Kybernetika

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Kybernetika, Vol. 11 (1975), No. 6, (391)--397

Persistent URL: `http://dml.cz/dmlcz/125730`

KYBERNETIKA — VOLUME 11 (1975), NUMBER 6

On G-machines Generating Intersection and Union of Generable Languages

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The article deals with the construction of G-machine generating to given two G-machines the union (if they satisfy certain necessary and sufficient conditions) and the intersection of their languages.

1. INTRODUCTION

The notion of a G-machine was introduced in [2] as a certain generalization of machines studied in [1] and [4]. A generable language is the set of all "words" generated by a G-machine (in the mentioned references "generable set" instead of "generable language" is used). The class of generable languages is closed under intersection, but not generally under union (see [3]). We shall deal with the construction of G-machines generating to given two G-machines the intersection and the union of their languages.

2. PRELIMINARIES

2.1. Denotation. $T = \{1, 2, \ldots\}$, $\bar{T} = \{0, 1, 2, \ldots\}$, $T_n = \{1, 2, \ldots, n\}$, $\bar{T}_n = \{0, 1, 2, \ldots, n\}$.

2.2. Let I be a finite set (including the empty set). Denote I^∞ the set of all nonvoid sequences of elements of I. These sequences are called *words*. For $w \in I^\infty$, $m \in T$, $w = (s_0, s_1, \ldots, s_{m-1})$ put $l(w) = m$. For $w \in I^\infty$, $w = (s_0, s_1, \ldots)$ put $l(w) = \infty$. The symbol $l(w)$ is called *the length of* w. Instead of $w = (s_0, s_1, \ldots, s_{m-1})$ and $w = (s_0, s_1, \ldots)$ we write $w = s_0 s_1 \ldots s_{m-1}$ and $w = s_0 s_1 \ldots$ or $w = \prod_{i=0}^{m-1} s_i$ and $w = \prod_{i=0}^{\infty} s_i$ respectively. Considering a word of finite or infinite length we use the denotation

$\prod_i s_i$. For $k \in T$ by the symbol $(s_0 s_1 \dots s_{m-1})^k$ we understand the word $s_0 s_1 \dots s_m s_{m+1} \dots$ $\dots s_{2m} s_{2m+1} \dots s_{km-1}$, where $s_{im+j} = s_j$ for all $i \in \overline{T}_{k-1}$ and all $j \in \overline{T}_{m-1}$. Further, by the symbol $(s_0 s_1 \dots s_{m-1})^\infty$ we understand the word $s_0 s_1 \dots s_m s_{m+1} \dots s_{nm} s_{nm+1} \dots$, where $s_{nm+j} = s_j$ for all $j \in \overline{T}_{m-1}$ and all $n \in \overline{T}$. For $m = 1$ we omit the brackets and write s_0^k, s_0^∞.

2.3. Convention. In the relation $C \subseteq I^\infty$ we suppose that every element from I is included in at least one sequence from C.

2.4. *A G-machine* is a triple $M = (S, I, \delta)$, where S is a nonvoid finite set, $I \subset S(I \neq S)$, δ is a mapping of I into the set of all nonvoid subsets of S. In the following, M is to be understood as G-machine $M = (S, I, \delta)$. Let $m \in T$. A word $\prod_{i=0}^{m-1} s_i$ or $\prod_{i=0}^{\infty} s_i$ is called *an output word* of the length m or ∞ respectively, if $s_0 \in I$, $s_{i+1} \in \delta(s_i) \cap I$ for all $i \in \overline{T}_{m-2}$ or for all $i \in \overline{T}$. An output word $w = \prod_i s_i$ is called *a word generated by M* if either $l(w) = \infty$ or $l(w) = m$ and there exists $v \in \delta(s_{m-1}) \cap (S - I)$. To distinguish that $\prod_i s_i$ is an output word of G-machine M we use the denotation $\prod_i s_i(\delta)$. The set of all words generated by M is denoted $L(M)$ and called *the language of M*. A set C, $C \subseteq I^\infty$ is called *a generable language* if there exists M such that $C = L(M)$.

2.5. A pair (s, v) is called *productive* if $s \in I$ and $v \in \delta(s) \cap I$ and *unproductive* if $s \in I$ and $v \in \delta(s) \cap (S - I)$. Denote P_δ the set of all productive pairs and N_δ the set of all unproductive pairs. Then $\delta = P_\delta \cup N_\delta$. For every $s \in I$ put $N^s = \{(s, v) \mid (s, v) \in N_\delta\}$. Choose from every $N_\delta \neq \emptyset$ an arbitrary (s, v^s) (a representative) and put $N_\delta^R = \bigcup_{s \in I,\, N^s \neq \emptyset} (s, v^s)$ and $\delta^R = P_\delta \cup N_\delta^R$. G-machine $M^R = (S, I, \delta^R)$ is said to be *result of reduction of M*.

2.6. G-machines $M_1 = (S_1, I_1, \delta_1)$ and $M_2 = (S_2, I_2, \delta_2)$ are said to be *equivalent* if $L(M_1) = L(M_2)$. Then we write $M_1 \sim M_2$.

2.7. Let I be a finite set, $C \subseteq I^\infty$ and $I \subset S(I \neq S)$, where S is a nonvoid finite set. Suppose $I \neq \emptyset$ (then according to Convention 2.3 $C \neq \emptyset$). Denote c an element (sequence) from C and by s_i the $(i + 1)$-th element of c, $c = \prod_i s_i$ for all $i \in \overline{T}_{m-1}$ if $l(c) = m \in T$ and for all $i \in \overline{T}$ if $l(c) = \infty$. For $c \in C$, $c = \prod_{i=0}^{m-1} s_i(m \in T)$ put $P(c) = \bigcup_k (s_k, s_{k+1})$ for all $k \in \overline{T}_{m-2}$ and $N(c) = (s_{m-1}, v)$, where v is an arbitrary element of $(S - I)$. For $c \in C$, $c = \prod_{i=0}^{\infty} s_i$ put $P(c) = \bigcup_k (s_k, s_{k+1})$ for all $k \in \overline{T}$. Denote $P = \bigcup_{c \in C} P(c)$, $N = \bigcup_{c \in C} N(c)$, $\delta[C] = P \cup N$. If $I = \emptyset$ put $\delta[C] = \emptyset$. Define G-machine $M[C] = (S, I, \delta[C])$.

3. G-MACHINES M_P AND M_U

3.1. Proposition. *$M \sim M^R$.*
(See [2], Corollary 2.)

3.2. Proposition. *Let I be a finite set, $C \subseteq I^\infty$. C is a generable language iff $C = L(M[C])$.*
(See [2], Theorem 6 and Corollary 3.)

3.3. Proposition. *Let $M_1 = (S_1, I_1, \delta_1)$, $M_2 = (S_2, I_2, \delta_2)$ be G-machines. Then the following statements (A), (B) are equivalent:*
(A) *$P_{\delta_1} = P_{\delta_2}$ and there exists $(s, v^1) \in N_{\delta_1}$ iff there exists $(s, v^2) \in N_{\delta_2}$.*
(B) *$M_1 \sim M_2$.*
(See [2], Corollary 5.).

3.4. Definition. Let $M_1 = (S_1, I_1, \delta_1)$, $M_2 = (S_2, I_2, \delta_2)$ be G-machines. Put

(1) $P_1 = \{(s, v) \mid$ there exist $\prod_{j=0}^{\infty} s_j$ and $i \in \overline{T}$ such that

$$\prod_{j=0}^{\infty} s_j(\delta_1) = \prod_{j=0}^{\infty} s_j(\delta_2) \quad \text{and} \quad s_i = s\,, \quad s_{i+1} = v\}\,.$$

(2) $P_2 = \{(s, v) \mid$ there exist indices $i, n \in \overline{T}$, $n > i + 1$, states $v^1 \in S_1$, $v^2 \in S_2$ and an output word $\prod_{j=0}^{n-1} s_j$ with $s_0 s_1 \dots s_i s_{i+1} \dots s_{n-1}(\delta_1)$, $s_0 s_1 \dots s_i s_{i+1} \dots s_{n-1}(\delta_2)$ where $s_i = s$, $s_{i+1} = v$ and $(s_{n-1}, v^1) \in N_{\delta_1}$, $(s_{n-1}, v^2) \in N_{\delta_2}\}$;

(3) $N' = \{(s, v^i) \mid (s, v^i) \in N_{\delta_i}$ and there exists $(s, v^j) \in N_{\delta_j}$ for all $i, j \in \{1, 2\}$, $i \neq j\}$;

(4) $\delta_P = P_1 \cup P_2 \cup N'$, $P_{\delta_P} = P_1 \cup P_2$, $N_{\delta_P} = N'$;

(5) $S_P \supseteq S$, where $S = \{s \mid$ there exists $(s, t) \in \delta_P\} \cup \{t \mid$ there exists $(s, t) \in \delta_P\}$, S_P is a nonvoid finite set;

(6) $I_P = \{s \mid$ there exists $(s, t) \in \delta_P\} \cup \{t \mid (s, t) \in P_{\delta_P}\}$.
Define G-machine $M_P = (S_P, I_P, \delta_P)$.

3.5. Theorem. *Let $M_1 = (S_1, I_1, \delta_1)$, $M_2 = (S_2, I_2, \delta_2)$ be G-machines, $C = L(M_2) \cap L(M_2)$. Then $L(M_P) = L(M[C])$.*

Proof. Suppose $P_{\delta_P} \neq \emptyset$, $(s, v) \in P_{\delta_P}$. By Definition 3.4 $(s, v) \in P_m$ for some $m \in \{1, 2\}$. First assume $(s, v) \in P_1$. From 2.4 and (1) of Definition 3.4 it follows there exists a word $w = \prod_{j=0}^{\infty} s_j$ which belongs to $L(M_1)$ and $L(M_2)$, thus $w \in C$. By 2.7 $(s, v) \in P(w)$ and $(s, v) \in P_{\delta[C]}$. Second let $(s, v) \in P_2$. By Definition 3.4 there exist $v^1 \in S_1$, $v^2 \in S_2$ and an output word w of the form given by (2). Using 2.4 $w = s_0 s_1 \ldots$ $\ldots s_{n-1} \in L(M_m)$ for all $m \in \{1, 2\}$, thus $w \in C$. By 2.7 $(s, v) \in P(w)$, $(s, v) \in P_{\delta[C]}$. Hence the inclusion

$$P_{\delta_P} \subseteq P_{\delta[C]} \tag{7}$$

holds true. Now suppose $(s, v) \in P_{\delta[C]}$. By 2.7 there exist $c \in C$ and $i \in \bar{T}$ such that $(s_i, s_{i+1}) \in P(c)$, $s_i = s$, $s_{i+1} = v$. Since $c \in L(M_1) \cap L(M_2)$ then there holds $(s_i, s_{i+1}) \in P_{\delta_m}$ for all $m \in \{1, 2\}$. First consider $c = \prod_{j=0}^{\infty} s_j$. Then from (2) and (4) of Definition 3.4 it follows immediately $(s_i, s_{i+1}) \in P_1$, $(s_i, s_{i+1}) \in P_{\delta_P}$. Second let $c = \prod_{j=0}^{n-1} s_j (n \in T)$. Since $c \in L(M_1) \cap L(M_2)$ from 2.4 it follows $s_0 s_1 \ldots s_i s_{i+1} \ldots$ $\ldots s_{n-1}(\delta_1) = s_0 s_1 \ldots s_i s_{i+1} \ldots s_{n-1}(\delta_2)$ and there exist $(s_{n-1}, v^1) \in N_{\delta_1}$, $(s_{n-1}, v^2) \in$ $\in N_{\delta_2}$. By Definition 3.4 $(s_i, s_{i+1}) \in P_2$, $(s_i, s_{i+1}) \in P_{\delta_P}$ and therefore $P_{\delta[C]} \subseteq P_{\delta_P}$. Using (7) we obtain

$$P_{\delta_P} = P_{\delta[C]}\,. \tag{8}$$

Further, suppose $N_{\delta_P} \neq \emptyset$, $(s, v) \in N_{\delta_P}$. By 2.4 and (3) there exists a word $w = s_0 \in$ $\in L(M_j)$, where $s_0 = s$ for all $m \in \{1, 2\}$. From here $c = s_0 \in L(M_1) \cap L(M_2)$ and by 2.7 there exists $v' \in N_{\delta[C]}$ such that for $s = s_0$ $(s, v') \in N(c)$ holds, thus $(s, v') \in$ $\in N_{\delta[C]}$. Hence the implication

$$\text{if } (s, v) \in N_{\delta_P} \quad \text{then there exists} \quad (s, v') \in N_{\delta[C]} \tag{9}$$

holds true. Now suppose $(s, v') \in N_{\delta[C]}$. By 2.4 and 2.7 there exist a word $c = s_0 \in C =$ $= L(M_1) \cap L(M_2)$, where $s_0 = s$ and $v^1 \in (S_1 - I_1)$, $v^2 \in (S_2 - I_2)$ such that $(s, v^1) \in$ $\in N_{\delta_1}$, $(s, v^2) \in N_{\delta_2}$. From (3) it follows $(s, v^1) \in N'$, $(s, v^1) \in N_{\delta_P}$ and therefore the implication

$$\text{if} \quad (s, v') \in N_{\delta[C]} \quad \text{then there exists} \quad (s, v) \in N_{\delta_P}\,, \tag{10}$$

where $v = v^1$ holds. By (8), (9), (10) and (A) of Proposition 3.3 we obtain $L(M_P) =$ $= L(M[C])$.

3.6. Corollary. *Let* $M_1 = (S_1, I_1, \delta_1)$, $M_2 = (S_2, I_2, \delta_2)$ *be G-machines,* $C = L(M_1) \cap L(M_2)$. *Then* $M_P \sim M[C] \sim M_P^R$.

3.7. Theorem. *Let* $M_1 = (S_1, I_1, \delta_1)$, $M_2 = (S_2, I_2, \delta_2)$ *be G-machines,* $C = L(M_1) \cap L(M_2)$. *Then* $C = L(M[C]) = L(M_P) = L(M_P^R)$.

Proof. Since $C = L(M_1) \cap L(M_2)$ is a generable language (see [3]) then by Proposition 3.2 $C = L(M[C])$ and the proof is completed.

3.8. Example. Using Definition 3.4 we shall construct to given G-machines M_1, M_2 the G-machine M_P, for which $L(M_P) = L(M_1) \cap L(M_2)$. G--machines $M_1 = (S_1, I_1, \delta_1)$, $M_2 = (S_2, I_2, \delta_2)$ are given as follows: $S_1 = \{a, b, c, x\}$, $I_1 = \{a, b, c\}$, $\delta_1 : [a \to \{a, x\},\ b \to \{a, b\},\ c \to \{c, x\}]$, $S_2 = \{a, b, y\}$, $I_2 = \{a, b\}$, $\delta_2 : [a \to \{y\},\ b \to \{a, b\},\ c \to \{b\}]$. Since $s_0 s_1 \ldots s_{n-1}(\delta_1) = s_0 s_1 \ldots s_{n-1}(\delta_2)$, where $s_j = b$ for every $n \in (T - \{1\})$ and $j \in \overline{T}_{n-1}$ then by (1) $(b, b) \in P_1$. Further, $ba(\delta_1) = ba(\delta_2)$, $(a, x) \in N_{\delta_1}$, $(a, y) \in N_{\delta_2}$, thus by (2) $(b, a) \in P_2$. The pairs (a, a), (c, c), (c, b) obviously do not belong to P_j for any $j \in \{1, 2\}$. Further, $(a, x) \in N_{\delta_1}$, $(a, y) \in N_{\delta_2}$ and by (3) $(a, x) \in N'$, $(a, y) \in N'$. Hence $\delta_P : [a \to \{x, y\},\ b \to \{a, b\}]$, $S_P = \{a, b, x, y\}$, $I_P = \{a, b\}$, $M_P = (S_P, I_P, \delta_P)$. By 2.5 $M_P^R = (S_P, I_P, \delta_P^R)$, where $\delta_P^R : [a \to \{x\},\ b \to \{a, b\}]$. Apparently $L(M_P^R) = \{b^\infty, b^k a, a | k \in T\} = L(M_1) \cap L(M_2)$.

3.9. Definition. Let $M_1 = (S_1, I_1, \delta_1)$, $M_2 = (S_2, I_2, \delta_2)$ be G-machines. Define G-machine $M_U = (S_U, I_U, \delta_U)$, where $S_U = S_1 \cup S_2$, $I_U = I_1 \cup I_2$, $\delta_U = \delta_1 \cup \delta_2$.

3.10. Theorem. *Let* $M_1 = (S_1, I_1, \delta_1)$, $M_2 = (S_2, I_2, \delta_2)$ *be G-machines,* $C = L(M_1) \cup L(M_2)$. *Then* $L(M_U) = L(M[C])$.

Proof. Suppose $(s, t) \in P_{\delta_U}$. Obviously $(s, t) \in (P_{\delta_1} \cup P_{\delta_2})$. There exists a word $w \in L(M_U)$ beginning with the output word $s_0 s_1(\delta_U)$, where $s_0 = s$, $s_1 = t$ (see [2], Corollary 1). By 2.7 $(s_0, s_1) \in P(w)$, thus $(s_0, s_1) \in P_{\delta[C]}$, $(s, t) \in P_{\delta[C]}$. Herefrom it follows

$$P_{\delta_U} \subseteq P_{\delta[C]} \,. \tag{11}$$

Now assume $(s, t) \in P_{\delta[C]}$. By 2.7 there exists a word $c \in C$ such that $(s, t) \in P(c)$. Since $C = L(M_1) \cup L(M_2)$ it must hold $(s, t) \in P_{\delta_j}$ at least for one $j \in \{1, 2\}$, therefore $(s, t) \in P_{\delta_U}$ and $P_{\delta[C]} \subseteq P_{\delta_U}$. Using (11) we obtain

$$P_{\delta_U} = P_{\delta[C]} \,. \tag{12}$$

Let $(s, t) \in N_{\delta_U}$. By 2.4 $w = s_0 = s \in L(M_U)$. Apparently $(s, t) \in N_{\delta_j}$ at least for one $j \in \{1, 2\}$. From 2.7 it follows there exists $(s, t') \in N(w)$, thus $(s, t') \in N_{\delta[C]}$ and the implication

$$\text{if} \quad (s, t) \in N_{\delta_U} \quad \text{then there exists} \quad (s, t') \in N_{\delta[C]} \tag{13}$$

holds true. Now suppose $(s, z) \in N_{\delta[C]}$. Then there exists $c \in L(M[C])$ such that $(s, z) \in N(c)$. Since $c \in L(M_1) \cup L(M_2)$, then $c \in L(M_j)$ and there exists (s, z^j) at least for one $j \in \{1, 2\}$. Hence the implication

$$\text{(14)} \qquad \text{if} \quad (s, t') \in N_{\delta[C]} \quad \text{then there exists} \quad (s, t) \in N_{\delta_U},$$

where $t' = z$, $t = z^j$ is satisfied. From (12), (13), (14) it follows the condition (A) of Proposition 3.3 is fulfilled, hence $M_U \sim M[C]$ and $L(M_U) = L(M[C])$.

3.11. Theorem. *Let $M_1 = (S_1, I_1, \delta_1)$, $M_2 = (S_2, I_2, \delta_2)$ be G-machines and let $C = L(M_1) \cup L(M_2)$ be a generable language. Then $C = L(M[C]) = L(M_U) = L(M_U^R)$.*

Proof. The statement is the consequence of Propositions 3.1, 3.2 and Theorem 3.10.

3.12. Proposition. *Let $M_1 = (S_1, I_1, \delta_1)$, $M_2 = (S_2, I_2, \delta_2)$ be G-machines. Then the following statements* (A), (B) *are equivalent:*

(A) *For every $i, j \in \{1, 2\}$, $i \neq j$ and for every $n \in T$*

(A′) *if $s_0 s_1 \dots s_{n-1}(\delta_j)$ and $(s_{n-1}, v) \in P_{\delta_i}$ then $s_0 s_1 \dots s_{n-1}(\delta_i)$ or $(s_{n-1}, v) \in P_{\delta_j}$ and*

(A″) *if $s_0 s_1 \dots s_{n-1}(\delta_j)$ and $(s_{n-1}, v^i) \in N_{\delta_i}$ then $s_0 s_1 \dots s_{n-1}(\delta_i)$ or there exists $(s_{n-1}, v^j) \in N_{\delta_j}$.*

(B) *$L(M_1) \cup L(M_2)$ is a generable language.*

(See [3]).

3.13. Corollary. *Let $M_1 = (S_1, I_1, \delta_1)$, $M_2 = (S_2, I_2, \delta_2)$ be G-machines. Then the following statements* (A), (B), (C) *are equivalent:*

(A) *For every $i, j \in \{1, 2\}$, $i \neq j$ and for every $n \in T$*

(A′) *if $s_0 s_1 \dots s_{n-1}(\delta_j)$ and $(s_{n-1}, v) \in P_{\delta_i}$ then $s_0 s_1 \dots s_{n-1}(\delta_i)$ or $(s_{n-1}, v) \in P_{\delta_j}$ and*

(A″) *if $s_0 s_1 \dots s_{n-1}(\delta_j)$ and $(s_{n-1}, v^i) \in N_{\delta_i}$ then $s_0 s_1 \dots s_{n-1}(\delta_i)$ or there exists $(s_{n-1}, v^j) \in N_{\delta_j}$.*

(B) *$L(M_1) \cup L(M_2)$ is a generable language.*

(C) *$L(M_1) \cup L(M_2) = L(M_U) = L(M_U^R) = L(M[L(M_1) \cup L(M_2)])$.*

3.14. Example. Let G-machines $M_1 = (S_1, I_1, \delta_1)$, $M_2 = (S_2, I_2, \delta_2)$ be given as follows: $S_1 = \{a, b, c, x\}$, $I_1 = \{a, b, c\}$, $\delta_1 : [a \to \{a, x\},\ b \to \{b, c, x\},\ c \to \{a\}]$,

$S_2 = \{a, c, d, y\}$, $I_2 = \{a, c, d\}$, $\delta_2 : [a \to \{a, y\},\ c \to \{a\},\ d \to \{c, d\}]$. First, we shall examine the condition (A) of Corollary 3.13. Let $k, m \in T$. Then the following holds:

$$a^k(\delta_1), (a, a) \in P_{\delta_2}, a^k(\delta_2);\ a^k(\delta_1), (a, y) \in N_{\delta_2}, (a, x) \in N_{\beta_1}\ ;$$
$$b^k c(\delta_1), (c, a) \in P_{\delta_2}, (c, a) \in P_{\delta_1};\ b^k ca^m(\delta_1), (a, a) \in P_{\delta_2}, (a, a) \in P_{\delta_1}\ ;$$
$$b^k ca^m(\delta_1), (a, y) \in N_{\delta_2}, (a, x) \in N_{\delta_1};\ a^k(\delta_2), (a, a) \in P_{\delta_1}, a^k(\delta_1)\ ;$$
$$a^k(\delta_2), (a, x) \in N_{\delta_1}, (a, y) \in N_{\delta_2};\ ca^k(\delta_2), (a, a) \in P_{\delta_1}, (a, a) \in P_{\delta_2}\ ;$$
$$dca^k(\delta_2), (a, x) \in N_{\delta_1}, (a, y) \in N_{\delta_2};\ dca^k(\delta_2), (a, a) \in P_{\delta_1}, (a, a) \in P_{\delta_2}\ ;$$
$$dca^k(\delta_2), (a, x) \in N_{\delta_1}, (a, y) \in N_{\delta_2}\ .$$

From the above G-machines M_1, M_2 satisfy the condition (A) of Corollary 3.13, therefore $L(M_1) \cup L(M_2)$ is a generable language and $L(M_1) \cup L(M_2) = L(M_U) = L(M_U^R)$ holds true. By 2.4 $L(M_1) \cup L(M_2) = \{a^\infty, a^k, b^\infty, b^k, ca^k, b^k ca^\infty, b^k ca^m, ca^\infty, dca^\infty, dca^k, d^\infty\}$. By Definition 3.9 $S_U = S_1 \cup S_2 = \{a, b, c, d, x, y\}$, $I_U = I_1 \cup I_2 = \{a, b, c, d\}$, $\delta_U = (\delta_1 \cup \delta_2) : [a \to \{a, x, y\},\ b \to \{b, c, x\},\ c \to \{a\},\ d \to \{c, d\}]$, $M_U = (S_U, I_U, \delta_U)$. By 2.5 $\delta_U^R : [a \to \{a, x\},\ b \to \{b, c, x\},\ c \to \{a\},\ d \to \{c, d\}]$, $M_U^R = (S_U, I_U, \delta_U^R)$. It is easy to verify that $L(M_U) = L(M_U^R) = \{a^\infty, a^k, b^\infty, b^k, ca^k, b^k ca^\infty, b^k ca^m, ca^\infty, dca^\infty, dca^k, d^\infty\} = L(M_1) \cup L(M_2)$.

(Received January 3, 1973.)

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