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Kybernetika, Vol. 18 (1982), No. 1, 50--65

Persistent URL: http://dml.cz/dmlcz/125591

KYBERNETIKA — VOLUME 18 (1982), NUMBER 1

OPTIMAL FEEDBACK CONTROL OF A CLASS OF DISTRIBUTED-PARAMETER SYSTEMS WITH INCOMPLETE MEASUREMENT

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Necessary conditions are discussed for a minimization of an integral performance criterion for control of a class of counter-current distributed-parameter systems with distributed control variable and with boundary control. The system dynamics are described by hyperbolic system of first-order partial differential equations. For the system with a quadratic performance criterion to drive it from one steady state to another a linear feedback control law has been derived. Part of state variables cannot be measured and part can be only incompletely measured.

1. INTRODUCTION

The optimal control theory of lumped-parameter systems has been well developed. This theory has resulted in analytical solution for optimal control of linear systems subject to performance criteria such as minimum integral-square-error minimum time and minimum fuel [1]. For other lumped-parameter systems and other performance criteria, efficient computational algorithms have been devised to obtain numerical solutions. For distributed-parameter systems, comparable results are available for many cases with complete measurement [2], [3], etc. Gilles [4] present a control for a class of distributed-parameter systems with incomplete measurement. In this paper, various controls, such as a spatially distributed control and boundary control, of a class of counter-current distributed-parameter system with incomplete measurement whose system dynamics are described by a hyperbolic system of first-order partial differential equations are studied. A linear feedback control law has been derived.

Theoretical results can be used for optimal control of tubular heat exchangers, tubular chemical reactors, absorption and distillation columns, tunnel kilns, glass bath tubs, sequence stirred chemical reactors, etc. A typical application of the discussed theory is the optimal control of tubular chemical reactor, because in a chemical reactor the concentration cannot be measured and the temperature can be incompletely measured. Illustrative calculations are given for a tubular plug flow heat exchanger.

2. SYSTEM EQUATIONS

A distributed-parameter system of the following is considered

$$(1)\qquad \frac{\partial \mathbf{x}(z,t)}{\partial t} = \mathbf{A}(z,t)\,\mathbf{x}(z,t) + \mathbf{A}_z(z,t)\,\frac{\partial \mathbf{x}(z,t)}{\partial z} + \mathbf{B}_w(z,t)\,\mathbf{w}(z,t)$$

where

t is the dimensionless time-like variable, $0 \leqq t \leqq t_1$,
z is the dimensionless spatial coordinate, $0 \leqq z \leqq 1$,
$\mathbf{x}(z,t) = [x_1(z,t),\ x_2(z,t), \dots, x_n(z,t)]^T$ is the state vector of n components,
$\mathbf{w}(z,t) = [w_1(z,t),\ w_2(z,t), \dots, w_h(z,t)]^T$ is the distributed control vector of h components, $h \leqq n$,

$$\mathbf{A}(z,t) = \begin{bmatrix} a_{11}(z,t) & a_{12}(z,t) & \dots & a_{1n}(z,t) \\ a_{21}(z,t) & a_{22}(z,t) & \dots & a_{2n}(z,t) \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ a_{n1}(z,t) & a_{n2}(z,t) & \dots & a_{nn}(z,t) \end{bmatrix} \text{ is an } n \times n \text{ matrix,}$$

$$\mathbf{A}_z(z,t) = \begin{bmatrix} a_{z1}(z,t) & 0 & \dots & 0 \\ 0 & a_{z2}(z,t) & \dots & 0 \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ 0 & 0 & \dots & a_{zn}(z,t) \end{bmatrix} \text{ is an } n \times n \text{ diagonal matrix,}$$

$$\mathbf{B}_w(z,t) = \begin{bmatrix} b_{11}(z,t) & b_{12}(z,t) & \dots & b_{1h}(z,t) \\ b_{21}(z,t) & b_{22}(z,t) & \dots & b_{2h}(z,t) \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ b_{n1}(z,t) & b_{n2}(z,t) & \dots & b_{nh}(z,t) \end{bmatrix} \text{ is an } n \times h \text{ matrix.}$$

Eqn. (1) represents a hyperbolic system. Elements of $\mathbf{A}(z,t)$ and $\mathbf{B}_w(z,t)$ matrix are continuous. Elements of $\mathbf{A}_z(z,t)$ matrix are continuously differentiable on z and t. Elements of distributed control vector $\mathbf{w}(z,t)$ are continuous and $\mathbf{w}(z,t) \in W$. W is a simply connected open region in h-dimensional Euclidean space. Boundary control is continuous, it can be differentiated and it has a value in n-dimensional Euclidean space.

The initial conditions are

$$\mathbf{x}(z, 0) = \mathbf{x}_0(z) . \tag{2}$$

The boundary conditions are

$$x_f(0, t) = u_f^0(t) \qquad f = 1, 2, \ldots, c , \tag{3}$$

$$x_i(0, t) = x_i^0(t) = 0 , \quad i = c + 1 , \quad c + 2, \ldots, d , \tag{4}$$

$$x_j(1, t) = u_j^1(t) , \qquad j = d + 1 , \quad d + 2, \ldots, e , \tag{5}$$

$$x_k(1, t) = x_k^1(t) = 0 , \quad k = e + 1 , \quad e + 2, \ldots, n . \tag{6}$$

Let us consider vectors given by

$$\mathbf{u}^0(t) = [u_1^0(t), u_2^0(t), \ldots, u_c^0(t)]^T ,$$
$$\mathbf{u}^1(t) = [u_{d+1}^1(t), u_{d+2}^1(t), \ldots, u_e^1(t)]^T ,$$
$$\mathbf{x}^0(t) = [x_{d+1}(0, t), x_{d+2}(0, t), \ldots, x_e(0, t), x_{e+1}(0, t), \ldots, x_n(0, t)]^T ,$$
$$\mathbf{x}^1(t) = [x_1(1, t), x_2(1, t), \ldots, x_c(1, t), x_{c+1}(1, t), \ldots, x_d(1, t)]^T .$$

$\mathbf{u}^0(t)$, $\mathbf{u}^1(t)$ are boundary control vectors. We have assumed that if control $\mathbf{w}(z, t)$ and initial and boundary conditions (2), (3), (4), (5), (6) are given then Eqn. (1) has a single solution.

We shall assume that measurements are available as follows [4]

$$y_r(t) = \int_0^1 \sum_{p=1}^{n} p_{rp}(z)\, x_p(z, t)\, \mathrm{d}z , \quad (r = 1, 2, \ldots, s) , \tag{7}$$

$$\mathbf{y}(t) = \int_0^1 \mathbf{P}(z)\, \mathbf{x}(z, t)\, \mathrm{d}z , \tag{8}$$

where

$$\mathbf{y}(t) = [y_1(t), y_2(t), \ldots, y_s(t)]^T ,$$

$$\mathbf{P}(z) = \begin{bmatrix} p_{11}(z) & p_{12}(z) & \ldots & p_{1n}(z) \\ p_{21}(z) & p_{22}(z) & \ldots & p_{2n}(z) \\ \cdot & \cdot & \ldots & \cdot \\ \cdot & \cdot & \ldots & \cdot \\ \cdot & \cdot & \ldots & \cdot \\ p_{s1}(z) & p_{s2}(z) & \ldots & p_{sn}(z) \end{bmatrix} \quad \text{is an } s \times n \text{ matrix.}$$

Elements of matrix $\mathbf{P}(z)$ are given by the measurements performed.

3. NECESSARY CONDITIONS FOR OPTIMAL CONTROL

The problem is to find optimal control law which minimizes the integral performance criterion

(9)

$$I = \int_0^{t_1} \mathbf{y}^T(t)\,\boldsymbol{\mu}(t)\,\mathbf{y}(t)\,\mathrm{d}t + \int_0^{t_1}\int_0^1 \mathbf{w}^T(z,t)\,\varrho^w(z,t)\,\mathbf{w}(z,t)\,\mathrm{d}z\,\mathrm{d}t +$$
$$+ \int_0^{t_1} \mathbf{x}^{0T}(t)\,\boldsymbol{\mu}^0(t)\,\mathbf{x}^0(t)\,\mathrm{d}t + \int_0^{t_1} \mathbf{x}^{1T}(t)\,\boldsymbol{\mu}^1(t)\,\mathbf{x}^1(t)\,\mathrm{d}t + \int_0^{t_1} \mathbf{u}^{0T}(t)\,\varrho^0(t)\,\mathbf{u}^0(t)\,\mathrm{d}t +$$
$$+ \int_0^{t_1} \mathbf{u}^{1T}(t)\,\varrho^1(t)\,\mathbf{u}^1(t)\,\mathrm{d}t\,,$$

where $\boldsymbol{\mu}(t)$, $\boldsymbol{\mu}^0(t)$, $\boldsymbol{\mu}^1(t)$ are positive semi-definite weighting factors and $\varrho^w(z, t)$, $\varrho^0(t)$, $\varrho^1(t)$ are positive weighting factors.

Substituting Eqn. (8) into Eqn. (9) one obtains

$$I = \int_0^{t_1}\int_0^1\int_0^1 [\mathbf{P}(\xi)\,\mathbf{x}(\xi,t)]^T\,\boldsymbol{\mu}(t)\,\mathbf{P}(z)\,\mathbf{x}(z,t)\,\mathrm{d}\xi\,\mathrm{d}z\,\mathrm{d}t + \tag{10}$$
$$+ \int_0^{t_1}\int_0^1 \mathbf{w}^T(z,t)\,\varrho^w(z,t)\,\mathbf{w}(z,t)\,\mathrm{d}z\,\mathrm{d}t + \int_0^{t_1} \mathbf{x}^{0T}(t)\,\boldsymbol{\mu}^0(t)\,\mathbf{x}^0(t)\,\mathrm{d}t +$$
$$+ \int_0^{t_1} \mathbf{x}^{1T}(t)\,\boldsymbol{\mu}^1(t)\,\mathbf{x}^1(t)\,\mathrm{d}t + \int_0^{t_1} \mathbf{u}^{0T}(t)\,\varrho^0(t)\,\mathbf{u}^0(t)\,\mathrm{d}t + \int_0^{t_1} \mathbf{u}^{1T}(t)\,\varrho^1(t)\,\mathbf{u}^1(t)\,\mathrm{d}t\,,$$

where ξ – the dimensionless spatial coordinate, $0 \leqq \xi \leqq 1$.

We consider the case where the final time t_1 is fixed, the final state $\mathbf{x}(z, t_1)$ is free.

The Hamiltonian functions are defined as

$$H[z,t,\mathbf{x}(z,t),\boldsymbol{\lambda}(z,t),\mathbf{w}(z,t)] = \int_0^1 [\mathbf{P}(\xi)\,\mathbf{x}(\xi,t)]^T\,\boldsymbol{\mu}(t)\,\mathbf{P}(z)\,\mathbf{x}(z,t)\,\mathrm{d}\xi + \tag{11}$$
$$+ \mathbf{w}^T(z,t)\,\varrho^w(z,t)\,\mathbf{w}(z,t) + \boldsymbol{\lambda}^T(z,t)\,[\mathbf{A}(z,t)\,\mathbf{x}(z,t) + \mathbf{B}_w(z,t)\,\mathbf{w}(z,t)]\,,$$

$$H^0[\mathbf{u}^0(t),\mathbf{u}^1(t),\mathbf{x}^0(t),\mathbf{x}^1(t),\boldsymbol{\lambda}^0(t),\boldsymbol{\lambda}^1(t)] = \mathbf{u}^{0T}(t)\,\varrho^0(t)\,\mathbf{u}^0(t) + \tag{12}$$
$$+ \mathbf{u}^{1T}(t)\,\varrho^1(t)\,\mathbf{u}^1(t) + \mathbf{x}^{0T}(t)\,\boldsymbol{\mu}^0(t)\,\mathbf{x}^0(t) + \mathbf{x}^{1T}(t)\,\boldsymbol{\mu}^1(t)\,\mathbf{x}^1(t) -$$
$$- \sum_{f=1}^{c} a_{zf}(0,t)\,\lambda_f(0,t)\,u_f^0(t) + \sum_{j=d+1}^{e} a_{zj}(1,t)\,\lambda_j(1,t)\,u_j^1(t)\,,$$

where

$\boldsymbol{\lambda}(z, t)$ is the adjoint vector on n components,

$\boldsymbol{\lambda}^0(t) = [\lambda_1^0(t), \lambda_2^0(t), \ldots, \lambda_c^0(t)]^T = [\lambda_1(0,t), \lambda_2(0,t), \ldots, \lambda_c(0,t)]^T$,

$\boldsymbol{\lambda}^1(t) = [\lambda^1_{d+1}(t), \lambda^1_{d+2}(t), \ldots, \lambda^1_e(t)]^T = [\lambda_{d+1}(1, t), \lambda_{d+2}(1, t), \ldots, \lambda_e(1, t)]^T$,

a_{zf}, $(f = 1, 2, \ldots, c)$, a_{zj}, $(j = d + 1, d + 2, \ldots, e)$ are the elements of matrix $\mathbf{A}_z$.

The adjoint vector satisfies the adjoint partial differential equation

$$\frac{\partial \boldsymbol{\lambda}(z, t)}{\partial t} = \frac{\partial}{\partial z}\left[\mathbf{A}_z^T(z, t)\, \boldsymbol{\lambda}(z, t)\right] - \frac{\partial H}{\partial x} \tag{13}$$

with final and boundary conditions

$$\boldsymbol{\lambda}(z, t_1) = 0\,, \tag{14}$$

$$a_{zj}(0, t)\, \lambda_j(0, t) = \frac{\partial H^0}{\partial x_j(0, t)}\,, \tag{15}$$

$$a_{zk}(0, t)\, \lambda_k(0, t) = \frac{\partial H^0}{\partial x_k(0, t)}\,, \tag{16}$$

$$a_{zf}(1, t)\, \lambda_f(1, t) = -\frac{\partial H^0}{\partial x_f(1, t)}\,, \tag{17}$$

$$a_{zi}(1, t)\, \lambda_i(1, t) = -\frac{\partial H^0}{\partial x_i(1, t)}\,. \tag{18}$$

The performance criterion, Eqn. (10), can be rewritten as

$$I = \int_0^{t_1}\int_0^1 [H - \boldsymbol{\lambda}^T(\mathbf{A}\mathbf{x} + \mathbf{B}_w\mathbf{w})]\, \mathrm{d}z\, \mathrm{d}t + \int_0^{t_1}\Big[H^0 + \sum_{f=1}^{c} a_{zf}(0, t)\, \lambda_f(0, t)\, u_f^0(t) - \\ - \sum_{j=d+1}^{e} a_{zj}(1, t)\, \lambda_j(1, t)\, u_j^1(t)\Big]\, \mathrm{d}t\,.$$

After taking the first variation of Eqn. (19), integrating by parts, and then setting $\delta I = 0$ for arbitrary variations in the state and control variables, we obtain the following necessary conditions [5], [6]:

$$\frac{\partial H}{\partial \mathbf{w}(z, t)} = 0\,, \tag{20}$$

$$\frac{\partial H^0}{\partial u_f^0(t)} = 0\,, \quad f = 1, 2, \ldots, c\,, \tag{21}$$

$$\frac{\partial H^0}{\partial u_j^1(t)} = 0\,, \quad j = d + 1,\ d + 2, \ldots, e\,. \tag{22}$$

4. LINEAR FEEDBACK CONTROL LAW FOR DISTRIBUTED CONTROL

We consider the case where $\mu^0(t) = \mu^1(t) = 0$, $x_j(0, t) = x_j(1, t) = 0$. The optimal control function is

$$\mathbf{w}(z, t) = -\tfrac{1}{2}\varrho^{w^{-1}}(z, t)\, \mathbf{B}_w^T(z, t)\, \boldsymbol{\lambda}(z, t)\,. \tag{23}$$

For the lumped system the adjoint vector $\boldsymbol{\lambda}(z, t)$ can be expressed as a homogeneous linear function of the state [1]. Since the distributed system in Eqn. (1) may be considered as the limit of large approximating lumped system in which spatial derivatives are replaced by differences, it follows that an equivalent result must hold for the function $\boldsymbol{\lambda}(z, t)$ in the distributed system. The proper representation in which to seek a solution is then

$$\boldsymbol{\lambda}(z, t) = \int_0^1 \mathbf{N}(z, \xi, t)\, \mathbf{x}(\xi, t)\, \mathrm{d}\xi\,. \tag{24}$$

It follows from the lumped system result, and is proved independently that $\mathbf{N}$ is symetric in its indices and spatial arguments

$$\mathbf{N}(z, \xi, t) = \mathbf{N}^T(\xi, z, t)\,. \tag{25}$$

Eqn. (13) can be rewritten as

$$\frac{\partial \boldsymbol{\lambda}}{\partial t} = \frac{\partial}{\partial z}\left[\mathbf{A}_z^T(z, t)\, \boldsymbol{\lambda}(z, t)\right] - \mathbf{A}^T(z, t)\, \boldsymbol{\lambda}(z, t) - \int_0^1 \mathbf{P}^T(z)\, \mu(t)\, \mathbf{P}(\xi)\, \mathbf{x}(\xi, t)\, \mathrm{d}\xi\,. \tag{26}$$

Substituting Eqn. (24) into Eqn. (25) and using Eqn. (1), one obtains

$$\begin{aligned} \int_0^1 \frac{\partial \mathbf{N}(z, \xi, t)}{\partial t}\, \mathbf{x}(\xi, t)\, \mathrm{d}\xi = &-\int_0^1 \mathbf{N}(z, \xi, t)\left[\mathbf{A}(\xi, t)\, \mathbf{x}(\xi, t) + \mathbf{A}_z(\xi, t)\frac{\partial \mathbf{x}(\xi, t)}{\partial \xi} + \right. \\ &\left. + \mathbf{B}_w(\xi, t)\, \mathbf{w}(\xi, t)\right] \mathrm{d}\xi + \int_0^1 \frac{\partial}{\partial z}\left[\mathbf{A}_z^T(z, t)\, \mathbf{N}(z, \xi, t)\right] \mathbf{x}(\xi, t)\, \mathrm{d}\xi - \\ &- \int_0^1 \mathbf{A}^T(z, t)\, \mathbf{N}(z, \xi, t)\, \mathbf{x}(\xi, t)\, \mathrm{d}\xi - \int_0^1 \mathbf{P}^T(z)\, \mu(t)\, \mathbf{P}(\xi)\, \mathbf{x}(\xi, t)\, \mathrm{d}\xi\,. \end{aligned} \tag{27}$$

Substitution of Eqn. (23) and Eqn. (24) into Eqn. (27) yields

$$\begin{aligned} &\int_0^1 \frac{\partial \mathbf{N}(z,\xi, t)}{\partial t}\, \mathbf{x}(\xi, t)\, \mathrm{d}\xi - \int_0^1 \frac{\partial}{\partial z}\left[\mathbf{A}_z^T(z, t)\, \mathbf{N}(z, \xi, t)\right] \mathbf{x}(\xi, t)\, \mathrm{d}\xi - \\ &- \int_0^1 \frac{\partial}{\partial \xi}\left[\mathbf{N}(z, \xi, t)\, \mathbf{A}_z(\xi, t)\right] \mathbf{x}(\xi, t)\, \mathrm{d}\xi + \int_0^1 \mathbf{A}^T(z, t)\, \mathbf{N}(z, \xi, t)\, \mathbf{x}(\xi, t)\, \mathrm{d}\xi + \end{aligned} \tag{28}$$

$$+\int_0^1 \mathbf{N}(z,\xi,t)\,\mathbf{A}(\xi,t)\,\mathbf{x}(\xi,t)\,\mathrm{d}\xi - \tfrac{1}{2}\int_0^1\left[\int_0^1 \mathbf{N}(z,r,t)\,\mathbf{B}_w(r,t)\,\varrho^{w^{-1}}(r,t)\,\mathbf{B}_w^T(r,t)\,\times\right.$$

$$\left.\times\,\mathbf{N}(r,\xi,t)\,\mathbf{x}(\xi,t)\,\mathrm{d}r\right]\mathrm{d}\xi + \int_0^1 \mathbf{P}^T(z)\,\boldsymbol{\mu}(t)\,\mathbf{P}(\xi)\,\mathbf{x}(\xi,t)\,\mathrm{d}\xi =$$

$$= -\left[\mathbf{N}(z,\xi,t)\,\mathbf{A}_z(\xi,t)\,\mathbf{x}(\xi,t)\right]_0^1 .$$

If the following boundary condition on the gain matrix

$$-\mathbf{N}(z,1,t)\,\mathbf{A}_z(1,t)\,\mathbf{x}(1,t) + \mathbf{N}(z,0,t)\,\mathbf{A}_z(0,t)\,\mathbf{x}(0,t) = 0 \tag{29}$$

holds then satisfaction of Eqn. (28) will be obtained if

$$\frac{\partial \mathbf{N}(z,\xi,t)}{\partial t} - \frac{\partial}{\partial z}\left[\mathbf{A}_z(z,t)\,\mathbf{N}(z,\xi,t)\right] - \frac{\partial}{\partial \xi}\left[\mathbf{N}(z,\xi,t)\,\mathbf{A}_z(\xi,t)\right] + \tag{30}$$

$$+ \mathbf{A}^T(z,t)\,\mathbf{N}(z,\xi,t) + \mathbf{N}(z,\xi,t)\,\mathbf{A}(\xi,t) -$$

$$-\tfrac{1}{2}\int_0^1 \mathbf{N}(z,r,t)\,\mathbf{B}_w(r,t)\,\varrho^{w^{-1}}(r,t)\,\mathbf{B}_w^T(r,t)\,\mathbf{N}(r,\xi,t)\,\mathrm{d}r +$$

$$+ \mathbf{P}^T(z)\,\boldsymbol{\mu}(t)\,\mathbf{P}(\xi) = 0 .$$

Using relations in Eqns. (14), (24) and (25) for $\lambda_j(0,t) = \lambda_k(0,t) = \lambda_f(1,t) = \lambda_i(1,t) = 0$ we get final and boundary conditions of matrix $\mathbf{N}(z,\xi,t)$ as

(31) $\mathbf{N}(z,\xi,t_1) = 0$,

(32) $N_{fp}(1,\xi,t) = 0$, $f = 1,2,\ldots,c$,
$p = 1,2,\ldots,c,\ c+1,\ldots,d,\ d+1,\ldots,e,\ e+1,\ldots,n$,

(33) $N_{ip}(1,\xi,t) = 0$, $i = c+1,\ c+2,\ldots,d$,
$p = 1,2,\ldots,c,\ c+1,\ldots,d,\ d+1,\ldots,e,\ e+1,\ldots,n$,

(34) $N_{jp}(0,\xi,t) = 0$, $j = d+1,\ d+2,\ldots,e$,
$p = 1,2,\ldots,c,\ c+1,\ldots,d,\ d+1,\ldots,e,\ e+1,\ldots,n$,

(35) $N_{kp}(0,\xi,t) = 0$, $k = e+1,\ e+2,\ldots,n$,
$p = 1,2,\ldots,c,\ c+1,\ldots,d,\ d+1,\ldots,e,\ e+1,\ldots,n$,

(36) $N_{pf}(z,1,t) = 0$, $p = 1,2,\ldots,c,\ c+1,\ldots,d,\ d+1,\ldots,e,\ e+1,\ldots,n$,
$f = 1,2,\ldots,c$,

(37) $N_{pi}(z,1,t) = 0$, $p = 1,2,\ldots,c,\ c+1,\ldots,d,\ d+1,\ldots,e,\ e+1,\ldots,n$,
$i = c+1,\ c+2,\ldots,d$,

(38) $N_{pj}(z,0,t) = 0$, $p = 1,2,\ldots,c,\ c+1,\ldots,d,\ d+1,\ldots,e,\ e+1,\ldots,n$,
$j = d+1,\ d+2,\ldots,e$,

(39) $N_{pk}(z, 0, t) = 0\,,\quad p = 1, 2, \ldots, c,\ c + 1, \ldots, d,\ d + 1, \ldots, e,\ e + 1, \ldots, n,$
$k = e + 1,\ e + 2, \ldots, n.$

Using conditions (3)–(6) $[u_f^0(t) = u_j^1(t) = 0]$ and conditions (36)–(39) Eqn. (29) is satisfied.

In the present case, matrix $\mathbf{N}(z, \xi, t)$ takes the form

$$\mathbf{N}(z, \xi, t) = \mathbf{P}^T(z)\, \mathbf{K}(t)\, \mathbf{P}(\xi)\,. \tag{40}$$

The form of matrix $\mathbf{N}(z, \xi, t)$ can be obtained by using the dynamic programming when Lyapunov function is

$$J^* = \mathbf{y}^T(t)\, \mathbf{K}(t)\, \mathbf{y}(t)\,. \tag{41}$$

Substituting Eqn. (8) and Eqn. (40) into Eqn. (41) we obtain

$$J^* = \int_0^1 \int_0^1 \mathbf{x}^T(z, t)\, \mathbf{N}(z, \xi, t)\, \mathbf{x}(\xi, t)\, \mathrm{d}z\, \mathrm{d}\xi\,. \tag{42}$$

Using Eqns. (8), (24) and (40), the optimal control (23) can be written in the feedback form

$$\mathbf{w}(z, t) = \tfrac{1}{2}\varrho^{w^{-1}}(z, t)\, \mathbf{B}_w^T(z, t)\, \mathbf{P}^T(z)\, \mathbf{K}(t)\, \mathbf{y}(t)\,, \tag{43}$$

where $\mathbf{K}(t)$ is $s \times s$ matrix. It follows from the Eqn. (40) that $\mathbf{K}(t) = \mathbf{K}^T(t)$.

The matrix $\mathbf{P}(z)$ cannot be arbitrary.

Assuming the same as in the lumped-parameter system, the optimal gain becomes stationary as t_1 approach infinity, Eqn. (30) becomes $[\mathbf{A}_z, \mathbf{A}, \mathbf{B}_w, \varrho^w, \mu$ are time independent, $\mathbf{N}(z, \xi, t) \to \mathbf{N}_s(z, \xi)$, $\mathbf{K}(t) \to \mathbf{K}_s]$

(44)

$$-\frac{\partial}{\partial z}\left[\mathbf{A}_z^T(z)\, \mathbf{N}_s(z, \xi)\right] - \frac{\partial}{\partial \xi}\left[\mathbf{N}_s(z, \xi)\, \mathbf{A}_z(\xi)\right] + \mathbf{A}^T(z)\, \mathbf{N}_s(z, \xi) + \mathbf{N}_s(z, \xi)\, \mathbf{A}(\xi) -$$
$$- \tfrac{1}{2}\int_0^1 \mathbf{N}_s(z, r)\, \mathbf{B}_w(r)\, \varrho^{w^{-1}}(r)\, \mathbf{B}_w^T(r)\, \mathbf{N}_s(r, \xi)\, \mathrm{d}r + \mathbf{P}^T(z)\, \mu\, \mathbf{P}(\xi) = 0$$

with boundary conditions

(45) $N_{sfp}(1, \xi) = 0\,,\quad f = 1, 2, \ldots, c,$
$p = 1, 2, \ldots, c,\ c + 1, \ldots, d,\ d + 1, \ldots, e,\ e + 1, \ldots, n,$

(46) $N_{sip}(1, \xi) = 0\,,\quad i = c + 1,\ c + 2, \ldots, d,$
$p = 1, 2, \ldots, c,\ c + 1, \ldots, d,\ d + 1, \ldots, e,\ e + 1, \ldots, n,$

(47) $N_{sjp}(0, \xi) = 0\,,\quad j = d + 1,\ d + 2, \ldots, e,$
$p = 1, 2, \ldots, c,\ c + 1, \ldots, d,\ d + 1, \ldots, e,\ e + 1, \ldots, n,$

(48) $N_{skp}(0, \xi) = 0\,,\quad k = e + 1,\ e + 2, \ldots, n,$
$p = 1, 2, \ldots, c,\ c + 1, \ldots, d,\ d + 1, \ldots, e,\ e + 1, \ldots, n,$

(49) $N_{spf}(z, 1) = 0\,,\quad p = 1, 2, \ldots, c,\ c + 1, \ldots, d,\ d + 1, \ldots, e,\ e + 1, \ldots, n,$
$f = 1, 2, \ldots, c,$

(50) $N_{spi}(z, 1) = 0\,,\quad p = 1, 2, \ldots, c,\ c + 1, \ldots, d,\ d + 1, \ldots, e,\ e + 1, \ldots, n,$
$i = c + 1,\ c + 2, \ldots, d,$

(51) $N_{spj}(z, 0) = 0\,,\quad p = 1, 2, \ldots, c,\ c + 1, \ldots, d,\ d + 1, \ldots, e,\ e + 1, \ldots, n,$
$j = d + 1,\ d + 2, \ldots, e,$

(52) $N_{spk}(z, 0) = 0\,,\quad p = 1, 2, \ldots, c,\ c + 1, \ldots, d,\ d + 1, \ldots, e,\ e + 1, \ldots, n,$
$k = e + 1,\ e + 2, \ldots, n.$

5. LINEAR FEEDBACK CONTROL LAW FOR BOUNDARY CONTROL

We consider the case where $\boldsymbol{\mu}^0(t) = \boldsymbol{\mu}^1(t) = 0$, $\mathbf{w} = 0$.
The boundary control functions are

$$\mathbf{u}^0(t) = \frac{1}{2}\varrho^{0^{-1}}(t)\,\frac{\partial}{\partial \mathbf{u}^0}\left[\sum_{f=1}^{c} a_{zf}(0, t)\,\lambda_f(0, t)\,u_f^0(t)\right], \tag{53}$$

$$\mathbf{u}^1(t) = -\frac{1}{2}\varrho^{1^{-1}}(t)\,\frac{\partial}{\partial \mathbf{u}^1}\left[\sum_{j=d+1}^{e} a_{zj}(1, t)\,\lambda_j(1, t)\,u_j^1(t)\right]. \tag{54}$$

In order to obtain optimal feedback control, relation (24) is employed. Substituting Eqn. (24) into Eqn. (26), using relations of Eqn. (1), one obtains

(55)
$$\int_0^1 \frac{\partial \mathbf{N}(z, \xi, t)}{\partial t}\,\mathbf{x}(\xi, t)\,\mathrm{d}\xi = -\int_0^1 \mathbf{N}(z, \xi, t)\left[\mathbf{A}(\xi, t)\,\mathbf{x}(\xi, t) + \mathbf{A}_z(\xi, t)\,\frac{\partial \mathbf{x}(\xi, t)}{\partial \xi}\right]\mathrm{d}\xi +$$
$$+ \int_0^1 \frac{\partial}{\partial z}\left[\mathbf{A}_z^T(z, t)\,\mathbf{N}(z, \xi, t)\right]\mathbf{x}(\xi, t)\,\mathrm{d}\xi - \int_0^1 \mathbf{A}^T(z, t)\,\mathbf{N}(z, \xi, t)\,\mathbf{x}(\xi, t)\,\mathrm{d}\xi -$$
$$- \int_0^1 \mathbf{P}^T(z)\,\boldsymbol{\mu}(t)\,\mathbf{P}(\xi)\,\mathbf{x}(\xi, t)\,\mathrm{d}\xi\,.$$

After rearrangement we can write

$$\int_0^1 \frac{\partial \mathbf{N}(z, \xi, t)}{\partial t}\,\mathbf{x}(\xi, t)\,\mathrm{d}\xi - \int_0^1 \frac{\partial}{\partial z}\left[\mathbf{A}_z^T(z, t)\,\mathbf{N}(z, \xi, t)\right]\mathbf{x}(\xi, t)\,\mathrm{d}\xi - \tag{56}$$
$$- \int_0^1 \frac{\partial}{\partial \xi}\left[\mathbf{N}(z, \xi, t)\,\mathbf{A}_z(\xi, t)\right]\mathbf{x}(\xi, t)\,\mathrm{d}\xi + \int_0^1 \mathbf{A}^T(z, t)\,\mathbf{N}(z, \xi, t)\,\mathbf{x}(\xi, t)\,\mathrm{d}\xi +$$

$$+ \int_0^1 \mathbf{N}(z, \xi, t)\, \mathbf{A}(\xi, t)\, \mathbf{x}(\xi, t)\, d\xi + \int_0^1 \mathbf{P}^T(z)\, \mu(t)\, \mathbf{P}(\xi)\, \mathbf{x}(\xi, t)\, d\xi +$$

$$+ [\mathbf{N}(z, \xi, t)\, \mathbf{A}_z(\xi, t)\, \mathbf{x}(\xi, t)]_1^0 = 0 .$$

Using relations in Eqns. (14), (24) and (25) when $\lambda_j(0, t) = \lambda_k(0, t) = \lambda_f(1, t) = \lambda_i(1, t) = 0$ we get final and boundary conditions of matrix $\mathbf{N}(z, \xi, t)$ as in Eqns. (31)–(39).

Using conditions (3)–(6) and conditions (36)–(39) we can write

$$[\mathbf{N}(z, \xi, t)\, \mathbf{A}_z(\xi, t)\, \mathbf{x}(\xi, t)]_1^0 = \tag{57}$$

$$= \mathbf{N}(z, 1, t)\, \mathbf{A}_z(1, t)\, \mathbf{x}(1, t) - \mathbf{N}(z, 0, t)\, \mathbf{A}_z(0, t)\, \mathbf{x}(0, t) ,$$

$$[\mathbf{N}(z, \xi, t)\, \mathbf{A}_z(\xi, t)\, \mathbf{x}(\xi, t)]_1^0 = \tag{58}$$

$$= \mathbf{N}^1(z, 1, t)\, \mathbf{A}_z^1(1, t)\, \mathbf{u}^1(t) - \mathbf{N}^0(z, 0, t)\, \mathbf{A}_z^0(0, t)\, \mathbf{u}^0(t) ,$$

where

$$\mathbf{A}_z^0(\xi, t) = \begin{bmatrix} a_{z1}(\xi, t) & 0 & \dots & 0 \\ 0 & a_{z2}(\xi, t) & \dots & 0 \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ 0 & 0 & \dots & a_{zc}(\xi, t) \end{bmatrix} \quad \text{is an } c \times c \text{ matrix,}$$

$$\mathbf{A}_z^1(\xi, t) = \begin{bmatrix} a_{z(d+1)}(\xi, t) & 0 & \dots & 0 \\ 0 & a_{z(d+2)}(\xi, t) & \dots & 0 \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ 0 & 0 & \dots & a_{ze}(\xi, t) \end{bmatrix} \quad \text{is an } (e - d) \times (e - d) \text{ matrix,}$$

$$\mathbf{N}^0(z, \xi, t) = \begin{bmatrix} N_{11}(z, \xi, t) & N_{12}(z, \xi, t) & \dots & N_{1c}(z, \xi, t) \\ N_{21}(z, \xi, t) & N_{22}(z, \xi, t) & \dots & N_{2c}(z, \xi, t) \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ N_{n1}(z, \xi, t) & N_{n2}(z, \xi, t) & \dots & N_{nc}(z, \xi, t) \end{bmatrix} \quad \text{is an } n \times c \text{ matrix,}$$

$$\mathbf{N}^1(z, \xi, t) = \begin{bmatrix} N_{1(d+1)}(z, \xi, t) & N_{1(d+2)}(z, \xi, t) & \dots & N_{1e}(z, \xi, t) \\ N_{2(d+1)}(z, \xi, t) & N_{2(d+2)}(z, \xi, t) & \dots & N_{2e}(z, \xi, t) \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ N_{n(d+1)}(z, \xi, t) & N_{n(d+2)}(z, \xi, t) & \dots & N_{ne}(z, \xi, t) \end{bmatrix} \quad \text{is an } n \times (n - d) \text{ matrix.}$$

Substituting Eqns. (53) and (54) into Eqn. (58), one obtains

$$(59) \qquad [\mathbf{N}(z, \xi, t)\, \mathbf{A}_z(\xi, t)\, \mathbf{x}(\xi, t)]_0^1 =$$
$$= \mathbf{N}^1(z, 1, t)\, \mathbf{A}_z^1(1, t) \left[-\tfrac{1}{2}\varrho^{1^{-1}}(t)\, \mathbf{A}_z^1(1, t)\, \boldsymbol{\lambda}^1(t)\right] -$$
$$- \mathbf{N}^0(z, 0, t)\, \mathbf{A}_z^0(0, t)\, \tfrac{1}{2}\varrho^{0^{-1}}(t)\, \mathbf{A}_z^0(0, t)\, \boldsymbol{\lambda}^0(t)\,.$$

Substituting Eqn. (24) into Eqn. (59), one obtains

$$(60) \qquad [\mathbf{N}(z, \xi, t)\, \mathbf{A}_z(\xi, t)\, \mathbf{x}(\xi, t)]_0^1 =$$
$$= -\tfrac{1}{2}\int_0^1 \mathbf{N}^1(z, 1, t)\, \mathbf{A}_z^1(1, t)\, \varrho^{1^{-1}}(t)\, \mathbf{A}_z^1(1, t)\, \mathbf{N}^{11}(1, \xi, t)\, \mathbf{x}(\xi, t)\, \mathrm{d}\xi -$$
$$-\tfrac{1}{2}\int_0^1 \mathbf{N}^0(z, 0, t)\, \mathbf{A}_z^0(0, t)\, \varrho^{0^{-1}}(t)\, \mathbf{A}_z^0(0, t)\, \mathbf{N}^{00}(0, \xi, t)\, \mathbf{x}(\xi, t)\, \mathrm{d}\xi\,,$$

where

$$(61) \qquad \mathbf{N}^{00}(z, \xi, t) = \mathbf{P}^{00^T}(z)\, \mathbf{K}(t)\, \mathbf{P}(\xi)\,,$$

$$\mathbf{P}^{00}(z) = \begin{bmatrix} p_{11}(z) & p_{12}(z) & \dots & p_{1c}(z) \\ p_{21}(z) & p_{22}(z) & \dots & p_{2c}(z) \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ p_{s1}(z) & p_{s2}(z) & \dots & p_{sc}(z) \end{bmatrix} \quad \text{is an } s \times c \text{ matrix,}$$

$$(62) \qquad \mathbf{N}^{11}(z, \xi, t) = \mathbf{P}^{11^T}(z)\, \mathbf{K}(t)\, \mathbf{P}(\xi)\,,$$

$$\mathbf{P}^{11}(z) = \begin{bmatrix} p_{1(d+1)}(z) & p_{1(d+2)}(z) & \dots & p_{1e}(z) \\ p_{2(d+1)}(z) & p_{2(d+2)}(z) & \dots & p_{2e}(z) \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ p_{s(d+1)}(z) & p_{s(d+2)}(z) & \dots & p_{se}(z) \end{bmatrix} \quad \text{is an } s \times (e - d) \text{ matrix.}$$

Eqn. (56) can be written

$$(63) \qquad \frac{\partial \mathbf{N}(z, \xi, t)}{\partial t} - \frac{\partial}{\partial z}[\mathbf{A}_z^T(z, t)\, \mathbf{N}(z, \xi, t)] - \frac{\partial}{\partial \xi}[\mathbf{N}(z, \xi, t)\, \mathbf{A}_z(\xi, t)] +$$
$$+ \mathbf{A}^T(z, t)\, \mathbf{N}(z, \xi, t) + \mathbf{N}(z, \xi, t)\, \mathbf{A}(\xi, t) + \mathbf{P}^T(z)\, \boldsymbol{\mu}(t)\, \mathbf{P}(\xi) -$$
$$- \tfrac{1}{2}\mathbf{N}^1(z, 1, t)\, \mathbf{A}_z^1(1, t)\, \varrho^{1^{-1}}(t)\, \mathbf{A}_z^1(1, t)\, \mathbf{N}^{11}(1, \xi, t) -$$
$$- \tfrac{1}{2}\mathbf{N}^0(z, 0, t)\, \mathbf{A}_z^0(0, t)\, \varrho^{0^{-1}}(t)\, \mathbf{A}_z^0(0, t)\, \mathbf{N}^{00}(0, \xi, t) = 0\,.$$

Using Eqns. (8), (24) and (61), the optimal control (53) can be written in the feedback form

$$(64) \qquad \mathbf{u}^0(t) = \tfrac{1}{2}\varrho^{0^{-1}}(t)\, \mathbf{A}_z^0(0, t)\, \mathbf{P}^{00^T}(0)\, \mathbf{K}(t)\, \mathbf{y}(t)\,.$$

Using Eqns. (8), (24) and (62), the optimal control (54) can be written in the feedback form

$$\mathbf{u}^1(t) = -\tfrac{1}{2}\varrho^{1^{-1}}(t)\,\mathbf{A}_z^1(1, t)\,\mathbf{P}^{11^T}(1)\,K(t)\,\mathbf{y}(t)\,, \tag{65}$$

where $\mathbf{K}(t)$ is a symmetric $s \times s$ matrix. The solution of Eqns. (63) and (40) with initial and boundary conditions (31)–(39) is matrix $\mathbf{K}(t)$. Matrix $\mathbf{P}(z)$ cannot be arbitrary for boundary control.

Similarly as for distributed control the optimal gain becomes stationary as t_1 approaches infinity, Eqn. (63) becomes [$\mathbf{A}_z$, $\mathbf{A}$, ϱ^0, ϱ^1, μ are time independent, $\mathbf{N}(z, \xi, t) \to \mathbf{N}_s(z, \xi)$, $\mathbf{N}^1(z, \xi, t) \to \mathbf{N}_s^1(z, \xi)$, $\mathbf{N}^{11}(z, \xi, t) \to \mathbf{N}_s^{11}(z, \xi)$, $\mathbf{N}^0(z, \xi, t) \to$ $\to \mathbf{N}_s^0(z, \xi)$, $\mathbf{N}^{00}(z, \xi, t) \to \mathbf{N}_s^{00}(z, \xi)$, $\mathbf{K}(t) \to \mathbf{K}_s$]

(66)

$$-\frac{\partial}{\partial z}[\mathbf{A}_z^T(z)\,\mathbf{N}_s(z, \xi)] - \frac{\partial}{\partial \xi}[\mathbf{N}_s(z, \xi)\,\mathbf{A}_z(\xi)] + \mathbf{A}^T(z)\,\mathbf{N}_s(z, \xi) + N_s(z, \xi)\,\mathbf{A}(\xi) +$$
$$+ \mathbf{P}^T(z)\,\boldsymbol{\mu}\,\mathbf{P}(\xi) - \tfrac{1}{2}\mathbf{N}_s^1(z, 1)\,\mathbf{A}_z^1(1)\,\varrho^{1^{-1}}\,\mathbf{A}_z^1(1)\,\mathbf{N}_s^{11}(1, \xi) -$$
$$- \tfrac{1}{2}\mathbf{N}_s^0(z, 0)\,\mathbf{A}_z^0(0)\,\varrho^{0^{-1}}\,\mathbf{A}_z^0(0)\,\mathbf{N}_s^{00}(0, \xi) = 0$$

with boundary conditions (45)–(52).

As in [3] we can demonstrate that the control system is asymptotically stable.

6. TUBULAR PLUG FLOW HEAT EXCHANGER WITH WALL TEMPERATURE CONTROLLING

We now apply the theory to obtain optimal control of the tubular plug flow heat exchanger by the manipulation of the well temperature. It is assumed that the wall temperature can be varied with both position and time. The control is to drive the exchanger from an initial undesired steady state to a new steady state, optimizing a quadratic performance index. If we also have constant physical properties, perfect radial mixing, and no axial dispersion, the dynamics of the heat exchanger can be represented by the following scalar linear partial differential equation

$$\frac{\partial \vartheta(z, t)}{\partial t} = - \frac{\partial \vartheta(z, t)}{\partial z} + P[\vartheta_w(z, t) - \vartheta(z, t)]\,, \tag{67}$$

where

$t = vt'/L$ is the dimensionless time-like variable, $0 \leqq t \leqq t_1$,
t' is the time,
v is the velocity,
L is the length of the exchanger,
$z = z'/L$ is the dimensionless spatial coordinate, $0 \leqq z \leqq 1$,
z' is the distance from exchanger entrance,

$\vartheta(z, t)$ is the temperature of fluid in exchanger,
$\vartheta_w(z, t)$ is the wall temperature,
$P = \alpha F L / v F_z \varrho_p c_p$ is the ratio of heat exchanger to heat capacity,
α is the heat transfer coefficient between exchanger wall and fluid,
F is the perimeter of pipe wall,
F_z is the cross-sectional area for flow in exchanger,
ϱ_p is the density,
c_p is the specific heat.

Let us assume a constant inlet temperature ϑ_{00} and that the exchanger has been operated for a long period of time at the constant wall temperature ϑ_{w0}. Then we determine the initial temperature profile $\vartheta_0(z)$ from (67) by setting $\partial\vartheta(z, t)/\partial t = 0$, $\vartheta_w(z, t) = \vartheta_{w0}$, $\vartheta_0(0) = \vartheta_{00}$ and by solving we obtain

(68) $$\vartheta_0(z) = \vartheta_{w0} - (\vartheta_{w0} - \vartheta_{00})\, e^{-Pz}\,.$$

Let us assume that we now wish to operate the exchanger in a new steady state condition with a wall temperature ϑ_{w1}. The objective will be to determine how $\vartheta_w(z, t)$ should be changed from ϑ_{w0} to ϑ_{w1} so as to minimize the performance criterion, to be defined later. The temperature $\vartheta(z, t)$ cannot be completely measured. We define the state variable as a deviation about the final steady state profile

(69) $$\vartheta_1(z) = \vartheta_{w1} - (\vartheta_{w1} - \vartheta_{00})\, e^{-Pz}\,.$$

The state deviation is defined in the dimensionless form

(70) $$x_1(z, t) = \frac{\vartheta(z, t) - \vartheta_1(z)}{\vartheta_{w1} - \vartheta_{00}}$$

and the manipulated variable deviation is defined as

(71) $$w_1(z, t) = \frac{\vartheta_w(z, t) - \vartheta_{w1}}{\vartheta_{w1} - \vartheta_{00}}\,.$$

The normalized partial differential equation describing the system is

(72) $$\frac{\partial x_1(z, t)}{\partial t} = -\frac{\partial x_1(z, t)}{\partial z} - P[x_1(z, t) - w_1(z, t)]$$

with the boundary condition

(73) $$x_1(0, t) = 0\,.$$

Since the initial conditions is also a steady state, we obtain from Eqns. (72) and (73) for $\partial x_1/\partial t = 0$ and $w_1(z, 0) = w_{10}(0)$

(74) $$x_1(z, 0) = w_{10}(0)\,(1 - e^{-Pz})\,.$$

The factor $w_{10}(0)$ serves only to multiply all the temperature profiles; hence we can make $w_{10}(0) = 1$ without loss of generality

(75) $$x_1(z, 0) = 1 - e^{-Pz} .$$

The gains will remain unchanged.

The system is completely controllable. If $w_1 = 0$ is applied to the system, then $x_1(z, t)$ will be zero when $t \geqq 1$, because the normalized time required for all fluid (in the head exchanger at $t = 0$) to leave the heat exchanger is $t = 1$. We can therefore choose as performance criterion the following

(76) $$I = \int_0^{t_1} \mu_{11}\, y_1^2(t)\, dt + \int_0^{t_1}\int_0^1 \varrho_{11}^w(z)\, w_1^2(z, t)\, dz\, dt ,$$

where t_1 may be infinite, $\mu_{11} \geqq y$, $\varrho_{11}^w(z) > 0$ and where

(77) $$y_1(t) = \int_0^1 p_{11}(z)\, x_1(z, t)\, dz .$$

The Hamiltonian, the adjoint system and its boundary conditions are

(78) $$H = \int_0^1 p_{11}(\xi)\, x_1(\xi, t)\, \mu_{11}\, p_{11}(z)\, x_1(z, t)\, d\xi + \varrho_{11}^w(z)\, w_1^2(z, t) - P\, \lambda_1(z, t) \times \times \left[x_1(z, t) - w_1(z, t)\right] ,$$

(79) $$\frac{\partial \lambda_1(z, t)}{\partial t} = - \frac{\partial \lambda_1(z, t)}{\partial z} + P\, \lambda_1(z, t) - \int_0^1 p_{11}(z)\, \mu_{11}\, p_{11}(\xi)\, x_1(\xi, t)\, d\xi ,$$

(80) $$\lambda_1(z, t_1) = 0 ,$$

(81) $$\lambda_1(1, t) = 0 .$$

Using Eqn. (43) and assuming that t_1 approaches infinity, the optimal feedback control

(82) $$w_1(z, t) = - \frac{1}{2\varrho_{11}^w(z)}\, P\, p_{11}(z)\, K_{s11}\, y_1(t) ,$$

where K_{s11} will satisfy equations

(83) $$N_{s11}(z, \xi) = p_{11}(z)\, K_{s11}\, p_{11}(\xi) ,$$

(84)
$$\frac{\partial N_{s11}(z, \xi)}{\partial z} + \frac{\partial N_{s11}(z, \xi)}{\partial \xi} - 2P\, N_{s11}(z, \xi) - \tfrac{1}{2}P^2 \int_0^1 N_{s11}(z, r)\, \frac{1}{\varrho_{11}^w(r)}\, N_{s11}(r, \xi)\, dr + p_{11}(z)\, \mu_{11}\, p_{11}(\xi) = 0$$

with the boundary conditions

(85) $$N_{s11}(1, \xi) = 0 ,$$

(86) $$N_{s11}(z, 1) = 0 .$$

If

$$p_{11}(z) = 1 - e^{-\beta(1-z)} \tag{87}$$

and if $\varrho_{11}^{W}(z) = 1/4z$, $\beta = 100$, $\mu_{11} = 1$, $P = 1$ then $K_{s11} \doteq 0{,}4142135$.

If $\mu_{11} = 0$, then $K_{s11} = 0$ and therefore $w_1(z, t) = 0$. Physically, this is quite correct. The condition $\mu_{11} = 0$ means no penalty on $y_1(t)$, so that the system is steered to its final desired steady state condition using a minimum of control.

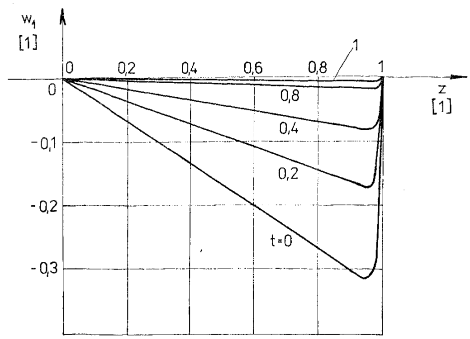

Fig. 1. Control variable profiles.

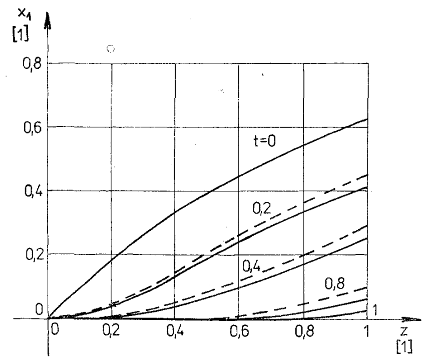

Fig. 2. State variable profiles.

Computational results are shown in Figures 1 and 2. The dashed profiles on Figure 2 are for $w_1 = 0$.

(Received May 20, 1981.)

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