Kybernetika

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Kybernetika, Vol. 6 (1970), No. 3, (189)--216

Persistent URL: http://dml.cz/dmlcz/125218

KYBERNETIKA ČÍSLO 3, ROČNÍK 6/1970

A Class of Models of Semi-Autonomous Subsystems

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Both a medium and an adjacent and unadjacent environment of an arbitrary subsystem of a system cannot be modelled without any regard to each other, in general. In the present paper a modelling of such dependences is ensured by means of certain pairs of rudimentary properties. Each of those pairs as a whole, being unresolved into its two components, does not influence its medium and environment, but is resolvable under certain conditions. Each rudimentary property originates from such a pair. The rudimentary properties constitute the single properties of the single elements which again constitute systems. A specification of a concept of event and a model of time and space complete the definition of the system. The interelement relations and their development, the information transfer or/and formation and signalling are represented in matrix form.

1. INTRODUCTION

Without regard to any laws, we can consider an arbitrary system or subsystem, an arbitrary state of the given subsystem, an arbitrary medium or environment of the said subsystem. But is it correct in general? If it is so or not, we want to try to avoid any incorrectness.

In the course of a design of a model the design can be changed till the model becomes similar to the reality to which it has to correspond. How to make it easier — that is the question.

The paper deals with a model of a system constituted of elements. The aim of the paper is to introduce a method which allows

1. modelling of subsystems of a system,
2. modelling of interactions among the single subsystems,
3. modelling of influences of the interactions already in the course of the constitution of the said subsystems.

Certainly, our model does not involve all possible systems. Thus, first of all, it ought to be said that our model of system is only one of many possible mathematical or physical means, each of which purports to represent reality. Our model of system is only a certain abstract approach to reality. It is characterized by certain pairs of rudimentary properties. Rudimentary properties originating out of such pairs can constitute single properties of elements which again can constitute a system of elements.

Secondly, we get over to a special consideration of one such method — the equivalence in influence (see § 3.2) of all single rudimentary properties of the same quality type on their medium and environment. That equivalence makes possible an interchangeability of such properties.

As the rudimentary properties are not specified, a certain class of models is considered. A response of an element to an action of its environment or medium depends on a choice of its strategy in general. Provided at least some of the applied elements be semi-autonomous, the subsystem can be semi-autonomous as well.

Our model leads to design a subsystem simultaneously with a design of its medium and environment — if any exists. The model enables certain tests of the design in each stage of it. The test can be carried out applying the mentioned unresolved pairs of rudimentary properties. A possibility to take advantage of the model presented is in the synthesis or analysis or demonstration of subsystems where both a medium and an environment play their part. We can study the principles according to which subsystems change with time and the influences which make them to do so.

The presented paper is established as follows:

The subject under discussion is outlined. The definition first suggests a system, then details the elements and their properties, continues with a subsystem and terminates with a system again. After the basic definition and nomenclature the general and special relations are repeated in a form of certain equalities. An example of resolving unresolved pairs is given. Matrices (e. g. [1], [2], [3]) are used to express an example of processing the model introduced.

2. BASIC DEFINITION OF THE SYSTEM

Definition 1. Let a system and modelling of the system be considered. Let the model of the given system be as follows:

1. There is a *system* constituted of *elements*. The system is *developing* a *sequence* of its own single *states*.

2. At least one of the elements of the system is a proper or improper part of an *observer* considered which is a part of the system. There are more than one observer possible in the system.

3. Each element possesses one or more *properties*. Each property is possessed by an element which is a *carrier* of it.

4. Let each element contain one or more *input channels* and one or more *output channels*.

a) Each pair of elements such that one channel at least of the one of those elements continues as a channel of the other element, are elements *mutually connected* or *contiguous* elements or elements in *contact*.

b) Both the channel of the one element (which continues as the channel of the mentioned other element) and the channel of the other element (which is a continuation of the channel of the given first element) constitute a *contact* of those two elements.

5. a) Let any two elements be given out of whatever but the same state of a system (point 24 g). If the said elements are in contact, they are either *outside-contiguous* or *inside-contiguous*.

b) If the two elements are outside-contiguous, then each of both is a *neighbour* of the other, it is *a part of the adjacent environment* of the other element.

c) If the two elements are inside-contiguous, then each of them is *a part of the medium* of the other, i. e. none of them is a neighbour of the other.

d) If the two elements are not of contact, i. e. if they are neither inside-contiguous nor outside-contiguous, then each of them is *a part of the nonadjacent environment* of the other.

e) The properties

– to be a part of the adjacent environment,
– to be a part of the medium,
– to be a part of the nonadjacent environment,

are *locational relations* between two elements, i. e. they are *locational properties* of any element with regard to the other.

f) The locational relations between any two elements can be *changed*. Each change of the contiguity of two elements is a *dislocation* of the elements.

6. Let both an arbitrary element and some other elements of a system be given out of whatever but the same state of the system (point 24 g).

a) The union of all these of the given other elements which are parts of adjacent environment of the given element in the given system is the (*whole*) *adjacent environment* of the said element in that state.

b) The union of all these of the given other elements which are parts of the medium of the given element in the given state of the system is the *medium* (*as a whole*) of the said element in that state.

c) The union of all these of the given other elements which are parts of nonadjacent environment of the given element in the given state of the system is the *nonadjacent environment* (*as a whole*) of the said element in that state.

7. a) Each single element *exerts action* (applies its *demands*) on its medium and on a part at least of its adjacent environment, by means both of its properties and by *informations transferred* or/and formed by its properties and their development (point 24 h) *used as signals.*

b) Each element *can be influenced* by means of both its *autonomy*, of properties of its medium and adjacent environment, and by *interpretations* taken for *informations transferred* by *events considered as signals.*

c) The union of the simultaneous actions of any element is its *activity.*

8. a) The *properties of a contact* depend not only on the two elements the channels of which constitute the contact considered, but also on the *development* in the past *of the contact*, on the *properties transferred* (and their development) through the contact, on the *development of the transfer*, and on the adjacent environments and the media of the two elements.

b) The states in which a *channel is an input* and those in which the same *channel is an output* or both depend on the properties of the contact.

c) The contact enables an *interaction* between the two elements with regard to its *transit properties* (point 13 a) for the single of both possible *transition directions*, in general.

d) Each two mutually contiguous elements, with a transit in one at least of both possible transit directions, are *connected* in that direction. The transit being dis-established the *connectivity is discontinued*.

e) Each action of each element is *decomposed* into *simultaneous component actions* of the single outputs of the element.

f) The *simultaneous* actions of the adjacent environment and the medium of the element, exerted on the single inputs of the element, are *composed* into one and only one *resulting action* of each single quality type on the element.

9. a) There exists a non-empty at most countable set of quality types of *rudimentary properties*, so that one and only one *opposite quality type* corresponds to each *quality type*.

b) A rudimentary property is a constituent part of a property of an element (of an *elementary property*). An element is the *union* of certain rudimentary properties. The element possesses each of them with a certain *probability*.

c) Any two rudimentary properties, being carried by elements, gain the locational relations of the elements. Those locational relations and their changes can be given by the corresponding probabilities.

d) Each combination of rudimentary properties, each property of different quality type, is an *allowable combination* of rudimentary properties to be *processed – accepted* or *lost – simultaneously* by the same element.

e) Each element includes one single *processor* for each single quality type.

f) There are certain *allowable quantities* of *stores* of rudimentary properties of the single quality types of an element, and methods for *maintaining of the limits* of the quantities for the single possible states of the element.

g) Between any two contiguous elements one single contact for each quality type is possible simultaneously.

h) If the single quality types are to be specified, this has to be done in an appendix to the presented definition.

10. a) Each rudimentary property originates from an *unresolved pair* of rudimentary properties (briefly "unresolved pair") of mutually opposite quality types, and either being a medium of the other.

b) Each unresolved pair is *undistinguishable* as a whole at any level of *resolution acuity*, i. e. it does not appear by its effects.

c) The single rudimentary properties of an unresolved pair *do not act* on the rest of their medium and environment because both actions cancel each other, but they both are *resolvable* by those medium and environment under certain *conditions*.

d) An unresolved pair being resolved, its two components appear as outside contiguous, the location of which is their *reference location*.

11. a) There is an *undefinite number* of unresolved pairs.

b) Every medium contains an undefinite number of unresolved pairs.

c) Every adjacent environment contains an undefinite number of unresolved pairs.

12. Any two elements, each of which possessing not a single rudimentary property of any common quality type, are *compatible* in their simultaneous processing, i. e. either one can be a part of the medium of the other one within the simultaneous processing of both. Any two elements, each of which possessing at least one rudimentary property of one at least common quality type, are *incompatible* in their simultaneous processing, i. e. neither one can be any part of the medium of the other within the simultaneous processing of both.

13. a) Each element, when passing through a transit, *gains or loses* (or *loses*) certain one or more or none of rudimentary properties out of unresolved pairs out of its medium being left. It means that certain rudimentary properties are assigned to be gained or lost by the single possible elements when transiting through a given transit.

b) The *gain* or *loss* depend not only on both elements possessing the given transit, and on the transiting element, but also on the development of the transit in the past and on the development of the transiting element.

c) Accepting the same two rudimentary properties, which have been resolved from any common unresolved pair, the element *loses both*, having enabled their *remerging*.

d) That remerging can be associated with the simultaneous resolving one or more unresolved pairs of rudimentary properties of one or more pairs of quality types. In that remerging and resolving consists the *conversion* of some events of several quality types into events partly at least of other types.

14. a) One or more simultaneous or subsequent rudimentary properties of one or more certain different quality types are *needed and satisfy to resolution* of one or more unresolved pairs, out of the common medium of the mentioned properties or out of their common adjacent environment. In other words, they are the cause of a *simple* or a *multiple resolution stir* (resolving perturbation).

b) The *resolution* means that the considered rudimentary properties, being a part of the medium to each other, become a part of adjacent environment to each other. Each of both components of any resolved pair becomes distinguishable.

c) The needed quantity of resolving influence and a mechanism of the resolution are given by the relations among the single quality types of rudimentary properties.

15. In a *special case of modelling* all rudimentary properties of each but the same quality type are mutually *equivalent* in their influences on their media and environments,

16. Let a set of elements out of whatever but the same state of a system be given.

a) If a sequence of the elements can be found so that the element of each order is in contanct with the element of the one less order or with the element of the one more order (including both of them), then the elements of the sequence considered constitute a *path*.

b) The element of the lowest order is the *initial element* of the path, and the element of the highest order is the *terminal element*, if the path proceeds through contacts in their transit direction.

c) Each element, which is neither initial nor terminal, is an *intermediate element.*

d) The path is continuous between the initial element and the terminal one. The given pair of the elements is *connected* through the given path.

e) The path is *limited* by its initial and terminal elements and acts as a *connection* between the two elements which are its *end elements.*

f) Any two elements of whatever but the same path can be *identical* (can *coincide*) or not.

g) Each mutually dislocation of the elements constituting a path or going to constitute a path is a *switching* of the path.

17. Each two paths with one common element at least are *mutually connected paths.*

18. a) Each set of elements of whatever but the same state of a system, between each two elements of which there is one path at least, is a *subsystem* of the system considered.

b) The simplest subsystem consists of one element.

c) The simplest element possesses one rudimentary property.

d) Any locational relations of elements of a subsystem constitute a *configuration* of those elements (of that subsystem).

19. Let a subsystem of a system be given. Then all these elements out of whatever but the same state of the system, which are not out of the given subsystem in that state,

a) such that each of the elements is a neighbour of one element at least of the given subsystem, constitute the *adjacent environment of* that *subsystem* in the given state;

b) such that each of the elements is inside-contiguous with one element at least of the given subsystem, constitute the *medium of* that *subsystem* in the given state;

c) such that each of the elements is neither out of medium nor out of adjacent environment of the subsystem, constitute the *nonadjacent environment of* that *subsystem* in the given state.

20. a) A subsystem can be *evaluated* according to one or more *criteria* of approach to any given *aims*.

b) A subsystem can be noted for one of its properties, which is *predominant* over the other properties of the subsystem according to a *classification* performed on the subsystem.

c) Each classifying element attaches one *relative priority class*, out of a nonempty at most countable set of relative priority classes, to each single element classified by it for each quality type and any *level* of classification.

d) The rudimentary properties of any but the same quality type can but need not be classified in accordance with the classification of their carrier elements.

e) A subsystem can process its informations. The subsystem can pursue an aim by that processing.

21. The union of all and only all subsystems considered in an arbitrary but the same state, is a *system*.

22. a) Each two locational changes of rudimentary properties in a system pass either in a *sequence* or in a *coincidence*.

b) Any rudimentary properties and any sequence or/and coincidence of locational changes of the given rudimentary properties, are an *event*. (Any configuration or development of any properties is an *event*.)

c) The medium and environment of an event are the *conditions* of the event.

23. A choice of *test properties* (to be used as *reference quantities* of the single quality types), and *reference processes*, and *comparison modes*, is to be carried out in an appendix to the present definition in order

a) to *compare* quality types and quantities of the single properties of the single elements in their influence on their medium and adjacent environment,

b) to *evaluate* the single states of the single subsystems,

c) to state *total influences* (*resultants*) of two or more elements.

24. Let a subsystem of a system be given.

a) The sequence of both single locational changes of rudimentary properties and all coincidences of such changes in the subsystem, its medium and adjacent environment, constitute the *relative time of the subsystem* (*subsystem time*).

b) Each such change or coincidence of such changes respectively is an *instant* of the relative time of the subsystem.

c) The time of any subsystem is *continuous in its instants*.

d) Each sequence of instants, beginning with any one instant and ending with any subsequent instant of the time of the same subsystem, is an *interval* of the time of the subsystem.

e) The union of all and only all times of all and only all subsystems of a given system is the *time of the system* (*system time*).

f) The time of the system includes its instants and intervals.

g) All and only all elements of any subsystem with their properties at any instant of time constitute a *state of the subsystem* at that instant.

h) The time sequence of the single states of any subsystem is the *development of the subsystem*.

i) The sequence of time-located time intervals of the presence of the rudimentary properties of whatever but the same quality type in the corresponding processor of any one element is a realization of a *flow* of the given properties through the given processor. This presence can be expressed by its probability.

j) The union of flows through the elements of any one subsystem is a *process*.

25. Let a subsystem be given.

a) The union of all and only all media of all and only all elements of the subsystem at any instant of time is the *instantaneous space taken by the subsystem* (*instantaneous subsystem space*).

b) The space of a subsystem is *continuous in the media* of the single elements of the subsystem.

c) The union of all and only all subsystem spaces at any instant of system time is the *space of the system* (*system space*).

Remark 1. At least one of the elements of the system is an observer considered. Each development starts with such element in order to allow to track the development of the model, which is of interest for us.

Remark 2. A limitation of the quantities of rudimentary properties of an element is needed, for example, in order to avoid a partition of an element into two elements, etc.

Remark 3. If the element can process at most one rudimentary property of each single quality type simultaneously, then any simultaneous processing of any further rudimentary property is blocked (denied, refused, rejected, etc.).

Remark 4. The property predominant over the other properties of a subsystem in a classification performed on the subsystem may be for example an ability to observe, to consider, to control, to operate, to process, to signal, etc., further its receptivity, controllability, autonomy, and others like those.

Remark 5. It is to be noted that the defined space and time are approximations only, models — of real ones.

3. GENERAL AND SPECIAL RELATIONS

In order to abbreviate the presented paper, a notation will be introduced.

As far as the symbolics of the properties is concerned, three systems of symbols are possible:

1. System of increases (the element gains some property when passing through a transit);

2. system of decreases (the element looses or loses some property when passing through a transit);

3. a combination of both of them.

In the following part of this paper the system of increases will be used.

A method will be given for attaching numbers to the elements, rudimentary properties and their probabilities in order to derive labels that allow to distinguish among the various elements, rudimentary properties and probabilities respectively. For example, let us introduce the following notation:

s	– number of all distinguished pairs of quality types;
$+m$	– index of one of the quality types; $+m = +1, +2, \ldots, +s$;
$-m$	– index of the quality type opposite to the $(+m)$-th one; $-m = -1, -2, \ldots, -s$;
q	– quality type, in general; $q = +1, +2, \ldots, +s, -1, -2, \ldots, -s$;
r_{+m}	– rudimentary property of the $(+m)$-th quality type;
r_{-m}	– rudimentary property of the $(-m)$-th quality type;
p_m	– unresolved pair of rudimentary properties of the m-th two mutually opposite quality types $+m$ and $-m$;
$p_{m(v)}$	– the v-th unresolved pair of rudimentary properties of the m-th two opposite quality types $+m$ and $-m$; $v = 1, 2, \ldots$;
$r_{+m(v)}$	– rudimentary property of the $(+m)$-th quality type out of the v-th unresolved pair;
C_q	– set of rudimentary properties of the q-th quality type;
R	– undefinite number of unresolved pairs;
e_i	– the i-th element;
$n = n(t)$	– number of elements considered at instant t;
c	– relative priority class of an element or of its rudimentary property;
g	– number of the used priority classes;
t	– instant of time;
t, q, c, i, k	– five-tuple of indices relating the labelled symbol to the q-th quality type and c-th relative priority class, if the relation originates in the i-th element and is directed towards the k-th element at instant t; (that of the indices t, q, c, i, k which relates to all its possible values, may be replaced by a dash, for example: $t, -, -, i, k$);
$a_{t,q,c,i,k}$	– the probability of activity of the q-th quality type and c-th relative priority class (for accepting the given activity) exerted by the i-th element on the k-th element at instant t;
$\bigcup_q$	– union with respect to q;
$\cup$	– in union with;
$=$	– is equal to; relation of equivalence;

$\Rightarrow$	– implication;
$\circ$	– is a medium of; locational relation to be a medium to something;
$\rightarrow$	– change into;
$\in$	– belong to;
σ	– index of any output of an element;
x	– level of the classification of any output.

3.1. Influence properties for the general case

According to Definition 1, there is

(1) $$p_{m(v)} \cup R = R \quad \text{(definition 1, point 11a)},$$

(2) $$[p_{m(v)} = (r_{+m(v)} \circ r_{-m(v)})] \Rightarrow \quad \text{(point 10a,b,c)},$$
$$\Rightarrow [(r_{+m(v)} \circ r_{-m(v)}) \rightarrow (r_{+m(v)} \cup r_{-m(v)})]$$
(resolving, point 10d),

(3) $$[(r_{+m(v)} \cup r_{-m(v)}) \rightarrow (r_{+m(v)} \circ r_{-m(v)})] \Rightarrow [(r_{+m(v)} \circ r_{-m(v)}) = p_{m(v)}]$$
(merging, point 13c),

(4) $$r_{+m} = r_{+m} \circ R \quad \text{(point 10b,c and 11b,c)},$$

(5) $$r_{-m} = r_{-m} \circ R \quad \text{(point 10b,c and 11b,c)},$$

(6) $$e_{t,-,-,i,-} = \bigcup_{m=1}^{s} \bigcup_{u} \bigcup_{v} (r_{+m(u)}\, a_{+m(u)}(t) \cup r_{-m(v)}\, a_{-m(v)}(t)),$$

where

$a_{+m(u)}(t)$ is the probability that the element $e_i = e_{t,-,-,i,-}$ is the carrier of rudimentary property $r_{+m(u)}$ at instant t;

$a_{-m(v)}(t)$ is the probability that the element e_i is the carrier of rudimentary property $r_{-m(v)}$ at instant t;

$u = 1, 2, \ldots,$

$v = 1, 2, \ldots,$ (point 9b).

3.2. Additional influence properties for the special case. Equivalences

In a set C_q of rudimentary properties of the q-th quality type, if within it the single rudimentary properties are considered to be equivalent to each other in relation to their influences on their media and environments,

(7) $$r_{q(u)} = r_{q(v)},$$

then the rudimentary properties must satisfy

(i) the reflexiveness:

(8) $$r_{q(u)} \in C_q\,; \quad u = 1, 2, \ldots: \quad r_{q(u)} = r_{q(u)}\,;$$

(ii) the symmetry:

(9) $$r_{q(u)}, r_{q(v)} \in C_q\,; \quad v = 1, 2, \ldots : \\ [r_{q(u)} = r_{q(v)}] \Rightarrow [r_{q(v)} = r_{q(u)}]\,;$$

(iii) the transitiveness:

(10) $$r_{q(u)}, r_{q(v)}, r_{q(z)} \in C_q\,; \quad z = 1, 2, \ldots : \\ [r_{q(u)} = r_{q(v)} \text{ and } r_{q(v)} = r_{q(z)}] \Rightarrow [r_{q(u)} = r_{q(z)}]\,.$$

Supposing the property of equivalence, we can write

(11) $$r_{q(u)} = r_q = r_{q(v)}\,, \\ p_{m(u)} = p_m = p_{m(v)}\,.$$

Theorem 1. *Let any element possess any one of rudimentary properties* $r_{+m(u)}$, $r_{-m(v)}$ *already. Now let the element additionally accept the other of them. Then the element looses or loses both rudimentary properties, if the equivalence is valid:*

(12) $$[(r_{+m(u)} \cup r_{-m(v)}) \to (r_{+m(u)} \circ r_{-m(v)})] \Rightarrow [(r_{+m(u)} \circ r_{-m(v)}) = p_m]\,.$$

Proof. Substituting $r_{+m(u)} = r_{+m(v)}$ in the left-hand side of (12) according to (7), it follows (3) that

$$[(r_{+m(v)} \cup r_{-m(v)}) \to (r_{+m(v)} \circ r_{-m(v)})]$$

and

$$(r_{+m(v)} \circ r_{-m(v)}) = p_{m(v)}\,.$$

Finally, using the second row of (11), it is valid that

$$p_{m(v)} = p_m\,.$$

Q. E. D.

The resolution of unresolved pairs can be

(i) successive (resolution stir), for example

(13) $$R \to (r_{+1(1)} \cup r_{-1(1)}) \to (r_{+1(1)} \cup r_{-1(1)} \cup r_{+2(1)} \cup r_{-2(1)}) \to \\ \to (r_{+1(1)} \cup r_{-1(1)} \cup r_{+2(1)} \cup r_{-2(1)} \cup r_{+1(2)} \cup r_{-1(2)}) \to \ldots,$$

(ii) or simultaneous, for example,

(14) $$R \to (r_{+1(1)} \cup r_{-1(1)} \cup r_{+2(1)} \cup r_{-2(1)} \cup r_{+1(2)} \cup r_{-1(2)})\,,$$

(iii) or combined of both.

The more pairs are resolved the more their description becomes unclear. An arrangement in matrix form or in table form is often helpful. For example, the influences of any given element on the other elements will be given in a row, the influences of the other elements on the given element will be given in a column. (In mathematics the row index precedes the column index usually.) Of course, the system of rows and that of columns can be changed mutually. (The corresponding change of the matrix equations and the transposition of several matrices are easy.)

Remark 6. In order to distinguish between elements of the system and elements of the matrix, the elements of the matrix are denoted as components.

The elements out of the medium or out of the adjacent environment of any given element can be characterized by the non-zero values of probabilities of the corresponding locational relations. A single channel of input and that of output, of both the given element and of a contiguous one, is to be considered for each single quality type, in general.

The necessary and sufficient conditions for a simple or multiple resolution stir are given by certain properties of medium or adjacent environment or by both, which are to be defined in an appendix, if necessary.

In the matrices the elements can be labelled in the order of their occurrences in the system.

4. EXAMPLE OF RESOLVING UNRESOLVED PAIR AND OF MERGING RUDIMENTARY PROPERTIES: TWO MUTUALLY ADJACENT FREE BODIES

Given two mutually adjacent free bodies (e. g. [4], p. 72) denoted for example B_1 and B_2. Let each of the bodies be an element. There are three interactions:

1. Interaction between the two bodies, B_1 and B_2.
2. Interaction between body B_1 and all the other world excepting body B_2.
3. Interaction between body B_2 and all the other world excepting body B_1.

Let the force exerted by body B_1 on body B_2 be constituted of one or more rudimentary properties r_{+m}. Let the force exerted by body B_2 on body B_1 be constituted of one or more rudimentary properties r_{-m}. As it is known, those forces are equal in magnitude but are opposite in direction. (The magnitude is given by the number of rudimentary properties constituting the considered force. The direction is given by the path connecting the two bodies – by the two bodies.) If the bodies merge, the said forces cancel each other and do not act on the environment of the united body. A single isolated force is, therefore, an impossibility. The forces occur in pairs, for example, if a body is partitioned into two or more parts.

Further, it is known (e. g. [5], p. 132), that we can replace connections of the body with its environment by the forces the environment exerts on the body. Thus, in order to establish the simplest model of the environment and the medium of any body, the rudimentary properties of certain quality types are to be applied – the forces of corresponding direction and magnitude. But each of those forces originates out of a pair of forces. Such pair does not imply "cause" and "effect", but a mutual simultaneous interaction. (See also the well known Third Newton's law of dynamics).

5. EXAMPLE OF PROCESSING THE GIVEN MODEL

An example of a realization of successful processing the defined model is the best evidence possible of our assertion.

According to the definition of a system given in the presented paper, our considerations are restricted to a system the total gain of any element of which is one rudimentary property at most of each single quality type at any instant. This gain occurs with a certain probability. (The deterministic development in some sense can be treated as a special case of a probabilistic one).

There are many possible methods for processing deterministic or probabilistic systems. The one presented in the following text applies a method which has been introduced in [6] for a service system.

5.1. Input processing of any given quality type

Let us solve our problem successively in the single quality types – type by type.

In general, we shall use the i-th row of a matrix in order to express outputs of the i-th element of the system and the k-th column of the matrix in order to express inputs of the k-th element. If the rows and the columns are associated with the single elements, the element e_i is a source and the element e_k is a processor considered. The numbers labelling the output rows and the input columns may but need not be different. Let n elements $(e_1, e_2, \ldots, e_n)$ and undefinite number of unresolved pairs (R) be processed. Then

$$i = 1, 2, \ldots, n + 1\,,$$
$$k = 1, 2, \ldots, n + 1\,, \quad i \stackrel{=}{\neq} k\,,$$

where

$$n = n(t)\,.$$

Let us introduce some matrices with the indices in the order given above already:

first index	t –	instant of time,
second index	q –	quality type,
third index	c –	relative priority class,
fourth index	–	row of the matrix,
fifth index	–	column of the matrix,

etc. others, if necessary.

Remark 7. As it is known, the matrix equations often can be written in several equivalent forms. Only one of them will be given here.

Definition 2. Let matrices

$$\mathbf{B}_1 = (b_{i,k,1}), \quad \mathbf{B}_2 = (b_{i,k,2}), \ldots, \mathbf{B}_\varkappa = (b_{i,k,\varkappa}), \ldots,$$

of an arbitrary but the same type be given, where i = all single row indices, k = all single column indices.

Then the union of the given matrices is the matrix $\mathbf{B} = (b_{i,k})$ of the said type too, the single components of which are the unions of the corresponding components of the united matrices:

$$(15) \qquad [\mathbf{B} = \bigcup_\varkappa \mathbf{B}_\varkappa; \quad \varkappa = 1, 2, \ldots] \Leftrightarrow [b_{i,k} = \bigcup_\varkappa{}_{i,k,\varkappa}]$$

(the union of matrices need not be commutative).

1 *Configuration matrix:*

$$(16) \qquad \mathbf{F}_{t,-,-,-,-} = \begin{array}{c} \\ e_1 \\ \vdots \\ e_i \\ \vdots \\ e_n \\ R \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & \ldots e_k & \ldots e_n & R \end{array} \\ \left[\begin{array}{cccc} \mathbf{F}_{1,1} & \ldots \mathbf{F}_{1,k} & \ldots \mathbf{F}_{1,n} & \mathbf{F}_{1,(n+1)} \\ \multicolumn{4}{c}{\cdots\cdots\cdots\cdots\cdots\cdots} \\ \mathbf{F}_{i,1} & \ldots \mathbf{F}_{i,k} & \ldots \mathbf{F}_{i,n} & \mathbf{F}_{i,(n+1)} \\ \multicolumn{4}{c}{\cdots\cdots\cdots\cdots\cdots\cdots} \\ \mathbf{F}_{n,1} & \ldots \mathbf{F}_{n,k} & \ldots \mathbf{F}_{n,n} & \mathbf{F}_{n,(n+1)} \\ \mathbf{F}_{(n+1),1} & \ldots \mathbf{F}_{(n+1),k} & \ldots \mathbf{F}_{(n+1),n} & \mathbf{F}_{(n+1),(n+1)} \end{array}\right] \end{array},$$

where

$$(17) \qquad \mathbf{F}_{i,k} = \mathbf{F}_{t,-,-,i,k} = [f_{t,-,-,i,k;1}; f_{t,-,-,i,k;2}];$$

$f_{t,-,-,i,k;1}$ is the probability that the elements e_i, e_k are medium each to the other;

$f_{t,-,-,i,k;2}$ is the probability that the elements e_i, e_k are out of adjacent environment either to the other;

$f_{t,-,-,i,k;3}$ is the probability the elements e_i, e_k are out of nonadjacent environment either to the other; that probability is not explicitly included in the configuration matrix as it is known that

$$(18) \qquad f_{t,-,-,i,k;1} + f_{t,-,-,i,k;2} + f_{t,-,-,i,k;3} = 1.$$

The configuration matrix is symmetric with respect to its principal diagonal. It replaces a geometric representation.

2. *Activity matrix* for the q-th quality type and the c-th relative input priority class at instant t:

$$
(19)\qquad \mathbf{A}_{t,q,c,-,-} = \begin{array}{c} \\ e_1 \\ \vdots \\ e_i \\ \vdots \\ e_n \\ R \end{array}
\begin{array}{c} e_1 \;\; \ldots e_k \;\; \ldots e_n \;\; R \\
\left[\begin{array}{c}
a_{1,1} \ldots a_{1,k} \ldots a_{1,n}\; a_{1,(n+1)} \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
a_{i,1} \ldots a_{i,k} \ldots a_{i,n}\; a_{i,(n+1)} \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
a_{n,1} \ldots a_{n,k} \ldots a_{n,n}\; a_{n,(n+1)} \\
0 \;\; \ldots 0 \;\; \ldots 0 \;\; 0
\end{array}\right]\end{array},
$$

where

$a_{i,k} = a_{t,q,c,i,k}$ is the probability of activity (of demand) of the q-th quality type and the c-th relative input priority class (according to a classification performed by the processing element e_k), exerted by the i-th element on the k-th element (element e_i is a source of activity);

$a_{i,i}$ is the probability of activity (of the q-th quality type and the c-th relative input priority class) exerted by the i-th element on itself;

$a_{i,(n+1)}$ is the probability of activity (of the g-th quality type and the c-th relative input priority class) exerted by the i-th element on unresolved pairs.

Of course, considering all input priority classes of the given quality type, the corresponding activity matrix is

$$
(20)\qquad \mathbf{A}_{t,q,-,-,-} = \sum_c \mathbf{A}_{t,q,c,-,-}\,.
$$

If we take all quality types into account, the activity matrix is

$$
(21)\qquad \mathbf{A}_{t,-,-,-,-} = \bigcup_q \mathbf{A}_{t,q,-,-,-}\,.
$$

If instead of the probability $a_{i,k} = a_{t,q,c,i,k}$ a number $|a|_{i,k} = |a|_{t,q,c,i,k}$ is considered, which is as great as the corresponding probability, the matrix $\mathbf{A}_{t,q,c,-,-}$ changes into a matrix of magnitudes of the probabilities of activity:

$$
(22)\qquad |\mathbf{A}|_{t,q,c,-,-} = \begin{array}{c} \\ 1 \\ \vdots \\ i \\ \vdots \\ n \\ n+1 \end{array}
\begin{array}{c} 1 \;\; \ldots k \;\; \ldots n \;\; n+1 \\
\left[\begin{array}{c}
|a|_{1,1} \ldots |a|_{1,k} \ldots |a|_{1,n}\;\; |a|_{1,(n+1)} \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
|a|_{i,1} \ldots |a|_{i,k} \ldots |a|_{i,n}\;\; |a|_{i,(n+1)} \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
|a|_{n,1} \ldots |a|_{n,k} \ldots |a|_{n,n}\;\; |a|_{n,(n+1)} \\
0 \;\; ..0 \;\; \ldots 0 \;\; 0
\end{array}\right]\end{array}.
$$

The i-th row of the activity matrix is a distribution of the activity of the i-th element on all elements, itsef inclusive. Those partial activities with probabilities $a_{i,k}$ are due to the rudimentary properties possessed by the element e_i. But the influence exerted by any element consists not only in properties and development of the single elements, but also in the informations encoded into interelementar influences considered (interpreted) as signals.

3. *Information-content matrix* at an instant t:

$$\Gamma_{t,-,-,i,-} = \begin{array}{c} \\ e_1 \\ \vdots \\ e_i \\ \vdots \\ e_n \\ R \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_k & e_n & R \end{array} \\ \left[\begin{array}{cccc} \Gamma_{1,1} \ldots & \Gamma_{1,k} \ldots & \Gamma_{1,n} & \Gamma_{1,(n+1)} \\ \ldots & \ldots & \ldots & \ldots \\ \Gamma_{i,1} \ldots & \Gamma_{i,k} \ldots & \Gamma_{i,n} & \Gamma_{i,(n+1)} \\ \ldots & \ldots & \ldots & \ldots \\ \Gamma_{n,1} \ldots & \Gamma_{n,k} \ldots & \Gamma_{n,n} & \Gamma_{n,(n+1)} \\ 0 \ldots & 0 \ldots & 0 & 0 \end{array}\right] \end{array}, \tag{23}$$

where

$$\Gamma_{i,k} = \begin{bmatrix} \gamma_{t,-,-,i,k;1,1} & \gamma_{t,-,-,i,k;1,2} \\ \gamma_{t,-,-,i,k;2,1} & \gamma_{t,-,-,i,k;2,2} \end{bmatrix}, \tag{24}$$

for example

$$\gamma_{t,-,-,i,k;-,-} = [c_{t,-,-,i,k;-,-}\,;\; d_{t,-,-,i,k;-,-}], \tag{25}$$

i. e.

$$\Gamma_{i,k} = \begin{bmatrix} c_{t,-,-,i,k;1,1};\; d_{t,-,-,i,k;1,1} & c_{t,-,-,i,k;1,2};\; d_{t,-,-,i,k;1,2} \\ c_{t,-,-,i,k;2,1};\; d_{t,-,-,i,k;2,1} & c_{t,-,-,i,k;2,2};\; d_{t,-,-,i,k;2,2} \end{bmatrix}. \tag{26}$$

Here

$\gamma_{t,-,-,i,k;-,-}$ is the content of information, if the element e_i is a source of information;

$c_{t,-,-,i,k;-,-}$ is the relative priority class (the i-th element performs a classification);

$d_{t,-,-,i,k;-,-}$ is the aim (destination);

the sixth index equals "1" – the i-th element transfers or/and forms information about itself;

the sixth index equals "2" – the i-th element transfers or/and forms information about the k-th element;

the seventh index equals "1" – information is not signalized to the k-th element;

the seventh index equals "2" – information is signalized to the k-th element.

If any element classifies its input rudimentary properties, the classification is denoted as input classification. If the same element classifies its outputs, the classification is denoted as output classification.

Remark 8. The matrix $\Gamma_{i,k}$ (the i-th element is the source of information) and the matrix $\Gamma_{k,i}$ (the k-th element is the source of information) need not be equivalent.

The information matrix implies the corresponding input strategy matrices (see below).

4. *Processing-element matrix*:

$$
(27)\qquad \mathbf{K}_{t,-,-,k,k} = \begin{array}{c} \\ e_1 \\ \vdots \\ e_i \\ {\scriptstyle i=k} \\ \vdots \\ e_n \\ R \end{array}
\begin{array}{c} e_1 \;\ldots\; e_k \;\ldots\; e_n \; R \\ \begin{bmatrix} 0 & \ldots 0 \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots 1 \ldots & 0 & 0 \\ & & & \\ \ldots & \ldots & \ldots & \ldots \\ & & & \\ 0 & \ldots 0 \ldots & 0 & 0 \\ 0 & \ldots 0 \ldots & 0 & 0 \end{bmatrix} \end{array} .
$$

The element e_k being a processing element, the principal-diagonal component of the k-th column and only it equals one, but all other components of the matrix are zeros.

5. *Processed-inputs matrix*:

$$
(28)\qquad \mathbf{U}_{t,-,-,-,k} = \begin{array}{c} \\ e_1 \\ \vdots \\ e_i \\ \vdots \\ e_n \\ R \end{array}
\begin{array}{c} e_1 \;\ldots\; e_k \;\ldots\; e_n \; R \\ \begin{bmatrix} 0 & \ldots 1 \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots 1 \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots 1 \ldots & 0 & 0 \\ 0 & \ldots 1 \ldots & 0 & 0 \end{bmatrix} \end{array} .
$$

The components of the k-th column equal one, all other components are zeros.

6. *Processed-priority-class matrix* (*input-strategy matrix*) of the k-th element:

$$
(29)\qquad \mathbf{C}_{t,q,c,k,-} = \begin{array}{c} \\ 1 \\ \vdots \\ i \\ \vdots \\ n \\ n+1 \end{array}
\begin{array}{c} 1 \;\;\ldots\; i \;\ldots\; n \quad n+1 \\ \begin{bmatrix} c_{11} & \ldots 0 & \ldots 0 & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots c_{ii} & \ldots 0 & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots 0 & \ldots c_{nn} & 0 \\ 0 & \ldots 0 & \ldots 0 & c_{(n+1),(n+1)} \end{bmatrix} \end{array} .
$$

Let the processed priority class be the c-th. Then

(30) $c_{ii} = \begin{cases} 1 & \text{if in the } i\text{-th row of the matrix } \mathbf{A}_{t,q,-,-,-} \text{ among the activities } a_{t,q,c,i,k} \text{ there is a non-zero probability of an activity both of the } q\text{-th quality type and of the } c\text{-th relative input priority class according to the classification performed by the } k\text{-th element}); \\ 0 & \text{otherwise.} \end{cases}$

Considering all priority classes of the inputs of the processing element, the input strategy matrix is

$$\mathbf{C}_{t,q,-,k,-} = \bigcup_{c=1}^{g} \mathbf{C}_{t,q,c,k,-} \,. \tag{31}$$

If all quality types are taken into account, the input-strategy matrix of the processing element is

$$\mathbf{C}_{t,-,-,k,-} = \bigcup_{q} \mathbf{C}_{t,q,-,k,-} \,. \tag{32}$$

7. *Matrix of influences of higher input priority classes (applicability matrix)*:

$$\mathbf{N}_{t,q,c,-,k} = \begin{array}{c} \\ 1 \\ \vdots \\ n \\ n+1 \end{array} \begin{array}{c} 1 \;\ldots\; n \quad n+1 \\ \begin{bmatrix} N \ldots 0 & 0 \\ \ldots\ldots\ldots & \\ 0 \ldots N & 0 \\ 0 \ldots 0 & N \end{bmatrix} \end{array} \tag{33}$$

where $N = N_{t,q,c,-,k}$.

The given matrix is a diagonal one with components N in its principal diagonal. The quantity N is the probability of blocking (occupying) the processing element by activities of priority classes higher than that which is just going to be processed. Thus the activities of the highest input priority class are to be processed first of all, continuing always with the next lower input priority class. The values N are equal to unity for the relative highest input priority class.

8. *Matrix of successive partial results*:

$$\mathbf{Y}_{t,q,c,-,k;i} = \begin{array}{c} \\ 1 \\ \vdots \\ i \\ \vdots \\ n \\ n+1 \end{array} \begin{array}{c} 1 \;\ldots\; i \;\ldots\; n \quad n+1 \\ \begin{bmatrix} y_1 \ldots 0 \ldots 0 & 0 \\ \ldots\ldots\ldots\ldots & \\ 0 \ldots y_i \ldots 0 & 0 \\ \ldots\ldots\ldots\ldots & \\ 0 \ldots 0 \ldots y_n & 0 \\ 0 \ldots 0 \ldots 0 & 0 \end{bmatrix} \end{array} . \tag{34}$$

A diagonal matrix with component $y_i = y_{t,q,c,i,k}$ in the i-th row. All those components relate to the same instant t and are partial results in a step-by-step (here row-by-row) method of evaluating the quantity y_n:

y_i partial result in course of the successive processing of the activities, row by row, exerted on the k-th element, if just the first i rows become processed for the c-th input priority class;

$y_n = y_{t,q,c,n,k}$ final result, if all n elements have been processed for the c-th input priority class; that quantity is involved in the n-th row.

Now we can express the quantity $N_{t,q,c+1,-,k}$ for the next lower input priority class (i. e. for the $(c+1)$-st one) with regard to the c-th class of the quantity $N_{t,q,c,-,k}$, having denoted the relative highest input priority class as the first class.

If the relative input priority classes are labelled $j = 1, 2, \ldots, c, c+1, \ldots$, then

$$N_{t,q,c+1,-,k} = \sum_{j=1}^{c} y_{t,q,j,n,k}\,, \tag{35}$$

where

$$N_{t,q,1,-,k} = 1\,. \tag{36}$$

The values of the single components of the matrix $\mathbf{Y}_{t,q,c,-,k;i}$ are computed successively, row by row.

9. *Matrix of last previous partial results:*

$$\mathbf{Y}_{t,q,c,-,k;i-1} = \begin{array}{r} \\ 1 \\ 2 \\ \vdots \\ i \\ \vdots \\ n \\ n+1 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & \ldots\, i & & \ldots\, n & n+1 \end{array} \\ \begin{bmatrix} y_0 & 0 & \ldots\, 0 & \ldots\, 0 & 0 \\ 0 & y_1 & \ldots\, 0 & \ldots\, 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots\, y_{i-1} & \ldots\, 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots\, 0 & \ldots\, y_{n-1} & 0 \\ 0 & 0 & \ldots\, 0 & \ldots\, 0 & 0 \end{bmatrix} \end{array}. \tag{37}$$

The matrix (37) is a diagonal matrix with component $y_{i-1} = y_{t,q,c,i-1,k}$ in the i-th row, and with

$$y_0 = 0\,. \tag{38}$$

The values of the single components of that matrix are also computed row by row. The matrix $\mathbf{Y}_{t,q,c,-,k;i-1}$ allows to construct the matrix $\mathbf{Y}_{t,q,c,-,k;i}$ (see (34)).

10. *Acceptance matrix:*

$$\mathbf{H}_{t,q,c,-,-} = \begin{array}{r} \\ e_1 \\ \vdots \\ e_i \\ \vdots \\ e_n \\ R \end{array} \begin{array}{c} \begin{array}{cccc} e_1 \;\ldots\; e_k & \ldots\; e_n & R \end{array} \\ \begin{bmatrix} h_{1,1} \ldots h_{1,k} & \ldots\, h_{1,n} & h_{1,(n+1)} \\ \ldots & \ldots & \ldots \\ h_{i,1} \ldots h_{i,k} & \ldots\, h_{i,n} & h_{i,(n+1)} \\ \ldots & \ldots & \ldots \\ h_{n,1} \ldots h_{n,k} & \ldots\, h_{n,n} & h_{n,(n+1)} \\ 0 \;\;\ldots\; 0 & \ldots\; 0 & 0 \end{bmatrix} \end{array}; \tag{39}$$

$h_{i,k} = h_{t\,q,c,i,k}$ – probability of the acceptance of the activity exerted with probability $a_{t,q,c,i,k}$ by the i-th element on the k-th one, if the activity is of the q-th quality type and of the c-th relative input priority class;

$h_{i,(n+1)} = h_{t,q,c,i,n+1}$ – probability of the acceptance of the activity exerted with probability $a_{t,q,c,i,n+1}$ by the i-th element on an undefinite number of unresolved pairs;

The probabilities $h_{i,k}$ $(k = 1, 2, \ldots, n + 1)$ are the probabilities of the corresponding *partial losses* of the i-th element in rudimentary properties of the q-th quality type and the c-th relative input priority class – losses incurred by the i-th element itself and influenced by the single k-th elements.

Considering all relative input priority classes of the q-th quality type, the acceptance matrix is

$$\mathbf{H}_{t,q,-,-,-} = \sum_c \mathbf{H}_{t,q,c,-,-} \,. \tag{40}$$

If we take into account all quality types, the acceptance matrix is

$$\mathbf{H}_{t,-,-,-,-} = \bigcup_q \mathbf{H}_{t,q,-,-,-} \,. \tag{41}$$

Now we can introduce two theorems:

Theorem 2. *Let matrices* (20), (27), (28), (29), (33), (34), (37) *be given. Then*

$$\begin{aligned} \mathbf{Y}_{t,q\,c,-,k;i}\mathbf{U}_{t,-,-,-,k} &= \mathbf{Y}_{t,q,c,-,k;i-1}\mathbf{U}_{t,-,-,-,k} + \\ &+ (\mathbf{N}_{t,q,c,-,k} - \mathbf{Y}_{t,q,c,-,k,i-1})\,\mathbf{C}_{t,q,c,k,-}\mathbf{A}_{tq,-,-,-}\mathbf{K}_{t,-,-,k,k} \,. \end{aligned} \tag{42}$$

Proof. The result of the matrix equation (42) is of the form

$$\begin{array}{r} \\ 1 \\ \vdots \\ i \\ \vdots \\ n \\ n+1 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & \ldots & k & \ldots & n & n+1 \end{array} \\ \left[\begin{array}{ccccc} 0 & \ldots & y_1 & \ldots & 0\;0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & y_i & \ldots & 0\;0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & y_n & \ldots & 0\;0 \\ 0 & \ldots & 0 & \ldots & 0\;0 \end{array}\right] \end{array} = \tag{43}$$

$$= \begin{array}{r} \\ 1 \\ \vdots \\ i \\ \vdots \\ n \\ n+1 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 \;\ldots & & k & & \ldots\; n \;\; n+1 \end{array} \\ \left[\begin{array}{ccc} 0 \;\ldots & c_{1,1}a_{1,k} & \ldots\; 0\;0 \\ \ldots & \ldots & \ldots \\ 0 \;\ldots & y_{i-1} + (N - y_{i-1})\,c_{i,i}a_{i,k} & \ldots\; 0\;0 \\ \ldots & \ldots & \ldots \\ 0 \;.. & y_{n-1} + (N - y_{n-1})\,c_{n,n}a_{n,k} & \ldots\; 0\;0 \\ 0 \;\ldots & 0 & \ldots\; 0\;0 \end{array}\right] \end{array},$$

where

$$a_{i,k} = a_{t,q,c,i,k}\,, \tag{44}$$

$$(45) \qquad N = N_{t,q,c,-,k} = \sum_{j=1}^{c-1} y_{t,q,j,n,k}\,; \quad c = 2, 3, \ldots;$$

and c_{ii} satisfies (30).

The form of equation (43) is in accordance with the method introduced in [6], if the matrices proposed in the present paper are applied. The method of proof is, therefore, the same Q. E. D.

Theorem 3. *Let the n-th row of the matrix equation* (42) *involve the quantity* $y_n = y_{t,q,c,n,k}$ (see (43)). *Let the matrix* (39) *be given. Then*

$$(46) \qquad \mathbf{H}_{t,q,c,-,-}\mathbf{K}_{-,-,-,k,k}\overline{(\mathbf{C}_{t,q,c,k,-}|\mathbf{A}|_{t,q,-,-,-}\mathbf{K}_{-,-,-,k,k})}\,\mathbf{U}_{t,-,-,-,k} = |y|_{t,q,c,n,k}\mathbf{C}_{t,q,c,k,-}\mathbf{A}_{t,q,-,-,-}\mathbf{K}_{-,-,-,k,k}\,,$$

where $\overline{(\mathbf{C}_{t,q,c,k,-}|\mathbf{A}|_{t,q,-,-,-}\mathbf{K}_{-,-,-,k,k})}$ *is the transpose of* $\mathbf{C}_{t,q,c,k,-}|\mathbf{A}|_{t,q,-,-,-}\,.\,\mathbf{K}_{-,-,-,k,k}$, and $|y|_{t,q,c,n,k}$ *is the number, which is as great as the probability* $y_{t,q,c,n,k}$.

Proof follows from the same method as that used for Theorem 2. Now the result is

$$(47) \qquad \begin{array}{r} \\ 1 \\ \vdots \\ i = k \\ \vdots \\ n \\ n+1 \end{array}
\begin{array}{c} 1 \ldots k \quad \ldots n \;\; n+1 \\ \begin{bmatrix} 0 \ldots h_{1,k} \ldots 0 \; 0 \\ \ldots\ldots\ldots\ldots \\ 0 \ldots h_{i,k} \ldots 0 \; 0 \\ \ldots\ldots\ldots\ldots \\ 0 \ldots h_{n,k} \ldots 0 \; 0 \\ 0 \ldots 0 \quad \ldots 0 \; 0 \end{bmatrix} \end{array}
\begin{array}{c} 1 \ldots k \qquad \ldots n \;\; n+1 \\ \begin{bmatrix} 0 \ldots 0 \qquad \ldots 0 \; 0 \\ \ldots\ldots\ldots\ldots \\ 0 \ldots \sum_{i=1}^{n} c_{i,i}|a|_{i,k} \ldots 0 \; 0 \\ \ldots\ldots\ldots\ldots \\ 0 \ldots 0 \qquad \ldots 0 \; 0 \\ 0 \ldots 0 \qquad \ldots 0 \; 0 \end{bmatrix} \end{array} =$$

$$= |y|_{t,q,c,n,k} \begin{array}{r} \\ 1 \\ \vdots \\ i \\ \vdots \\ n \\ n+1 \end{array}
\begin{array}{c} 1 \ldots k \qquad \ldots n \;\; n+1 \\ \begin{bmatrix} 0 \ldots c_{1,1}a_{1,k} \ldots 0 \; 0 \\ \ldots\ldots\ldots\ldots \\ 0 \ldots c_{i,i}a_{i,k} \ldots 0 \; 0 \\ \ldots\ldots\ldots\ldots \\ 0 \ldots c_{n,n}a_{n,k} \ldots 0 \; 0 \\ 0 \ldots 0 \qquad \ldots 0 \; 0 \end{bmatrix} \end{array}. \qquad \text{Q. E. D.}$$

5.2 Output processing of any given quality type

Each accepted rudimentary property is assigned with some information, for example, with its destination and demanded priority class. But the demanded classification need not be in accordance with that performed by the accepting element. The demanded destination need not be convenient or available for the accepting element. The signals transferring information need not be correctly coded or/and decoded. Etc.

Let us assume that the accepting element considers and weighs all mentioned possibilities, in general. The resultant of the weighing is a strategy of the element. That strategy is expressed by an instantaneous correspondence between the single input rudimentary properties and the single outputs of the element. Thus the strategy of the element, applied on its mediumial and neighbouring elements (out of adjacent environment), can (but need not) be expressed for any given class by means of an *output strategy matrix* ($\mathbf{W}_{t,q,-,k,-}$). The components of that matrix are the weights of the association of the single input rudimentary properties (accepted) with the single outputs. (It is possible to prepare estimates of the strategy matrices for more than one instant in advance.)

Having accepted a rudimentary property with the probability $h_{i,k}$, the k-th element distributes that property towards one of its outputs. In our symbolics that distribution appears as certain increments to the single components of the output row of the given element in the activity matrix $\mathbf{A}_{t+1,q,-,-,-}$ at the instant $t + 1$.

Let the probability $h_{i,k}$ be partitioned into the component increments of the k-th row with this weights: Into the increment $p_{i,\sigma}$ of the σ-th column of the i-th row – the probability $h_{i,k}$ is partitioned with the weight $w_{i,\sigma}$ ($\sigma = 1, 2, \ldots, n + 1$).

Lemma 1. *Let a system be given according to Definition* 1.

i) *Let the single outputs of the k-th element be divided into groups according to the single quality types. Generally, let us consider one output of each single quality type of the k-th element be directed towards each of the n considered elements and towards an undefinite number of unresolved pairs.*

ii) *Let the* $n + 1$ *considered outputs of the q-th quality type be divided into one or more relative output priority classes of the first classification level* ($x = 1$), *and let the* σ*-th output of the k-th element be attached to the* $c(1)$*-st of those output classes according to the weight* $\omega_{t,q,c(1),k,\sigma;1}$.

iii) *Let within any relative output priority class of the first classification level one or more relative output priority sub-classes be considered of the second classification level* ($x = 2$), *and let the* σ*-th output of the k-th element be attached to the* $c(2)$*-nd of those sub-classes according to the weight* $\omega_{t,q,c(2),k,\sigma;2}$.

iv) *Let within any relative output priority sub-class of the* $(x - 1)$*-st classification level one or more relative output priority sub-sub-classes of the x-th classification level be considered, and let the* σ*-th output of the k-th element be attached to the* $c(x)$*-th of those sub-sub-classes according to the weight* $\omega_{t,q,c(x),k,\sigma;x}$.

v) *Let*

$$\sum_{c(x)=1}^{g(x)} \omega_{t,q,c(x),k,\sigma;x} = 1\,, \tag{48}$$

where

(49) $$0 \leqq \omega_{t,q,c(x),k,\sigma;x} \leqq 1\,,$$

and $x = 1, 2, \ldots$; $g(x)$ *is the number of relative output priority sub-sub-classes at the x-th classification level considered within the* $c(x-1)$*-st sub-class.*

Usually

(50) $$\omega_{t,q,c(x),k,\sigma;x} = \begin{cases} 1 \\ 0 \end{cases}.$$

Then the resultant weight of attaching the σ-th output of the k-th element to the $c(x)$*-th output relative priority class in the x-th classification level is*

(51) $$\omega_{t,q,c(x),k,\sigma} = \omega_{t,q,c(1),k,\sigma;1}\,\omega_{t,q,c(2),k,\sigma;2} \ldots \omega_{t,q,c;(x-1),k,\sigma;(x-1)}\,\omega_{t,q,c(x),k,\sigma;x}\,.$$

Proof is evident.

Remark 9. Since the level, at which the classification begins to get discontinuous, one sub-sub-class only is to be considered always.

The information matrix $\Gamma_{t\,-,-,i,-}$ involves the classification of the outputs of the i-th element according to a classification performed by that i-th element. Input-strategy matrix $\mathbf{C}_{t,q,-,k,-}$ gives a classification of the inputs of the processing element according to a classification performed by that processing element. Lemma 1 leads to a classification of the single outputs of the processing element according to a classification performed by the same processing element. Using this lemma, we can introduce

11. *Output classification matrix* for the $c(x)$-th relative priority class:

(52) $$\Omega_{t,q,c(x),k,-} = \begin{array}{c|cccccccccc} & 1 & \ldots & \sigma & \ldots & k & \ldots & n & n+1 \\ \hline 1 & \omega_{1,1} & \ldots & \omega_{1,\sigma} & \ldots & \omega_{1,k} & \ldots & \omega_{1,n} & \omega_{1,(n+1)} \\ \vdots & \cdots & & & & & & & \cdots \\ k & \omega_{k,1} & \ldots & \omega_{k,\sigma} & \ldots & \omega_{k,k} & \ldots & \omega_{k,n} & \omega_{k,(n+1)} \\ \vdots & \cdots & & & & & & & \cdots \\ n & \omega_{n,1} & \ldots & \omega_{n,\sigma} & \ldots & \omega_{n,k} & \ldots & \omega_{n,n} & \omega_{n,(n+1)} \\ n+1 & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & 0 \end{array}$$

where

$$\omega_{k,\sigma} = \omega_{t,q,c(x),k,\sigma}$$

(see (51)).

Considering all output priority classes, the output classification matrix is

(53) $$\Omega_{t,q,-,k,-} = \bigcup_{c(x)} \Omega_{t,q,c(x),k,-}\,.$$

 If all quality types are taken into account, then

$$\Omega_{t,-,-,k,-} = \bigcup_{q} \Omega_{t,q,-,k,-} \,. \tag{54}$$

Remark 10. The output-classification matrices for the single processing elements lead to a new information matrix $\Gamma_{t+1,-,-,i,-}$ at the instant $t + 1$.

Now let us introduce some matrices more:

12. *Processed-outputs matrix* of the k-th element:

$$\mathbf{V}_{-,-,-,k,-} = \begin{array}{c} \\ e_1 \\ \vdots \\ e_k \\ \vdots \\ e_n \\ R \end{array} \begin{array}{c} e_1 \;\dots\; e_n \;\; R \\ \left[\begin{array}{ccc} 0 & \dots\; 0 & 0 \\ & \cdots\cdots & \\ 1 & \dots\; 1 & 1 \\ & \cdots\cdots & \\ 0 & \dots\; 0 & 0 \\ 0 & \dots\; 0 & 0 \end{array}\right] \end{array} . \tag{55}$$

The components of the k-th row are equal to unity, all other components are zeros.

13. *Matrix of probabilities to be partitioned* (of the k-th element):

$$\mathbf{L}_{t,q,c,-,k} = \begin{array}{c} \\ 1 \\ \vdots \\ n \\ n+1 \end{array} \begin{array}{c} 1 \quad \dots\; n \quad n+1 \\ \left[\begin{array}{cccc} h_{1,k} & \dots & 0 & 0 \\ & \cdots\cdots\cdots & & \\ 0 & \dots & h_{n,k} & 0 \\ 0 & \dots & 0 & 0 \end{array}\right] \end{array} . \tag{56}$$

The matrix (56) is a diagonal matrix with component $h_{i,k} = h_{t,q,c,i,k}$ in the i-th row. Here $h_{n+1,k} = 0$.

Now, let the accepted rudimentary properties associate with the single classified outputs of the processing element. (Each accepted rudimentary property is attached to a priority class of an output.)

14. *Distributing-weights matrix* (*output-strategy matrix*) of the processing (the k-th) element:

$$\mathbf{W}_{t,q,-,k,-} = \begin{array}{c} \\ 1 \\ \vdots \\ i \\ \vdots \\ n \\ n+1 \end{array} \begin{array}{c} 1 \quad \dots\; \sigma \quad \dots\; k \quad \dots\; n \quad n+1 \\ \left[\begin{array}{cccccccc} w_{1,1} & \cdots & w_{1,\sigma} & \cdots & w_{1,k} & \cdots & w_{1,n} & w_{1,(n+1)} \\ & & & \cdots\cdots\cdots\cdots & & & & \\ w_{i,1} & \cdots & w_{i,\sigma} & \cdots & w_{i,k} & \cdots & w_{i,n} & w_{i,(n+1)} \\ & & & \cdots\cdots\cdots\cdots & & & & \\ w_{n,1} & \cdots & w_{n,\sigma} & \cdots & w_{n,k} & \cdots & w_{n,n} & w_{n,(n+1)} \\ 0 & \dots & 0 & \dots & 0 & \dots & 0 & 0 \end{array}\right] \end{array} . \tag{57}$$

Here $w_{i,\sigma} = w_{t,q,-,i,\sigma}$ is the weight of a distribution of the probability $h_{t,q,c,i,k}$ (of activity accepted by the k-th element from the i-th element; see (34)) towards the σ-th output of the k-th element, i. e. towards the σ-th column of the considered matrix.

Further

(58) $$0 \leqq w_{t,q,-,i,\sigma} \leqq 1\,,$$

and

(59) $$\sum_{\sigma} w_{t,q,-,i,\sigma} = 1\,.$$

15. *Matrix weighted partitions of probabilities of acceptance by the k-th element (partition matrix):*

(60) $$\mathbf{P}_{t,q,c,k,-} = \begin{array}{c} \\ 1 \\ \vdots \\ i \\ \vdots \\ n \\ n+1 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & \ldots & \sigma & \ldots & k & \ldots & n & n+1 \end{array} \\ \left[\begin{array}{ccccccccc} p_{1,1} & \cdots & p_{1,\sigma} & \cdots & p_{1,k} & \cdots & p_{1,n} & p_{1,(n+1)} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ p_{i,1} & \cdots & p_{i,\sigma} & \cdots & p_{i,k} & \cdots & p_{i,n} & p_{i,(n+1)} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ p_{n,1} & \cdots & p_{n,\sigma} & \cdots & p_{n,k} & \cdots & p_{n,n} & p_{n,(n+1)} \\ 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & 0 \end{array}\right] \end{array}.$$

Here $p_{i,\sigma} = p_{t,q,c,i,\sigma}$ is the probability that the rudimentary property of the q-th quality type and the c-th input priority class, accepted by the k-th element from the i-th element with probability $h_{t,q,c,i,k}$, is directed towards the σ-th output of the processing (k-th) element, as an activity exerted through the given σ-th output. The probability $p_{t,q,c,i,\sigma}$ is then a probability of an instantaneous partial gain of the k-th element in a rudimentary property of the q-th quality type and the c-th input priority class acting through the σ-th output:

(61) $$p_{t,q,c,i,\sigma} = h_{t,q,c,i,k} w_{t,q,-,i,\sigma}\,.$$

If all input priority clasess are considered, then the partition matrix of the k-th element is

(62) $$\mathbf{P}_{t,q,-,k,-} = \bigcup_{c} \mathbf{P}_{t,q,c,k,-}\,.$$

Taking all quality types into account, the partition matrix of the k-th element is

(63) $$\mathbf{P}_{t,-,-,k,-} = \bigcup_{q} \mathbf{P}_{t,q,-,k,-}\,.$$

16. *Partial-activity-increment matrix* of the q-th quality type and the c-th relative input priority class, gained by the single k-th elements during the elapsing

 of the time interval $\langle t, t + 1\rangle$:

$$(64)\qquad \Lambda_{\langle t,t+1\rangle,q,c,-,-} = \begin{array}{c} \\ 1 \\ \vdots \\ k \\ \vdots \\ n \\ n+1 \end{array} \begin{array}{c} \begin{array}{ccccccccc} 1 & \dots & \sigma & \dots & k & \dots & n & & n+1 \end{array} \\ \begin{bmatrix} \lambda_{1,1} & \dots & \lambda_{1,\sigma} & \dots & \lambda_{1,k} & \dots & \lambda_{1,n} & \lambda_{1,(n+1)} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \lambda_{k,1} & \dots & \lambda_{k,\sigma} & \dots & \lambda_{k,k} & \dots & \lambda_{k,n} & \lambda_{k,(n+1)} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \lambda_{n,1} & \dots & \lambda_{n,\sigma} & \dots & \lambda_{n,k} & \dots & \lambda_{n,n} & \lambda_{n,(n+1)} \\ 0 & \dots & 0 & \dots & 0 & \dots & 0 & 0 \end{bmatrix} \end{array},$$

where $\lambda_{k,\sigma} = \lambda_{\langle t,t+1\rangle,q,c,k,\sigma}$ is the partial increment of the probability of activity of the q-th quality type and the c-th input relative priority class *gained* by the single k-th elements and exerted through their single σ-th outputs, respectively.

Theorem 4. *Let matrices* (56), (57), (60) *be given. Then*

$$(65)\qquad \mathbf{P}_{t,q,c,k,-} = \mathbf{L}_{t,q,c,k,-}\mathbf{W}_{t,q,-,k,-}\,.$$

Proof follows from (61).

Theorem 5. *Let matrices* (27), (55), (60), (64) *be given. Then*

$$(66)\qquad \mathbf{K}_{-,-,-,k,k}\Lambda_{t,q,c,-,-} = \mathbf{V}_{-,-,-,k,-}\mathbf{P}_{t,q,c,k,-}\,.$$

Proof. All probabilities in the σ-th column of the matrix $\mathbf{P}_{t,q,c,k,-}$ relate to the σ-th output of the k-th element.

Theorem 6. *Let*

$$(67)\qquad \Lambda_{\langle t,t+1\rangle,q,-,-,-} = \sum_{c} \Lambda_{\langle t,t+1\rangle,q,c,-,-}$$

be the partial activity increment matrix of the q-th quality type and of all relative input priority classes, which increments have been gained by the single elements during the time interval $\langle t, t + 1\rangle$. *Let the acceptance matrix* (40) *be given. Then the total increment in rudimentary properties of the q-th quality type, to be included in the activity matrix at the instant* $t + 1$, *is given by*

$$(68)\qquad \mathbf{A}_{t+1,q,-,-,-} = \mathbf{A}_{t,q,-,-,-} + \Lambda_{\langle t,t+1\rangle,q,-,-,-} - \mathbf{H}_{t,q,-,-,-}\,,$$

where

$$(69)\qquad n(t + 1) \mathrel{\overline{\overline{\neq}}} n(t)\,.$$

Proof. Each rudimentary property being gained by any element is lost by another element. The accepted rudimentary properties make increases and the transmitted rudimentary properties make decreases of the corresponding activities.

Theorem 7. *Let each element be a union of certain rudimentary properties* (6). *Then the resulting activity* $\mathbf{A}_{t+1,-,-,-,-}$ *of the system at the instant* $t + 1$ – *the state of the system* – *is the union of the activities of all and only all quality types considered in that system*:

$$\mathbf{A}_{t+1,-,-,-,-} = \bigcup_{q=+1}^{+m} \mathbf{A}_{t+1,q,-,-,-} \cup \bigcup_{q=-1}^{-m} \mathbf{A}_{t+1,q,-,-,-} \,. \tag{70}$$

Proof follows from Definition 2 and Theorem 6.

As defined, an element can influence its medium and adjacent environment only, unresolved pairs inclusive. Whether the medium or adjacent or nonadjacent environment is concerned, it is evident both from the activity matrix and from the configuration matrix. A new configuration matrix $\mathbf{F}_{t+1,-,-,-,-}$ results due to the processing performed at instant t.

To the new activity matrix $\mathbf{A}_{t+1,-,-,-,-}$ corresponds a new information-content matrix $\Gamma_{t+1,-,-,-,-}$, which takes its part in the influence on the further development of the system.

5.4. Development of the system. Equation of the system

Theorem 8. *Let a system according to Definition* 1 *and Theorem* 7 *be given. Then the time sequence of activity matrices of the system represents the development of the system*:

$$\mathbf{E} = \{\mathbf{A}_{t,-,-,-,-}\}\,, \quad t = 1, 2, \ldots \tag{71}$$

Proof. This Theorem has been derived in the preceding text of the paper.

As the matrix $\mathbf{A}_{t,-,-,-,-}$ is two-dimensional, the corresponding development $\mathbf{E}$ can be expressed by means of a three-dimensional matrix (e. g. see [7]), the third dimension of which is a time axis. Thus (71) is one of possible notations of a certain three-dimensional-matrix equation – the equation of the system.

6. CONCLUSION

A model has been established within which a design method operating on elements can be applied. It is only one of many possible models and is not intended to be anything more.

From the presented model follows that subsystems or systems can be constructed arbitrarily within certain limits only. But not even a medium or an environment can be chosen arbitrarily for a given subsystem. The model introduced allows a design of a subsystem merely with a design of its medium and adjacent or unadjacent environment.

(Received October 8, 1969.)

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VÝTAH

Třída modelů poloautonomních podsystémů

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Prostředí a blízké i vzdálené okolí libovolného podsystému libovolného systému obecně nelze modelovat bez závislostí, v jistých mezích, jednoho na druhém. V předložené práci je modelování takových závislostí zajištěno proti opomenutí pomocí určitých párů prvotních (nevyvinutých) vlastností. Každý z těch párů, pokud není rozložen ve své dvě složky, jako celek nepůsobí na své okolí a prostředí, ale je rozložitelný za určitých podmínek. Prvotními vlastnostmi pocházejícími z takových párů jsou tvořeny jednotlivé vlastnosti elementů, ze kterých zase je utvořen systém. Specifikaci pojmu jevu a modelem času a prostoru je dotvořena definice systému. Mezielementové vztahy a jejich změny jsou vyjádřeny pomocí matic.

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