means of a control law. The following three control laws are frequently used in the literature.

1. Regular Output Feedback

(3) $$\begin{bmatrix} u \\ u_d \end{bmatrix} = F \begin{bmatrix} y \\ x_d \end{bmatrix} + Gv$$

(4) $$\begin{cases} F = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \in R^{(q+n_d)\times(p+n_d)} \\ G = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \in R^{(q+n_d)\times r} \end{cases}$$

and

(5) $$I - F_{11}D = \text{unit of } R^{q \times q} .$$

This static control law acting on the extended system (1)–(2) is often used to study the dynamic output feedback for the original system (1), see e.g. Wonham [4]. The regularity condition (5) was introduced by Descusse and Malabre [1] in order to prevent the derivatives of v to appear in x. The class of control laws (3) satisfying (4)–(5) will be denoted by $\mathbb{C}_1(\Sigma)$.

2. Combined Dynamic Compensation

(6) $$u = P(w)\, y + Q(w)\, v$$

where

(7) $$P(w) \in R^{q\times p}\{w\} , \quad Q(w) \in R^{q\times r}(w)$$

and

(8) $$I - P(w)\, T(w) = \text{unit of } R^{q\times q}\{w\} .$$

This is a general compensation scheme which makes explicit the presence of a feed-forward and a feedback in the control law. It has been found useful in the polynomial equation approach, see e.g. Kučera [2]. In most cases, the regularity condition (8) is tacitly assumed. The class of control laws (6) satisfying (7)–(8) will be denoted by $\mathbb{C}_2(\Sigma)$.

3. Dynamic Precompensation

(9) $$u = K(w)\, v$$

where

(10) $$K(w) \in R^{q\times r}\{w\} .$$

This is a standard way of introducing a compensator when the transfer-function approach is adopted. The scheme is flexible and can be used to represent a combined compensation or a dynamic feedback alone, see e.g. Wolovich [3]. The class of control laws (9) which satisfy (10) will be denoted by $\mathbb{C}_3(\Sigma)$.

We shall say that two classes are *input-output equivqlent* if, for any control law of one class, we can find a control law in the other class such that their application to a given system will result in overall systems having the same transfer matrices.

This kind of equivalence reflects just the ability of two control laws to produce the same input-output behaviour. In particular, this equivalence says nothing about dynamical order, stability, sensitivity or other properties of control systems which depend on a particular realization. Nevertheless, this concept is useful when various feedforward/feedback configurations are studied from the input-output point of view. This is the case, for example, when solving the disturbance rejection, exact model matching or model following problems in dynamical systems.

2 .RESULT

The aim of this note is to show that the three control laws are input-output equivalent. This would not be surprising if the control laws were unrestricted by (5), (8) and (10). What is less obvious is that the *regularity* condition (5) or (8) is equivalent to the *properness* condition (10).

Claim. The classes $\mathbb{C}_1(\Sigma)$, $\mathbb{C}_2(\Sigma)$ and $\mathbb{C}_3(\Sigma)$ are input-output equivalent.

Proof. The easiest way is to establish the chain of implications

$$\mathbb{C}_1(\Sigma) \Rightarrow \mathbb{C}_2(\Sigma) \Rightarrow \mathbb{C}_3(\Sigma) \Rightarrow \mathbb{C}_1(\Sigma) .$$

a) To show that each element of $\mathbb{C}_1(\Sigma)$ can be realized as an element of $\mathbb{C}_2(\Sigma)$, consider a control law defined by (3)–(5). Using (1)–(3), calculate the transfer matrices from y an v to u. On identifying with (6), we obtain

$$P(w) = F_{11} + wF_{12}(I - wF_{22})^{-1} F_{21} \tag{11}$$

$$Q(w) = G_1 + wF_{12}(I - wF_{22})^{-1} G_2 .$$

Since $I - wF_{22}$ is bi-proper, both $P(w)$ and $Q(w)$ are proper. Moreover

$$I - P(0)\,T(0) = I - F_{11}D$$

is non-singular whence (8) holds. The control law (11) thus belongs to $\mathbb{C}_2(\Sigma)$.

b) To show that each element of $\mathbb{C}_2(\Sigma)$ can be realized as an element of $\mathbb{C}_3(\Sigma)$, consider a control law defined by (6)–(8). Calculate the transfer matrix from v to u and compare it with (9) to obtain

$$K(w) = [I - P(w)\,T(w)]^{-1}\,Q(w) . \tag{12}$$

The properties (7) – (8) then imply (10); hence the control law defined by (12) belongs to $\mathbb{C}_3(\Sigma)$.

c) Finally, let us show that each element of $\mathbb{C}_3(\Sigma)$ can be realized as an element of $\mathbb{C}_1(\Sigma)$. Given any proper $K(w)$, let

$$K(w) = D_0 + wC_0(I - wA_0)^{-1} B_0$$

for some realization (A_0, B_0, C_0, D_0). Then

$$F_{11} = 0 \quad F_{12} = C_0 \quad G_1 = D_0 \tag{13}$$

$$F_{21} = 0 \quad F_{22} = A_0 \quad G_2 = B_0$$

defines a control law of the form (3). Moreover, $I - F_{11}D$ is the identity. Hence the control law (13) belongs to $\mathbb{C}_1(\Sigma)$. □

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