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On Two Methods of Inversion of Z-Transforms

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Two methods of inversion of Z-transforms have been recently suggested by Azar and Badgett. The purpose of this note is to show that both methods are closely related and to compare their applicability with that of known ones.

INTRODUCTION

Let $f(t)$ be a continuous function of exponential growth in the interval $\langle 0, \infty)$ and thus with existing Laplace transform. Let ε be a variable in the interval $\langle 0, 1)$. Then, as it is well known, the modified Z-transform

$$F(z, \varepsilon) = f(\varepsilon)\, z^{-1} + f(1 + \varepsilon)\, z^{-2} + \ldots \tag{1}$$

exists for each ε. With respect to supposed continuity, $f(0)$ is defined as $f(0+)$. Thus, the ordinary Z-transform of the sequence $\{f(n)\}$ is

$$F(z) = zF(z, 0)\,. \tag{2}$$

Usually, $f(t)$ has rational Laplace transform and thus $\{f(n + \varepsilon)\}$ has rational modified Z-transform, or, by other words, $f(t)$ is solution of a linear differential equation with constant coefficients and $\{f(n + \varepsilon)\}$ is solution of a corresponding finite difference equation.

From (1) and the known initial value theorem, there follows

$$f(\varepsilon) = \lim_{z \to \infty} z\, F(z, \varepsilon)\,. \tag{3}$$

THE TWO METHODS

We are defining

$$F_k(z, \varepsilon) = f(k + \varepsilon)\, z^{-1} + f(k + 1 + \varepsilon)\, z^{-2} + \ldots \tag{4}$$

for $k = 0, 1, 2, \ldots$, so that $F(z, \varepsilon) = F_0(z, \varepsilon)$. Then by the initial value theorem

$$f(k + \varepsilon) = \lim_{z \to \infty} z F_k(z, \varepsilon) \tag{5}$$

and it can easily be shown that following recurrence relation holds:

$$F_{k+1}(z, \varepsilon) = z\, F_k(z, \varepsilon) - f(k + \varepsilon)\,. \tag{6}$$

Defining

$$\Phi_k(z, \varepsilon) = z\, F_k(z, \varepsilon)\,, \tag{7}$$

one gets from (5), (7)

$$f(k + \varepsilon) = \lim_{z \to \infty} \Phi_k(z, \varepsilon) \tag{8}$$

and from (6), (7)

$$\Phi_{k+1}(z, \varepsilon) = z[\Phi_k(z, \varepsilon) - f(k + \varepsilon)]\,. \tag{9}$$

(8) and (9) with $\varepsilon = 0$ have been derived by Badgett [2]. It may be noted that $\Phi_0(z, \varepsilon)$ is the function used in soviet literature instead of the modified Z-transform [3].

Defining

$$A(z, k + \varepsilon) = z^{1-\varepsilon-k}\, F_k(z, \varepsilon)\,, \tag{10}$$

one gets from (5), (10)

$$f(k + \varepsilon) = \lim_{z \to \infty} z^{k+\varepsilon}\, A(z, k + \varepsilon)\,. \tag{11}$$

This (with $k + \varepsilon = t/T$, where t is the real time and T the sampling period) is the formula of Azar [1]. It may be noted that $A(e^s, \varepsilon)$ is the generalized discrete Laplace transform introduced by Ragazzini and Zadeh [3].

PRACTICAL APPLICABILITY

Although interesting per se, the formulae of Azar and Badgett seem to be for the time being of limited practical use. The respective functions, as it is seen from (4), are related with the remainder of the modified Z-transform expansion. This remainder is not used in the applications. To compute $f(k + \varepsilon)$ from (5) (or from (8) or (11)), the recurrence relation (6) is to be used at the first to compute $F_k(z, \varepsilon)$. But $f(k + \varepsilon)$ can also be computed without the aid of the functions $F_k(z, \varepsilon)$, by the known method of long division, or, what is practically the same, by known recurrence formulas (see e.g. [4], p. 136), rediscovered recently by Jenkins [5].

To obtain the explicit expression for $f(k+\varepsilon)$, the inverse modified Z-transform

$$f(k+\varepsilon) = \frac{1}{2\pi i} \int F(z, \varepsilon)\, z^k \, \mathrm{d}z \tag{12}$$

or the solution of the related difference equation is to be used (assuming rational Z-transform). The knowledge of all roots of the characteristic equation is needed in both cases, but this is no serious obstacle in the age of automatic computers.

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VÝTAH

O dvou metodách inverze transformací Z

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V této poznámce se srovnávají navzájem a se známými metodami dvě metody inverze transformací Z navržené Azarem a Badgettem.

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