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Decomposition Translations and Syntax Directed Translation Schemata

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Some important kinds of decomposition translation of context-free languages are concerned and proven not to give more than syntax-directed translation schemata.

1. INTRODUCTION

The notion of "*well-translation*" was introduced in [1] and [2]. Its further formalization and properties are explored in [3], [4] and [5]. On the one hand, its relation to the special context-free grammars (namely LR(k) and LL(k) grammars, [3]), on the other hand, its realization on the pushdown automata (transducers, [3], [5], or assemblers, [4]) is studied. The method of formalization used in [3] bears some resemblance to the way to the semantics formalization of context-free languages due to E. T. Irons (e.g. in [6]). The method of formalization used in [4] and [5] is characterized by the authors to be a generalization of the notion of context-free grammars and has proven to be very useful as to getting interesting results concerning the existence of the hierarchy of syntax-directed translations depending on the maximal number of metavariables in a transcription rule on the one hand ([4]), and the connection with the homomorphism of the word sets over a finite alphabet and other properties of translations, but also of the single languages (e.g. unambiguity), on the other hand ([5]).

The original definition of the "well-translation" is contiguous to a mapping of the rule set of the input language grammar into that of the output language grammar while certain conditions are satisfied (cf. [1], [2]). After the phrase marker of the input sentence has been found the vertices of which are labelled by input grammar rules, the vertices are labelled by output grammar rules obtained with help of the mentioned mapping. Simultaneously the labelling of edges is changed properly and thus the phrase marker of the desired translation has been obtained.

Let now a procedure, so called *decomposition procedure*, be given, which provides a decomposition of each element of some set of phrase markers over the given input grammar. The elements of this decomposition are subtrees of the considered tree. If several mappings of the rule set of the input grammar are given instead of one and if we know which of them is to be used in transforming the just considered subtree, the decomposition transformation and decomposition translation have arised, which were introduced in [7] pursuant to the demand of linguists for the natural languages investigation.

The question arises what kind of relation will hold between the decomposition trantlation and the well-translation (or the syntax-directed translation as its formalization). The answer is given by theorem 3.1 for certain important decomposition procedures.

2. DECOMPOSITION TRANSLATION

Definition 2.1. *A context free grammar* (CFG) G is a system

(1) $$G = (V, \Sigma, P, S),$$

where V and Σ are disjoint finite sets of *metavariables* and *terminal symbols*, respectively. $S \in V$ is the *sentence symbol*. P is the finite set of *rules* r of the form $A \to \alpha$, the so called *context-free rules*, where $A \in V$ and $\alpha \in (V \cup \Sigma)^*$. Then we write $A = \mathscr{L}(r)$, $\alpha = \mathscr{R}(r)$. If $(A \to \alpha) \in P$, $\{\beta, \gamma\} \subset (V \cup \Sigma)^*$, then we write $\beta A \gamma \underset{G}{\Rightarrow} \beta\alpha\gamma$. The relation $\overset{*}{\underset{G}{\Rightarrow}}$ is defined by $\alpha \overset{*}{\underset{G}{\Rightarrow}} \alpha$ for each $\alpha \in (V \cup \Sigma)^*$, and whenever $\alpha \overset{*}{\underset{G}{\Rightarrow}} \beta$ and $\beta \underset{G}{\Rightarrow} \gamma$, then $\alpha \overset{*}{\underset{G}{\Rightarrow}} \gamma$. The language generated by G, denoted $L(G)$, is $\{w \mid w \in \Sigma^* \wedge S \overset{*}{\underset{G}{\Rightarrow}} w\}$. Then $L(G)$ is a *context-free language* (CFL) and each w is its *phrase*.

Note. Sometimes we shall write $\Rightarrow$ and $\overset{*}{\Rightarrow}$ instead of $\underset{G}{\Rightarrow}$ and $\overset{*}{\underset{G}{\Rightarrow}}$, respectively, if G is obvious or will be constructed completely later.

Definition 2.2. A sequence

(2) $$\alpha_0, \alpha_1, \ldots, \alpha_m$$

of strings such that the following conditions hold:

a) $\alpha_i \in (V \cup \Sigma)^*$ for each i, $0 \leqq i \leqq m$;

b) $\alpha_0 = S$, $\alpha_m \in \Sigma^*$;

c) $\alpha_{i-1} \underset{G}{\Rightarrow} \alpha_i$ for each i, $1 \leqq i \leqq m$

is called a *derivation* of the phrase α_m in the grammar (1).

More generally the name derivation in G is used also for a sequence (2) satisfying a), c) and

b′) $\alpha_0 \in V$

or only a), c). (If necessary we speak of an α_0 – derivation of α_m, e.g. an S-derivation.)

Definition 2.3. Let (2) be an S-derivation of $\alpha_m \in \Sigma^*$ and let M be an oriented tree, the vertices of which are labelled by the rules of P of the grammar (1) and whose edges are labelled by the natural numbers in such a way, that the following holds:

a) The root is labelled by a rule r such that $\mathscr{L}(r) = S$;

b) let

$$X_0 \to w_0 X_1 w_1 X_2 \dots w_{k-1} X_k w_k \tag{3}$$

be a rule of P, where $X_i \in V$, $w_i \in \Sigma^*$ for each i, $0 \leqq i \leqq k$, $k \geqq 0$. Let (3) labels some vertex of M. Then exactly k oriented edges emit this vertex. Their labelling by the numbers $1, 2, \dots, k$ shows their correspondence to the metavariables $X_1, X_2, \dots, X_k$ of (3). The end vertex of the edge i is then labelled by a rule r with $\mathscr{L}(r) = X_i$, $1 \leqq i \leqq k$.

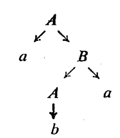
Then M is called a *phrase marker* of (2) (but also of the phrase α_m) over the grammar (1).

Note. If v is a vertex of M from the definition 2.3 then let $\varphi(v)$ denote the rule labelling v. Also we denote $L(M) = \alpha_m$.

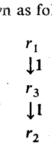
Example 2.1. Let $G = (\{A, B\}, \{a, b\}, P, A)$ be a CFG, where $P = \{r_1, r_2, r_3, r_4\}$ and $r_1 = A \to aB$, $r_2 = A \to b$, $r_3 = B \to Aa$, $r_4 = B \to b$. Let us consider the derivation

$$A \Rightarrow aB \Rightarrow aAa \Rightarrow aba\,. \tag{4}$$

The generation tree of (4) (cf. [11]) is



while the phrase marker of (4) may be drawn as follows:



Note 1. Such a mapping of the set of all the generation trees onto the set of all the phrase markers over the same context-free grammars that the tree and marker of the same derivation correspond each other is a one-one mapping. Thus the phrase marker describes fully the syntactical structure.

Note 2. The following property of the phrase marker will also be important in what follows: Context-free rules can be a part of a system which is more general than context-free grammar. E.g. some of these rules may be used only under certain conditions (cf. e.g. [9], [12]). Nevertheless, the phrase marker M describes the structure of $L(M)$, though it cannot be said to be a phrase marker over a context free grammar. In such a case, the phrase marker does not show the conditions underlaying the use of rules. In order that we may show these conditions we must have a more general tool (cf. e. g. [10] for context-sensitive languages).

Note 3. A phrase marker can be used to describe the syntactical structure also in case we have the more general case of derivation, namely one satisfying a), b′), c) in the definition 2.2.

Definition 2.4. Let K_M be (here and also in the following text) the set of all the vertices of the tree M. Let M be a phrase marker and L such a subset of K_M that it contains the root of M, and let $v \in L$. Let us denote $M(v)$ the maximal oriented subtree of M such that the following holds:

a) v is the root of $M(v)$;
b) if $w \neq v$, w is a vertex of $M(v)$, and $w \in L$, then w is an end vertex of $M(v)$ (i.e. no edge emits it).

Then the set

$$\{M(v) \mid v \in L\}$$

is called a *decomposition of* M (induced by L).

Note 1. It is obvious that

$$\bigcup_{v \in L} K_{M(v)} = K_M$$

and $K_{M(v)} \cap K_{M(w)}$ contains at most one element for $v \neq w$. Further, if w is a vertex of $M(v)$, then the only path connecting in M the root of M with w goes through v (or ends here if $w = v$).

Definition 2.5. Let h be such a mapping of P of (1) into P' of CFG $G' = (V, \Delta, P', S)$ that the following holds: if $r \in P$ is of the form (3), then

$$h(r) = X_0 \to w'_0 Z_1 w'_1 Z_2 \dots w'_{k-1} Z_k w'_k, \tag{5}$$

where $X_i = Z_{\Pi(i)}$ for all i, $1 \leqq i \leqq k$, and Π is a permutation of k objects (corresponding to the considered rule r and mapping h) and $w'_i \in \Delta^*$ for all i, $1 \leqq i \leqq k$. Then h is called a "*well-transformation*" of G into G'.

Note. In the original definition of the well-transformation (cf. [1], [2]) also a semantical demand of the meaning conservation was formulated in a convenient way. Further there was there a proper mapping of V of G into (generally from V different) V' of G'. This difference of metavariable denotation is unnecessary for our formal approach and after all, it would be cumbersome for the further formalization we want to reach. Also, on the contrary to [1] and [2], we have not two kinds of terminal symbols (namely proper and auxiliary ones), which are also unnecessary here. The verification of the meaning conservation is getting over the scope of this paper. It would force us into dealing with some method of context-free language semantics formalization. (The method developed in [8] seems to be useful in studying a generalization of syntax-directed translations, but this will be subject of another paper).

Definition 2.6. Let h be a well-transformation of (1) into $G' = (V, \Delta, P', S)$ and let M be a phrase marker over G. For each $v \in K_M$ let us replace $\varphi(v)$ by $h(\varphi(v))$ and simultaneously, as to the edge labelling, i is replaced by $\Pi(i)$, where Π is the permutation corresponding to $\varphi(v)$ and h. The oriented labelled tree $\tilde{M} = f(M)$ arising from M in this way is a phrase marker over G'. The mapping f of the set $\mathfrak{M}$ of all the phrase markers over G into the set of all the phrase markers over G' is called a "*well-translation*".

Note. If G is unambiguous, then the set of pairs

$$(6) \qquad \{(x, y) \mid \exists M(M \in \mathfrak{M} \wedge x = L(M) \wedge y \in L(f(M)))\}$$

is a mapping of $L(G)$ into $L(G')$. But even if G is not unambiguous we define (not at variance with the definition 2.6):

Definition 2.7. Let G, G' and h be the same as in the definition 2.6. Then the set (6) is called a "*well-translation*" of the CFL $L(G)$ into the CFL $L(G')$ (induced by the well-transformation h).

Note. The process described in the definition 2.6 may be used also in case we have an "incomplete" phrase marker M introduced in note 3 after the example 2.1. The result is then of course an incomplete phrase marker $\tilde{M}$ describing the structure of a string in $(V \cup \Delta)^*$; this string contains metavariables, viz. the same as $L(M)$ does, and each of them in the equal number of occurrences as in $L(M)$.

Definition 2.8. For $m \geqq 1$ let $h_1, \ldots, h_m$ be well-transformations of (1) into context-free grammars $G_1, \ldots, G_m$, resp., where $G_i = (V, \Delta_i, P_i, S)$ for each i, $1 \leqq i \leqq m$. Let M be a phrase marker over G and let

$$(7) \qquad \{M_1, \ldots, M_n\}$$

be a decomposition of M. Let $j = j(i)$ be a function mapping $\{1, 2, \ldots, n\}$ into $\{1, 2, \ldots, m\}$.

Holding each subtree M_i of (7) for an incomplete phrase marker (in the sense of the note 3 after the example 2.1) let us replace it by the corresponding $\tilde{M}_i$ (using $h_{j(i)}$ in the sense of the note foregoing this definition). At the same time, the mappings h_i

 are supposed to satisfy the following implication:

(8) $$(K_{M_p} \cap K_{M_q} = \{v\} \wedge \varphi(v) = r) \supset (h_{j(p)}(r) = h_{j(q)}(r)) \quad \text{for all} \quad p, q, 1 \leqq p, q \leqq m\,.$$

The phrase marker $\tilde{M}$ arising from M in this way is called a *decomposition transformation of M*.

Definition 2.9. Let $\mathscr{P}$ be a decomposition procedure, whose range is some set $\mathfrak{M}$ of phrase markers over (1), i.e. a procedure yielding a decomposition of the form (7) for each $M \in \mathfrak{M}$. Let further $h_1, \ldots, h_m$ be weell-transformations from definition 2.8 and lastly let the corresponding function $j(i)$ be given for each $M \in \mathfrak{M}$. The mapping f yielding the decomposition transformation $\tilde{M}$ for each $M \in \mathfrak{M}$ on the strength of the corresponding decomposition and function $j(i)$ is called a *decomposition transformation of the grammar* (1).

Note. A question arises what are the properties of a decomposition translation being an analogy of the well-translation of CFL (cf. definition 2.7). The difficulty consists in what is a convenient description of both the input and output language, for we have only a subset of the whole set of all phrase markers on the one hand and more output CFG's on the other hand. All these grammars have the same common set of metavariables and also the same sentence symbol. The union of all terminal alphabets Δ_i might be hold for the output terminal alphabet and then also the union of all the rule sets P_i for the output rule set. But the output grammar arising in this way is not à priori a CFG for some restrictions follow from the construction of the decomposition transformation of a phrase marker as to the use of the context-free rules of P_i.

We shall see that for some important cases of decomposition procedures these restrictions may be formulated with the help of certain kind of "ordering" of the rule set and that do not go out the family of CFL's.

Definition 2.10. Let G, and G_i, h_i, $1 \leqq i \leqq m$, be the same as in the definition 2.8. Let $\mathfrak{M}$ and f be the same as in the definition 2.9. Then the set (6) of pairs (x, y) is called a *decomposition translation from $L(G)$ into $L(G')$*, where $G' = (V, \bigcup_{i=1}^{m} \Delta_i, \bigcup_{i=1}^{m} P_i, S)$ (more exactly a *decomposition translation induced by the* considered *decomposition transformation of G*).

3. TWO KINDS OF DECOMPOSITION TRANSLATION

Definition 3.1. Let (1) and $G' = (V, \Delta, P', S)$ be CFG's such that there exists at least one well-transformation h of G into G' and at least one well-transformation of G' into G. Then every system $H = (V, \Sigma, \Delta, R, S)$, where R is such a set of elements of the form $A \to (\alpha, \beta, \Pi)$ that the following conditions hold:

1. $A \to (\alpha, \beta, \Pi) \in R \supset \exists h$ (h is a well-transformation of G into G' $\wedge$ $A \to \beta = h(A \to \alpha) \wedge \Pi$ is the permutation determined by h and the rule $A \to \alpha$);

2. $A \to \alpha \in P \supset \exists\beta\, \exists\Pi(A \to (\alpha, \beta, \Pi) \in R) \wedge A \to \beta \in P' \supset \exists\alpha\, \exists\Pi(A \to (\alpha, \beta, \Pi) \in$
$\in R)$,

is called a *syntax-directed translation scheme* (SDTS). The elements of R are called *translation rules.*

Definition 3.2. A *form* of SDTS H from the definition 3.1 is a triple (γ, δ, Π) where

a) $\gamma \in (V \cup \Sigma)^*$, $\delta \in (V \cup \Delta)^*$;

b) the number of metavariables in the strings γ and δ must be equal, say k in each; then

c) Π must be a permutation of k objects and

d) for all i, $1 \leqq i \leqq k$, the ith variable of γ (from the left) is the same as the $\Pi(i)$th variable of δ.

Note. The right side of a translation rule is a form of H. We may omit the permutation in a form record if it is obvious which variables of γ and δ correspond.

Definition 3.3. Let $(\gamma_1, \delta_1, \Pi_1)$ be a form of H from definition 3.1 and A the ith variable of γ_1; let $A \to (\alpha, \beta, \Pi)$ be a translation rule of H. Replacing the ith variable of γ_1 by α and the $\Pi_1(i)$th variable of δ_1 by β we get a form $(\gamma_2, \delta_2, \Pi_2)$, where Π_2 is the permutation such that variables of γ_1 other than the ith correspond to the same symbol in δ_2 as in δ_1 and each variable of α corresponds to the variable of β to which it corresponded according to Π. (The exact formal definition of Π_2 see in [4] or [5].) Then we write

$$(\gamma_1, \delta_1, \Pi_1) \underset{H}{\Rightarrow} (\gamma_2, \delta_2, \Pi_2).$$

Definition 3.4. Given an SDTS H as in definition 3.1 let us define the relation $\overset{*}{\underset{H}{\Rightarrow}}$ by $(\gamma, \delta, \Pi) \overset{*}{\underset{H}{\Rightarrow}} (\gamma, \delta, \Pi)$ and if $(\gamma_1, \delta_1, \Pi_1) \overset{*}{\underset{H}{\Rightarrow}} (\gamma_2, \delta_2, \Pi_2)$ and $(\gamma_2, \delta_2, \Pi_2) \underset{H}{\Rightarrow} (\gamma_3, \delta_3, \Pi_3)$, then $(\gamma_1, \delta_1, \Pi_1) \overset{*}{\underset{H}{\Rightarrow}} (\gamma_3, \delta_3, \Pi_3)$. The set

$$\{(x, y) \mid x \in \Sigma^* \wedge y \in \Delta^* \wedge (S, S) \overset{*}{\underset{H}{\Rightarrow}} (x, y)\}$$

is called the *syntax directed translation* (SDT) T defined by H; the denotation is $T(H)$. Finally we denote dom $(T) = L(G)$ and range $(T) = L(G')$. We say also that G, G' and P, P' arised by a fission of H and R, resp.

Note: Both dom (T) and range (T) is a CFL for each SDT T defined by some SDTS. If H is given as in definition 3.1, then R can contain two rules of the forms $A \to (\alpha, \beta, \Pi)$ and $A \to$ $\to (\alpha, \beta', \Pi')$. Thus, generally, the mapping of the set of all the phrase markers over G onto the set of all the phrase markers over G' induced by SDTS H is many-many. — Examples of SDTS's and SDT's see in [4] and [5].

Definition 3.5. Let G and G_i, h_i, $1 \leqq i \leqq m$, be the same as in the definition 2.8.

Let $S \overset{*}{\underset{G}{\Rightarrow}} \alpha$ be a derivation, where $\alpha = w_0 X_1 w_1 X_2 \ldots w_{k-1} X_k w_k$, $X_i \in V$ for all i, $1 \leqq i \leqq k$, $w_i \in \Sigma^*$ for all i, $0 \leqq i \leqq k$, $k > 0$. Let $\mathfrak{M}$ be a set of phrase markers M over G such that there exists a decomposition

$$\{M_0, M_1, \ldots, M_k\} \tag{9}$$

of M such that

a) M_0 is the (incomplete) phrase marker of the mentioned derivation $S \overset{*}{\underset{G}{\Rightarrow}} \alpha$;

b) for each i, $1 \leqq i \leqq k$, M_i is a phrase marker of a derivation $X_i \overset{*}{\underset{G}{\Rightarrow}} \beta_i$, where $\beta_i \in \Sigma^*$.

Then (9) is called an *index decomposition*, the corresponding decomposition transformation of G (where now $j(i)$'s are functions whose domain is $\{0, 1, \ldots, k\}$ and range is $\{1, \ldots, m\}$, cf. the definition 2.9) is called an *index transformation*, and α is called the *index*.

Definition 3.6. Let G and G_i, h_i, $1 \leqq i \leqq m$, be the same as in the definition 2.3. Let $W = \{A_1 = S, A_2, \ldots, A_k\} \subseteq V$ be a set of the so called *selected metavariables*. For an arbitrary phrase marker M over G, each decomposition

$$\{M_1, M_2, \ldots, M_n\} \tag{10}$$

of M induced by the set

$$\{v \mid v \in K_M \wedge \mathscr{L}(\varphi(v)) \in W\}$$

is called a *decomposition in virtue of selected metavariables*. For each M over G, let the corresponding function $j(i)$ (cf. the definition 2.8) satisfies the following condition: If the roots of M_p and M_q of (10) are both labelled by rulles with equal left sides, then $j(p) = j(q)$. Then the corresponding decomposition transformation of G is called a *transformation in virtue of selected metavariables.*

Note 1. The decomposition considered in the definition 3.5 has an equal number of elements for each considered M. On the contrary, the number of subtrees of the decomposition considered in the definition 3.6 is not bounded. Nevertheless, both kinds of transformations will be able to be investigated by the same convenient method.

Note 2. As to the importance of the index transformation see [7]. The transformation in virtue of selected metavariables appears as a natural generalization of the index transformation. It enables us to translate in different ways such different parts of the input phrase that have been generated from different but in certain sense important metavariables; at the same time, these parts can be nested to an arbitrary depth.

In order that the proof of the theorem 3.1, which is the substantiality of this paper, may be easier, we prove at first two lemmas.

Lemma 3.1. *Let* CFG (1) *be given and let*

$$W = \{A_1 = S, A_2, \ldots, A_k\} \subseteq V.$$

Let a sequence of rule sets

$$P_1, P_2, \ldots, P_{2k-1}, P_{2k}$$

be given such that

1) $\bigcup_{j=1}^{2k} P_j = P$;
2) P_{2i-1} *is the set of all the rules* $r \in P$ *such that* $\mathscr{L}(r) = A_i$;
3) *if* $r \in P_{2i}$, *then* $\mathscr{L}(r) \notin W$.

Let $L \subseteq \Sigma^*$ *be a set of such strings that can be generated from* S *by a derivation* (2) *such that if for some* j, $0 \leqq j < m$, $\alpha_j \underset{G}{\Rightarrow} \alpha_{j+1}$ *by means of a rule belonging to some set* P_{2i-1}, $1 \leqq i \leqq k$, *then there is* $l \geqq j$ *such that the segment* $\alpha_j, \alpha_{j+1}, \ldots, \alpha_l$ *of the considered derivation satisfies the following properties*:

a) $\alpha_j \in (\Sigma \cup W)^*$;
b) $\alpha_l \in \Sigma^* \vee (\alpha_{l+1} \in (\Sigma \cup W)^* \wedge \alpha_{l+1} \notin \Sigma^*)$;
c) $\alpha_t \underset{G}{\Rightarrow} \alpha_{t+1}$ *by means of a rule of* P_{2i} *for all* $t, j < t \leqq l$ *if* $\alpha_l \notin \Sigma^*$ *else* $j < t < l$.

Then L *is a* CFL.

Proof. The phrase marker of a derivation satisfying the above formulated properties has the following characteristic property:

(I) The vertices of an oriented path emitting a vertex v which is labelled by a rule of P_{2i-1} and containing (but v) no vertex labelled by a rule of some P_{2j-1}, $1 \leqq j \leqq k$, are labelled only by rules of P_{2i}.

Now, we shall ensure that two rules $r \in P_{2i}$, $r' \in P_{2j}$, where $i \neq j$, need not be used in any derivation close upon each other. This can be reached by introducing new metavariables in the following manner: For each i, $1 \leqq i \leqq k$, let the metavariables of

$$V - W = \{B_1, \ldots, B_t\}$$

be replaced in the rules of $P_{2i-1} \cup P_{2i}$ by the metavariables $B_1^{(i)}, \ldots, B_t^{(i)}$, and at the same time

$$\{B_1^{(i)}, \ldots, B_t^{(i)}\} \cap \{B_1^{(j)}, \ldots, B_t^{(j)}\} = \emptyset \quad \text{for} \quad i \neq j.$$

Let

$$V' = W \cup \bigcup_{i=1}^{k} \{B_1^{(i)}, \ldots, B_t^{(i)}\}$$

and let P' be the set of all the rules obtained from the rules of P by the mentioned replacing of metavariables.

Now, we label the vertices of all such phrase markers over G that satisfy (I) by the corresponding rules of P' instead of those of P. At the same time, we have to ensure the conservation of (I), for the original sets P_{2i}, $1 \leqq i \leqq k$, need not be pairwise disjoint. Thus we get a one-one mapping of the set of all such phrase markers over G that satisfy (I) onto the set of all phrase markers over $G' = (V', \Sigma, P', S)$, and $L = L(G')$. The lemma is proven.

Lemma 3.2. *Each decomposition translation induced by an index transformation is also a decomposition translation induced by a transformation in virtue of selected metavariables.*

Proof. Let T be a decomposition translation from $L(G)$ induced by the index transformation of the definition 3.5. At first, we add new metavariables $\bar{S}, Y_1, \ldots, Y_k$ to the set V of CFG G and put

$$\bar{V} = V \cup \{\bar{S}, Y_1, \ldots, Y_k\}$$

($\bar{S}, Y_1, \ldots, Y_k$ are supposed not to belong to V). Also, the rule set P of G will be completed by new rules; let us denote $\bar{\alpha} = w_0 Y_1 \ldots w_{k-1} Y_k w_k$ (cf. α in the definition 3.5) and put

$$\bar{P} = P \cup \{\bar{S} \to \bar{\alpha}\} \cup \{Y_i \to \beta \mid 1 \leqq i \leqq k \wedge (X_i \to \beta) \in P\}.$$

Note that all X_i, $1 \leqq i \leqq k$, need not be distinct; nevertheless, for the sake of simplification, all Y_i, $1 \leqq i \leqq k$, are supposed to be distinct.

To each $M \in \mathfrak{M}$ with a decomposition (9) let the phrase marker $\bar{M}$ be conformed in the following way:

a) the root of $\bar{M}$ is labelled by the rule $\bar{S} \to \bar{\alpha}$;

b) the maximal subtree $\bar{M}_i$ of $\bar{M}$ such that its root is the end vertex of the edge i emitting the root of $\bar{M}$ differs from M_i only by the labelling of the root: if the root of M_i is labelled by $X_i \to \beta$, then the root of $\bar{M}_i$ is labelled by $Y_i \to \beta$.

Obviously

1. $\{\bar{M} \mid M \in \mathfrak{M}\}$ is the set of all the phrase markers over the grammar $\bar{G} = (\bar{V}, \Sigma, \bar{P}, \bar{S})$ and

2. $\{\bar{M}_0, \bar{M}_1, \ldots, \bar{M}_k\}$ is a decomposition of $\bar{M}$ induced by the set

$$\{v \mid \mathscr{L}(\varphi(v)) = \bar{S} \vee \exists i(1 \leqq i \leqq k \wedge \mathscr{L}(\varphi(v)) = Y_i)\}.$$

From the foregoing it ensues $\bar{M}_0$ looks as follows: it has its root (labelled by the only rule r with $\mathscr{L}(r) = \bar{S}$) and k edges emmiting this root; the end vertex of the edge which is labelled by i is labelled by a rule of the form $Y_i \to \beta$, for all i, $1 \leqq i \leqq k$.

Now, we define the grammars $\bar{G}_j$ and the well-transformations $\bar{h}_j$ of $\bar{G}$ into $\bar{G}_j$ for each j, $1 \leqq j \leqq m$. According to the definition 2.8, all these $\bar{G}_j$ must have the common set $\bar{V}$ of metavariables. Further

(i) for each $r \in P$ and each j, $1 \leqq j \leqq m$, if $r' = h_j(r)$, then $r' = \bar{h}_j(r)$.

(ii) Let $t = j(0)$; let $\tilde{M}_0$ be the phrase marker corresponding to M_0 by h_t and let $\bar{\alpha}'$ arises by the replacing of each X_i in $L(\tilde{M}_0)$ by Y_i, for $1 \leqq i \leqq k$. Then

$$\bar{h}_t(\bar{S} \to \bar{\alpha}) = \bar{S} \to \bar{\alpha}' .$$

For $j \neq t$, let $\bar{h}_j(\bar{S} \to \bar{\alpha})$ be arbitrary (it will never be used in any derivation).

(iii) Let $1 \leqq i \leqq k$; let us put $t = j(i)$. Then

$$\forall(Y_i \to \beta)\left(\left((Y_i \to \beta) \in \bar{P} \wedge h_t(X_i \to \beta) = X_i \to \beta'\right) \supset \bar{h}_t(Y_i \to \beta) = Y_i \to \beta'\right).$$

For $j \neq t$ let $\bar{h}_j(Y_i \to \beta)$ be arbitrary (they also will never be used in any derivation.)

For each j, $1 \leqq j \leqq m$, let us put

$$\bar{G}_j = (\bar{V}, \Delta_j, \bar{P}_j, \bar{S}) ,$$

where $\bar{P}_j$ is the set of all the rulles obtained from $\bar{P}$ by means of $\bar{h}_j$ according to (i), (ii) and (iii).

The corresponding decomposition transformation of $\bar{G}$ is obviously a transformation in virtue of selected metavariables, viz $\bar{S}, Y_1, \ldots, Y_k$ and the decomposition translation of $L(\bar{G})$ induced by this transformation is identical with T. The lemma is proven.

Theorem 3.1. *Let T be a translation induced by a transformation in virtue of selected metavariables. Then there exists an* SDTS *H such that $T = T(H)$.*

Proof. Let T be the translation of $L(G)$ induced by the transformation of the definition 3.6. For each i, $1 \leqq i \leqq k$, let us define

$$(11) \qquad P_{2i-1} = \{h_{j(i)}(r) \mid r \in P \wedge \mathscr{L}(r) = A_i\} ,$$

$$(12) \qquad P_{2i} \;\; = \{h_{j(i)}(r) \mid r \in P \wedge \mathscr{L}(r) \notin W\} .$$

Let $\mathfrak{M}$ be the set of all the phrase markers over G, $M \in \mathfrak{M}$, and $\tilde{M}$ the corresponding decomposition transformation of M.

Among the derivation that M is the phrase marker of, let us choose such one,

$$(13) \qquad \alpha_0, \alpha_1, \ldots, \alpha_s$$

that the following condition holds:

(II) for each i, $1 \leqq i \leqq s$, if $\alpha_{i-1} \underset{G}{\Rightarrow} \alpha_i$ by means of a rule r with $\mathscr{L}(r) \in W$, then $\alpha_{i-1} \in (\Sigma \cup W)^*$.

Then, in getting through the derivation (13), separate subtress of (10) are constructed in such a way that the "whole" subtree M_p is obtained (except such vertices that are roots of another subtrees) before any other M_q is started to be constructed. This implies the derivation (13) is splitting in a sequence of segments of the form

(14) $$\alpha_p, \alpha_{p+1}, \ldots, \alpha_q, 0 \leqq p \leqq q \leqq s,$$

satisfying the following conditions:

1. $\alpha_p \in (\Sigma \cup W)^* \wedge \alpha_p \notin \Sigma^*$;
2. $\alpha_q \in \Sigma^* \vee \alpha_{q+1} \in (\Sigma \cup W)^* \wedge \alpha_{q+1} \notin \Sigma^*$;
3. only rules r with $\mathscr{L}(r) \notin W$ are used in $\alpha_i \underset{G}{\Rightarrow} \alpha_{i+1}$ for all i, where $p < i \leqq q$ if $\alpha_q \notin \Sigma^*$, else $p < i < q$.

Let us formulate once more the important properties of the mappings h_j, which will be used in what follows:

(15) Given an arbitrary $r \in P$, then for each j, $1 \leqq j \leqq m$,
a) $\mathscr{L}(r) = \mathscr{L}(h_j(r))$ and
b) $\mathscr{R}(r), \mathscr{R}(h_j(r))$ contain the same metavariables and each of them with an equal number of occurrences; the corresponding permutation determines then which occurrences correspond each other.

Thus we can construct a "derivation" (in G' of the definition 2.10)

(16) $$\beta_0, \beta_1, \ldots, \beta_s$$

by induction on the basis of (13) in the following manner: we put $\beta_0 = \alpha_0$ $(= S)$. The following two properties (i) and (ii) hold for the strings α_0 and β_0 trivially:

(i) they contain the same metavariables and each of them in an equal number of occurrences;

(ii) if both strings contain q metavariables, then there exists a permutation Π of q objects which determines their one-one correspondence.

Let in (13) and (16) α_i and β_i resp. have been obtained, for some i, $0 \leqq i < s$. Let (i) and (ii) hold for these α_i and β_i and let Π be the permutation mentioned in (ii). Let $p \leqq i \leqq q$, i.e. α_i is one member of (14), and let (14) be supposed to yield the subtree M_z of the decomposition (10). If $\alpha_i \underset{G}{\Rightarrow} \alpha_{i+1}$ by rewriting the tth metavariable in α_i by means of the rule r, then let $\beta_i \Rightarrow \beta_{i+1}$ by rewriting the $\Pi(t)$ th metavariable in β_i by means of the rule $h_{j(z)}(r)$. (15) implies then (i), (ii) hold true also for α_{i+1} and β_{i+1} (cf. e.g. [7], p. 301).

Thus, to each segment (14), a segment

(17) $$\beta_p, \beta_{p+1}, \ldots, \beta_q$$

of the derivation (16) corresponds and satisfies the following properties (where $\Delta = \bigcup_{i=1}^{m} \Delta_i$):

1'. $\beta_p \in (\Delta \cup W)^* \wedge \beta_p \notin \Delta^*$;
2'. $\beta_q \in \Delta^* \vee \beta_{q+1} \in (\Delta \cup W)^* \wedge \beta_{q+1} \notin \Delta^*$;
3'. if $\beta_p \Rightarrow \beta_{p+1}$ by means of a rule of P_{2j-1}, then only rules of P_{2j} are used in $\beta_i \Rightarrow \beta_{i+1}$ for all i, where $p < i \leqq q$ if $\beta_q \notin \Delta^*$, else $p < i < q$.

Thus, the derivation (16) satisfies the conditions of the lemma 3.1, where P_i, $1 \leqq i \leqq 2k$, are given by (11) and (12).

On the contrary, to each derivation (16) in G' satisfying 1', 2', 3', there exists at least one derivation (13) in G satisfying (II) and yielding again (16) by way of induction as described above. Besides, some segment (14) satisfying 1, 2, 3 corresponds at the same time to each segment (17) satisfying 1', 2', 3'. This assertion follows from (15) and from (11) and (12). Thus, all derivations (16) satisfying the mentioned conditions are obtained from all derivation (13) by means of the construction described above. Thus, according to lemma 3.1 the set of output phrases of T is a CFL.

The desired CFG in the proof of the lemma 3.1 is found by means of sequential introduction of the sets of new metavariables replacing those not belonging to W and being contained in the rules of P_{2i-1}, P_{2i}; more exactly, for each i, $1 \leqq i \leqq k$, a new set $\{B_1^{(i)}, \ldots, B_t^{(i)}\}$ is added. In this manner, from each rule $h_{j(i)}(r)$, $1 \leqq i \leqq k$, containing metavariables not belonging to W, a new rule is obtained, which differs from the original one only by the denotation of the metavariables originally not belonging to W. The new grammar

$$G''' = (V', \bigcup_{i=1}^{m} \Delta_i, P', S)$$

having arised in this way has the following characteristic property: If we retain the denotation of the sets (11), (12) also after the metavariables have been renamed then for any derivation in G''' an "equivalent" derivation (i.e. one with the same phrase marker)

$$(18) \qquad \beta'_0, \beta'_1, \ldots, \beta'_s$$

can be found which has the following property: If $i \neq j$, then any rule $r \in P_{2i}$ is not applied close upon any rule $r' \in P_{2j}$.

Let us denote

$$R'_1 = \{(r, h_{j(i)}, (r)) \mid r \in P \wedge \mathscr{L}(r) = A_i \wedge \mathscr{R}(r) \in (\Sigma \cup W)^* \wedge 1 \leqq i \leqq k\} .$$

For all i, $1 \leqq i \leqq k$, and for all $h_{j(i)} \in P_{2i}$, let us denote by $[h_{j(i)}(r)]^{(i)}$ the rule we get from $h_{j(i)}(r)$ by renaming of metavariables. According to (15), each such renaming of metavariables induces also a renaming of metavariables in r, from which we get k

different rules $r^{(i)}$, $1 \leqq i \leqq k$. Now we denote

$$R_2' = \{(r^{(i)}, [h_{j(i)}(r)]^{(i)} \mid r \in P \wedge (r, h_{j(i)}(r)) \notin R_1' \wedge 1 \leqq i \leqq k\}.$$

Finitely, let $h_{j(i)}(r) \in P_{2i-1}$ (i.e. $\mathscr{L}(r) = A_i$) and $\mathscr{R}(r) \in (\Sigma \cup W)^*$. Let r' and $h'_{j(i)}(r)$ be the rules obtained by the metavariable renaming from r and $h_{j(i)}(r)$, resp. We denote

$$R_3' = \bigcup_{i=1}^{k} \{(r', h'_{j(i)}(r)) \mid r \in P \,\&\, \mathscr{L}(r) = A_i \,\&\, \mathscr{R}(r) \notin (\Sigma \cup W)^*\},$$

$$R' = R_1' \cup R_2' \cup R_3',$$

$$G'' = (V', \Sigma, \{\mathbf{r} \mid (\mathbf{r}, \bar{\mathbf{r}}) \in R'\}, S),$$

where V' is the same as in G'''. (15) holds true if we replace there r and $h_j(r)$ by $\mathbf{r}$ and $\bar{\mathbf{r}}$ of G'', resp. where $(\mathbf{r}, \bar{\mathbf{r}}) \in R'$. Thus to each derivation (18) in G''' we can construct by induction the corresponding derivation

$$\alpha_0', \alpha_1', \ldots, \alpha_s' \tag{19}$$

in G'' such that the relation between (19) and (18) is the analogy of that between (13) and (16).

Now if we denote

$$R = \{A \to (\mathscr{R}(\mathbf{r}), \mathscr{R}(\bar{\mathbf{r}}), \Pi) \mid (\mathbf{r}, \bar{\mathbf{r}}) \in R' \wedge A = \mathscr{L}(\bar{\mathbf{r}}) \wedge \Pi \text{ is the corresponding permutation}\},$$

then $H = (V', \Sigma, \bigcup_{i=1}^{m} \Delta_i, R, S)$ is the desired SDTS. The theorem is proven.

Note. Now, from the lemma 3.2 and theorem 3.1, it follows: ***Both the decomposition translation induced by an index transformation and the decomposition translation induced by a translation in virtue of selected metavariables is an SDT.***

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VÝTAH

Rozkladové překlady a schémata syntaxí řízeného překládání

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Jsou vyšetřovány dva důležité případy rozkladového překládání bezkontextových jazyků, a to jednak indexové překládání, jednak překládání indukované transformací na základě vybraných metaproměnných. Je dokázáno, že první z nich se dá převést na druhý a že oba se dají realizovat jistým způsobem formalizace dobrého překládání, totiž tzv. schématy syntaxí řízeného překládání.

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