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Kybernetika, Vol. 31 (1995), No. 6, 613--621

Persistent URL: http://dml.cz/dmlcz/125271

KYBERNETIKA — VOLUME 31 (1995), NUMBER 6, PAGES 613-621

THE MATCHING PROBLEM FOR BEHAVIORAL SYSTEMS

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In this article the Matching Problem is considered in a general behavioral context. Conditions for the existence of solution are found under suitable hypothesis.

1. INTRODUCTION

Behavioral systems have been introduced by Willems in [8] as a tool for modeling a very general class of phenomena. In this note we consider a Matching Problem for general behavioral systems that can be viewed as a generalization of the Model Matching Problem studied by several authors in more classical system theoretic contexts (see e. g. [1] and [2] for a list of references and a more general discussion of the problem). Essentially, the problem consists in designing a compensator for a given system, in such a way that the composition, or interconnection, of the compensator and the system matches the behavior of an assigned model.

The starting point of our approach consists in viewing I/O behavioral systems as pairs of maps in a suitable category (see [5]). In this way the category theoretical notions of pullback and pushout can be used for defining a notion of series composition between systems (essentially equivalent to the notion of interconnection considered in [8] XII), for which the above mentioned problem makes sense.

Section 2 is devoted to a brief description of the notions of pullback and pushout in a category and Section 3 is devoted to the definition of series composition of behavioral systems and AR-systems. In Section 4, after stating the Matching Problem for I/O behavioral systems, we give a necessary condition for the existence of solutions, that is also sufficient in a particular case. Section 5 is devoted to the case of AR-system, and, after the Matching Problem has been stated in a suitable way, a necessary condition that turns out to be sufficient in a particular case is given. Such result is shown to be a generalization of known results about the Model Matching Problem for finite dimensional, linear systems having strictly proper transfer function.

The results of this note have partially appeared in [3] and [4].

2. PRELIMINARIES AND NOTATIONS

In order to develop our construction in the next Section, we need first to introduce a suitable mathematical framework. To this aim, let us recall that a *Category* consists of a collection of *Objects*, denoted by capital letters like $A, B, \ldots$, and *Maps* between the objects, represented as $f : A \longrightarrow B$, and of a *Composition Law*, that assigns a map $gf : A \rightarrow C$ to any pair of maps $f : A \rightarrow B$ and $g : B \rightarrow C$. The above data have to satisfy the following two axioms:
i) for any object A there exists an identity map $i_A : A \rightarrow A$, such that $i_A f = f$ and $gi_A = g$ for any $f : B \rightarrow A$ and $g : A \rightarrow B$;
ii) the composition law is associative.

Familiar examples of categories are, e. g., the category *Set*, whose objects are the ordinary sets and whose maps are the ordinary maps between sets, together with the usual composition law, and the category $\mathcal{K}$-Vect, consisting of the vector spaces over a field $\mathcal{K}$ and of the $\mathcal{K}$-linear maps between them, together with the the usual composition law (see [5] for further examples and a description of the role of category theory in the mathematical practice). Dynamical systems defined by linear differential equations may form a category as described in [6].

Given a map $f : A \longrightarrow B$, the object A is said to be the domain of f, or $A = \mathrm{Dom}(f)$, and the object B is said to be the codomain of f, or $B = \mathrm{Codom}(f)$. A pair of maps $f : A \longrightarrow B$ and $g : C \longrightarrow B$ with the same codomain in a category $\mathcal{C}$ will be denoted by $(A \xrightarrow{f} B \xleftarrow{g} C)$. Given one pair of this kind, one can consider the class of all pairs of maps with the same domain $g' : D \rightarrow A$ and $f' : D \rightarrow C$, denoted by $(A \xleftarrow{g'} D \xrightarrow{f'} C)$, such that $fg' = gf'$. Such class may contain a distinguished element whose properties are described by the following Definition.

Definition 1. Given a pair $(A \xrightarrow{f} B \xleftarrow{g} C)$, its pullback is a pair $(A \xleftarrow{g'} D \xrightarrow{f'} C)$ such that
i) $fg' = gf'$ and
ii) if $(A \xleftarrow{g''} D' \xrightarrow{f''} C)$ is another pair for which i) holds, there exists a unique map $h : D' \rightarrow D$ such that $g'' = g'h$ and $f'' = f'h$. (See the commutative diagram below.)

$$\begin{tikzcd}
& B & \\
A \arrow[ur, "f"] & & C \arrow[ul, "g"'] \\
& D \arrow[ul, "g'"'] \arrow[ur, "f'"] & \\
& D' \arrow[u, "h"] \arrow[uul, "g''"] \arrow[uur, "f''"'] &
\end{tikzcd}$$

Proposition 1. (See [5] Chap. 2 Section 4.) If $(A \xleftarrow{g'} D \xrightarrow{f'} C)$ and $(A \xleftarrow{g''} D' \xrightarrow{f''} C)$ are both pullbacks of $(A \xrightarrow{f} B \xleftarrow{g} C)$, then there exists in $\mathcal{C}$ an isomorphism $i : D' \to D$ such that $g'' = g'i$ and $f'' = f'i$.

Therefore pullbacks, if they exist, are essentially unique. In the category *Set* pullbacks can easily be constructed. To this aim, given $(A \xrightarrow{f} B \xleftarrow{g} C)$, let us consider the cartesian product $A \times C$ and the canonical projections $pr_A : A \times C \to A$ and $pr_C : A \times C \to C$. Calling D the subset of $A \times C$ defined by $D = \{(a,c) \in A \times C \textit{ such that } f(a) = g(b)\}$ and denoting respectively by g' and by f' the restrictions of pr_A and of pr_C to D, it is possible to show that $(A \xleftarrow{g'} D \xrightarrow{f'} C)$ is a pullback. This construction extends to the category $\mathcal{K}$-Vect simply remarking that all objects involved are vector spaces and all maps are linear.

Let us consider now a pair of maps with common domain $(A \xleftarrow{f} B \xrightarrow{g} C)$, then one can consider the class of all pairs of maps with common codomain $(A \xrightarrow{g'} D \xleftarrow{f'} C)$ such that $g'f = f'g$. Such class may contain a distinguished element whose properties are described by the following Definition.

Definition 2. Given a pair $(A \xleftarrow{f} B \xrightarrow{g} C)$, its pushout is a pair $(A \xrightarrow{g'} D \xleftarrow{f'} C)$ such that
i) $g'f = f'g$ and
ii) if $(A \xrightarrow{g''} D' \xleftarrow{f''} C)$ is another pair for which i) holds, there exist a unique map $h : D \to D'$ such that $g'' = hg'$ and $f'' = hf'$.

Proposition 2. (See [5] Chap. 2 Section 4.) If $(A \xrightarrow{g'} D \xleftarrow{f'} C)$ and $(A \xrightarrow{g''} D' \xleftarrow{f''} C)$ are both pushout of $(A \xleftarrow{f} B \xrightarrow{g} C)$, then there exists in $\mathcal{C}$ an isomorphism $i : D' \to D$ such that $g'' = g'i$ and $f'' = f'i$.

Proposition 2 states that pushouts, if they exist, are essentially unique. In the category *Set* pushouts can easily be constructed. Given $(A \xleftarrow{f} B \xrightarrow{g} C)$, let us consider the set $A \cup C$ and the canonical injections $i_A : A \to A \cup C$ and $i_C : C \to A \cup C$. Calling D the quotient set of $A \cup C$ obtained by identifying two points $a \in A \cup C$ and $c \in A \cup C$ if there exists $b \in B$ such that $a = i_A f(b)$ and $b = i_C g(b)$ and denoting respectively by g' and by f' the maps obtained by composing respectively i_A and i_C with the canonical projection $pr : A \cup C \to D$, it is possible to show that $(A \xrightarrow{g'} D \xleftarrow{f'} C)$ is a pushout. This construction extends to the category $\mathcal{K}$-Vect simply substituting $\cup$ with $\oplus$ and remarking that all objects involved are vector spaces and all maps are linear.

3. COMPOSITION OF I/O BEHAVIORAL SYSTEMS

Following the approach of [8], a behavioral dynamical system Σ is described by a triple $(T, \mathbf{U}, \mathcal{B})$, where T is the discrete-time axis and $\mathcal{B}$ is a subset of the set $\mathbf{U}^T$

of all $\mathbf{U}$-valued trajectories. If the set $\mathbf{U}$ can be viewed as a cartesian product $\mathbf{U} = U \times Y$, a convenient way of representing Σ is by means of the pair of maps $(U^T \overset{p}{\leftarrow} \mathcal{B} \overset{q}{\rightarrow} Y^T)$, where p and q are the restriction to $\mathcal{B}$ of the canonical projections from $U^T \times Y^T$ onto U^T and Y^T respectively. It is understood that two triples $(T, U \times Y, \mathcal{B})$ and $(T, U \times Y, \mathcal{B}')$, with $\mathcal{B}$ and $\mathcal{B}'$ contained in $U \times Y$, as well as the associated pairs of maps $(U^T \overset{p}{\leftarrow} \mathcal{B} \overset{q}{\rightarrow} Y^T)$ and $(U^T \overset{p'}{\leftarrow} \mathcal{B}' \overset{q'}{\rightarrow} Y^T)$ describe the same system if $\mathcal{B}$ and $\mathcal{B}'$ are the same subset of $U^T \times Y^T$. That is, more precisely, if there exists an isomorphism $i : \mathcal{B} \rightarrow \mathcal{B}'$ such that $p = p'i$ and $q = q'i$.

If in a suitable set of axioms we can think of U and Y, respectively, as of the input signal space and the output signal space of Σ, it is quite natural to consider the problem of defining an operation corresponding to the action of taking the outputs of one system of the above kind as inputs of a second one. A way of doing so by using the notion of pullback is described in the following Definition.

Definition 3. Given two I/O dynamical systems $\Sigma = (U^T \overset{f_1}{\leftarrow} \mathcal{B} \overset{f_2}{\rightarrow} Y^T)$ and $\Sigma' = (Y^T \overset{g_1}{\leftarrow} \mathcal{B}' \overset{g_2}{\rightarrow} Z^T)$, the series composition of Σ and Σ' is the dynamical system $\Sigma'' = (T, U \times Z, \mathcal{B}'')$, where $\mathcal{B}''$ is defined by taking the pullback $(\mathcal{B} \overset{g'}{\leftarrow} \mathcal{B}'' \overset{f'}{\rightarrow} \mathcal{B}')$ of $(\mathcal{B} \overset{f_2}{\rightarrow} Y^T \overset{g_1}{\leftarrow} \mathcal{B}')$ and identifying the generic point $x \in \mathcal{B}''$ with the point $(g'f_1(x), f'g_2(x)) \in (U \times Z)^T$. (See the diagram below.)

$$\begin{tikzcd}
U^T & & Y^T & & Z^T \\
& \mathcal{B} \arrow[ul, "f_1"] \arrow[ur, "f_2"'] & & \mathcal{B}' \arrow[ul, "g_1"'] \arrow[ur, "g_2"] & \\
& & \mathcal{B}'' \arrow[ul, "g'"'] \arrow[ur, "f'"] & &
\end{tikzcd}$$

The notion formalized above coincides, for I/O behavioral systems,with the more general notion of interconnection of two systems considered in [8] XII. The series composition of Σ_1 and Σ_2 introduced above will be denoted by $\Sigma_1 \cap \Sigma_2$. this notation is motivated by the fact that, if $U^T = Z^T$, $\mathcal{B}''$ coincides with $\mathcal{B} \cap \mathcal{B}'$. Clearly, in this setting, the composition is independent from the order and $\Sigma_1 \cap \Sigma_2 = \Sigma_2 \cap \Sigma_1$.

When the system Σ is described by behavioral equations, that is $\mathcal{B}$ is defined as the set $\{(u, y) \in (U \times Y)^T \textit{ such that } f_1(u) = f_2(y)\}$, we have a situation dual to the previous one, since we may represent Σ as a pair $(U^T \overset{f_1}{\rightarrow} E \overset{f_2}{\leftarrow} Y^T)$ of maps with common codomain. In this case we give the following Definition.

Definition 4. Given two I/O dynamical systems described by behavioral equations $\Sigma = (U^T \overset{f_1}{\rightarrow} E \overset{f_2}{\leftarrow} Y^T)$ and $\Sigma' = (Y^T \overset{g_1}{\rightarrow} E' \overset{g_2}{\leftarrow} Z^T)$, the series composition of Σ and Σ' is the dynamical system described by behavioral equations $\Sigma'' = (U^T \overset{g'f_1}{\rightarrow} E'' \overset{f'g_2}{\leftarrow}$

Z^T), where the pair $(E \xrightarrow{g'} E'' \xleftarrow{f'} E')$ is the pushout of $(E \xleftarrow{f_2} Y^T \xrightarrow{g_1} E')$. (See the diagram below.)

$$\begin{tikzcd}
& & E'' & & \\
& E \arrow[ur, "g'"] & & E' \arrow[ul, "f'"'] & \\
U^T \arrow[ur, "f_1"] & & Y^T \arrow[ul, "f_2"'] \arrow[ur, "g_1"] & & Z^T \arrow[ul, "g_2"']
\end{tikzcd}$$

Let us now restrict our attention to the class of linear AR-systems, in the sense of [8] and [7], extensively studied also in [6]. We can associate with a $q \times (p+m)$-matrix $R(s)$, whose entries belong to $\Re[s]$ (i.e. are polynomials with real coefficients in the indeterminate s), a set of autoregressive equations

$$R(s)\, w(t) = 0, \tag{1}$$

where s denote the shift operator acting on the signal space $(\Re^{p+m})^T$. The set of solutions $w(t) \in (\Re^{p+m})^T$ of Equation 1 defines a behavioral system and autoregressive systems can be characterised as equivalence classes, up to unimodular left factors, of polynomial matrices.

Writing the the $q \times (p+m)$ polynomial matrix $R(s)$ as $R(s) = (Q(s)\, P(s))$, where $Q(s)$ and $P(s)$ are polynomial matrices of dimension $q \times p$ and $q \times m$ respectively, a convenient way of representing the AR-system Σ, determined by $R(s)$, is by means of linear maps $((\Re^m)^T \xrightarrow{Q} (\Re^q)^T \xleftarrow{P} (\Re^p)^T)$, where Q and P are induced by $Q(s)$ and $P(s)$ respectively. We can now introduce a notion of composition for AR-systems.

Definition 5. Given two linear, I/O, AR-systems Σ_1 and Σ_2 represented respectively by $(\Re^m)^T \xrightarrow{Q_1} (\Re^q)^T \xleftarrow{P_1} (\Re^p)^T)$ and $(\Re^q)^T \xrightarrow{Q_2} (\Re^s)^T \xleftarrow{P_2} (\Re^p)^T)$, their series composition is the AR-system represented by $((\Re^m)^T \xrightarrow{Q_2'Q_1} (\Re^n)^T \xleftarrow{P_1'P_2} (\Re^p)^T)$, where $(\Re^r)^T \xrightarrow{Q_2'} (\Re^n)^T \xleftarrow{P_1'} (\Re^s)^T)$ is the pushout (see [5]) of $(\Re^r)^T \xleftarrow{P_1} (\Re^r)^T \xrightarrow{Q_2} (\Re^s)^T)$. In particular, $Q_2'(s)\, P_1(s) = P_1'(s)\, Q_2(s)$ is the least common left multiple of $P_1(s)$ and $Q_2(s)$.

Remark that, if $p = q$ (respectively $m = q$) and the square matrix $P(s)$ (respectively $Q(s)$) is nonsingular, we can associate to the AR-system Σ defined by $(Q(s)\, P(s))$ the rational matrix $G(s) = P^{-1}(s)\, Q(s)$ (respectively $G(s) = Q^{-1}(s)\, P(s)$), that, if it turns out to be strictly proper, can be interpreted as a Kalman transfer matrix. This shows that AR-systems can be viewed as a generalization of Left Fractional Representations for conventional linear input/output, finite dimensional systems.

4. THE MATCHING PROBLEM

The notion of series composition defined in the previous Section allows to state, in the framework of the behavioral approach, the following general, system theoretic problem, which, under various formulations, originated a large literature (see e. g. [1] and [3] for a list of references and a more general discussion of the problem).

Problem 1. (Matching Problem for Behavioral Systems) Given two behavioral systems $\Sigma_1 = (T, U \times Y, \mathcal{B}_1)$ and $\Sigma_2 = (T, U \times Z, \mathcal{B}_2)$ find a behavioral systems $\Sigma_3 = (T, Z \times Y, \mathcal{B}_3)$ such that the composite system $\Sigma_2 \cap \Sigma_3$ coincides with Σ_1.

Assume that the *Matching Problem* is solvable. Then there exists a system Σ_3, represented, for instance, by $\Sigma_3 = (Z^T \xleftarrow{p_3} \mathcal{B}_3 \xrightarrow{q_3} Y^T)$, such that $\Sigma_2 \cap \Sigma_3 = \Sigma_1$. By definition of composition we have that the equality $p_1 = p_2 p_3'$, where p_3' is defined by saying that $(\mathcal{B}_2 \xleftarrow{p_3'} \mathcal{B} \xrightarrow{q_2'} \mathcal{B}_3)$ is the pullback of $(\mathcal{B}_2 \xrightarrow{q_2} Z^T \xleftarrow{p_3} \mathcal{B}_3)$, holds. So we have the following necessary condition for the solution of the *Matching Problem*.

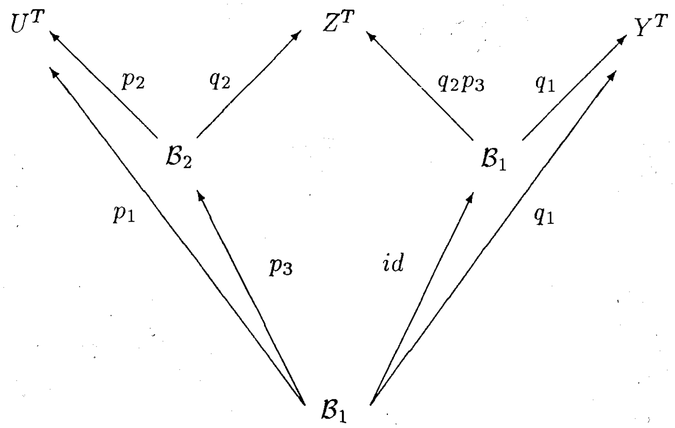
Proposition 3. Given a behavioral system Σ_1, represented by $(U^T \xleftarrow{p_1} \mathcal{B}_1 \xrightarrow{q_1} Y^T)$, and a system Σ_2, given by $(U^T \xleftarrow{p_2} \mathcal{B}_2 \xrightarrow{q_2} Z^T)$, the Matching Problem is solvable only if

$$\operatorname{Im} p_1 \subseteq \operatorname{Im} p_2. \tag{2}$$

A more complete result is provided, under a suitable restrictive hypothesis, by the following Proposition.

Proposition 4. Let Σ_1 and Σ_2 be given as in Proposition 3 and assume that in the representation $(U^T \xleftarrow{p_2} \mathcal{B}_2 \xrightarrow{q_2} Z^T)$ of Σ_2 the map q_2 is injective. Then the Matching Problem has a solution if and only if condition (2) is satisfied.

Proof. In the hypothesis $\operatorname{Im} p_1 \subseteq \operatorname{Im} p_2$ it is possible to factor p_1 through p_2. So, let $p_3 : \mathcal{B}_1 \longrightarrow \mathcal{B}_2$ be any map such that $p_2 p_3 = p_1$. By the injectivity of q_2 it easily turns out that $(\mathcal{B}_2 \xleftarrow{p_3} \mathcal{B}_1 \xrightarrow{id} \mathcal{B}_1)$ is the pullback of $(\mathcal{B}_2 \xrightarrow{q_2} Z^T \xleftarrow{q_2 p_3} \mathcal{B}_1)$. Moreover, the map $j : \mathcal{B}_1 \longrightarrow Z^T \times Y^T$ given by $j(b) = (q_2 p_3(b), q_1(b)$ is easily seen to be injective, as $(q_2 p_3(b), q_1(b)) = (q_2 p_3(b'), q_1(b'))$ implies $(p_3(b), q_1(b) = (p_3(b'), q_1(b'))$, hence $(p_1(b), q_1(b)) = (p_1(b'), q_1(b'))$ and $b = b'$. Thus, the pair $(Z^T \xleftarrow{q_2 p_3} \mathcal{B}_1 \xrightarrow{q_1} Y^T)$ defines a behavioral system, say $\Sigma_3 = (T, Z \times Y, \mathcal{B}_1)$. Now, by the commutativity of the diagram below, one has $\Sigma_1 = \Sigma_2 \cap \Sigma_3$. □



Necessary and sufficient conditions for the existence of solution to the Model Matching Problem could be worked out, in a less elegant way, under different hypothesis, less restrictive than those in the above Proposition. However, in the general case a complete characterization of the existence of solution for the Matching Problem does not seem easy to obtain.

The result of **Proposition** 4 can be applied to the problem of factoring a given system Σ through a subsystem.

Proposition 5. If Σ is represented by $(U^T \xleftarrow{p} \mathcal{B} \xrightarrow{q} Y^T)$, then Σ can be factored, in particular, through any subsystem characterized by a subset $\mathcal{B}'$ of $\mathcal{B}$ such that $p(\mathcal{B}) = p(\mathcal{B}')$ and $q|_{\mathcal{B}'}$ is injective.

5. THE MATCHING PROBLEM FOR AR-SYSTEMS

Assume now that Σ is the system defined by the following set of autoregressive equations

$$R(s)w(t) = 0$$

represented by the pair of linear maps $((\Re^m)^T \xrightarrow{Q} (\Re^q)^T \xleftarrow{P} (\Re^p)^T)$ where Q and P are induced by $Q(s)$ and $P(s)$ respectively. We can state the following Problem

Problem 2. (Matching Problem for AR-systems) Given two AR-systems $\Sigma_1 = ((\Re^p)^T \xrightarrow{Q_1} (\Re^n)^T \xleftarrow{P_1} (\Re^q)^T)$ and $\Sigma_2 = ((\Re^m)^T \xrightarrow{Q_2} (\Re^s)^T \xleftarrow{P_2} (\Re^q)^T)$ find, if possible, an AR-system $\Sigma_3 = ((\Re^p)^T \xrightarrow{Q_3} (\Re^r)^T \xleftarrow{P_3} (\Re^m)^T)$ such that, representing the composite system $\Sigma_2 \cap \Sigma_3$ as $\Sigma_2 \cap \Sigma_3 = ((\Re^p)^T \xrightarrow{Q_3Q_2'} (\Re^{n'})^T \xleftarrow{P_2P_3'} (\Re^q)^T)$, one has $n' = n$ and there exists a square nonsingular, $n \times n$ rational matrix $V(s)$ such that $Q_1(s) = Q_3(s)\,Q_2'(s)\,V(s)$ and $P_1(s) = P_2(s)P_3'(s)\,V(s)$.

Remark that in case $Q_1(s)$ and $Q_2(s)$ are square and nonsingular and $G_1(s) = Q_1^{-1}(s)\,P_1$ and $G_2(s) = Q_2^{-1}(s)\,P_2$ are strictly proper, the above formulation of the

Matching Problem requires that the compensated system and the model have the same transfer matrix (compare with [1]).

Similarly to the case studied in the previous Section, we have for the above problem the following key result.

Proposition 6. Solution for the Matching Problem for AR-systems Σ_1 and Σ_2, exists only if

$$\operatorname{Ker} P_2 \subseteq \operatorname{Ker} P_1. \tag{3}$$

Proof. If the *Matching Problem* is solvable by means of an AR-system $\Sigma_3 = ((\Re^p)^T \xrightarrow{Q_3} (\Re^r)^T \xleftarrow{P_3} (\Re^m)^T)$, the relation $P_1(s) = V(s)\,P_3(s)\,P_2(s)$ implies $\operatorname{Ker} P_2 \subseteq \operatorname{Ker} P_1$. □

Proposition 7. Assume that the matrix $Q_2(s)$ in the representation $(\Re^m)^T \xrightarrow{Q_2} (\Re^s)^T \xleftarrow{P_2} (\Re^q)^T)$ of Σ_2 is full row rank. Then the Matching Problem for AR-systems has a solution if and only if, representing Σ_1 by means of $((\Re^p)^T \xrightarrow{Q_1} (\Re^n)^T \xleftarrow{P_1} (\Re^q)^T)$ condition (3) holds.

Proof. Necessity has already been discussed. Assume that $\operatorname{Ker} P_2$ is contained into $\operatorname{Ker} P_1$, then there exists an $n \times s$ rational matrix $W(s)$ such that $P_1(s) = W(s)\,P_2(s)$. Write $W(s)$ as $W(s) = V(s)\,P_3(s)$, where $P_3(s)$ is an $n \times s$ polynomial matrix and $V(s)$ is a square $n \times n$ matrix of the form $V(s) = \operatorname{diag}(s^{-\alpha_1}, s^{-\alpha_2}, \ldots, s^{-\alpha_n})$. Since $Q_2(s)$ is full row rank,

$(\Re^n)^T \xrightarrow{id} (\Re^n)^T \xleftarrow{P_3} (\Re^s)^T)$ is easily seen to be the push-out of $(\Re^n)^T \xleftarrow{P_3 Q_2} (\Re^m)^T \xrightarrow{Q_2} (\Re^s)^T)$. Hence, from the commutative diagram we have that the AR-system $\Sigma_3 = (\Re^p)^T \xrightarrow{V^{-1}Q_1} (\Re^n)^T \xleftarrow{P_3 Q_2} (\Re^m)^T)$ solves the problem we are considering.

$$\begin{tikzcd}
 & & & (\Re^n)^T & & & \\
 & & & (\Re^n)^T \arrow[u, "V"] & & & \\
 & & (\Re^n)^T \arrow[ur, "id"] & & (\Re^s)^T \arrow[ul, "P_3"'] & & \\
(\Re^p)^T \arrow[uuurrr, "Q_1"] \arrow[urr, "V^{-1}Q_1"'] & & & (\Re^m)^T \arrow[ul, "P_3Q_2"] \arrow[ur, "Q_2"'] & & & (\Re^q)^T \arrow[uuulll, "P_1"'] \arrow[ull, "P_2"]
\end{tikzcd}$$

Remark that, in case $Q_1(s)$ and $Q_2(s)$ are square and nonsingular, letting $G_1(s) = Q_1(s)^{-1}P_1(s)$ and $G_2(s) = Q_2(s)^{-1}P_2(s)$, the condition of Proposition 7 coincides with the condition $\operatorname{Ker} G_2 \subseteq \operatorname{Ker} G_1$, which, in turn, is well known to be equivalent to the existence of a rational matrix $G_3(s)$ such that $G_1(s) = G_3(s)G_2(s)$. In this way, Proposition 6 is seen to be a generalization of a well known result about the factorization of transfer matrices. Analogously, if $P_1(s)$ and $P_2(s)$ are square and nonsingular, letting $F_1(s) = P_1^{-1}Q_1(s)$ and $F_2(s) = P_2^{-1}Q_2(s)$ is, in the hypothesis of Proposition 7 we have $\operatorname{Im} F_1 \subseteq \operatorname{Im} F_2$, which assures the possibility of factoring $F_1(s)$ through $F_2(s)$ (compare with the Exact Model Matching Problem considered e.g. in [1]).

6. CONCLUSION AND FUTURE DIRECTIONS OF WORK

The *Matching Problem* has been considered in a general behavioral context and conditions for the existence of solutions have been found under suitable hypothesis. As mentioned earlier, a complete characterization, in the general case, of the existence of solutions does not seem easy to obtain. Using the possibility of transforming disturbance decoupling problems into model matching problems, the results of Section 4 and Section 5 can be used for studying decoupling problems for general behavioral systems and for AR-systems. Concerning AR-systems, the set of solutions to the *Matching Problem* can be investigated using algebraic tools in order to give cc ıditions for the existence of solutions having particular dynamical properties.

(Received February 24, 1995.)

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