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Jaromír Štěpán

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Some Problems of System Identification

JAROMÍR ŠTĚPÁN

The paper deals with the influence of the form of the input signals upon the accuracy of the results of the identification. It is shown, that the system identification in open control loop cannot mostly result in sufficiently accurate estimates nor in coefficients $\bar{a}_3$. Perspectives of identification methods are discussed. The possibilities of system identification in closed control loop are demonstrated by an example. The treatment sets out from the least squares estimator.

1. INTRODUCTION

At present there exists an extensive control systems theory, however, the utilization of this theory in the treatment of practical problems is rather limited [1]. In this connection one often speaks of a gap between theory and practice of control system design. One of the main causes of this situation is the identification of systems. So far, no sufficiently reliable method of identification has been established, not even for the simplest types of systems, and it is questionable altogether wether such a method can be found at the present state of measuring technique. Authors of papers and books employ the usual phrase: "Let us consider a system described by a differential equation (or a transfer function)." In practical problems, however we can always find only an estimate of the transfer function. Further calculations of the synthesis of control systems, which necessarily must set out from the results of identification, can be reasonable only if we know the accuracy of the estimates of the transfer function coefficients. This paper represents an attemp to analyze this problem. Especially we shall try to analyse the connection between the variances of the estimates and the properties of the functions to be identified.

Single-input-single-output systems will be discussed, which can be described by transfer functions of the type

(2,1) $$F_s(p) = \frac{x(p)}{y(p)} = \frac{1}{N(p)} = \frac{1}{\sum_{i=0}^{n} a_i p^i},$$

where $x(p)$ is the Laplace transform (L-transform) of the output signal and $y(p)$ the L-transform of the input signal. Only deterministic input signals and initial zero conditions will be considered.

Let us start from the following response curves shifted so that $x(\infty) = 0$:

$x(t)$ – ideal response pertinent to the transfer function (2,1),
$\tilde{x}(t)$ – measured response,
$\bar{x}(t)$ – substitute response calculated by the approximation (evaluation) of the response $\tilde{x}(t)$ and pertinent to the transfer function,

(2.2) $$\bar{F}_s(p) = \frac{1}{\bar{N}(p)} = \frac{1}{\sum_{i=0}^{\bar{n}} \bar{a}_i p^i}.$$

Let us consider the linearized substitute function $\bar{x}(t)$ in the form

(2.3) $${}^s\bar{x}(t) = \sum_i a_i \, {}^s v^{(i)}(t)$$

where

(2.4) $$\mathscr{L}\{{}^s v^{(i)}(t)\} = \frac{p^i}{N^2(p)} S(p).$$

$S(p)$ is L-transform, which describes the form of the input signal.

Further on only two forms of input signals are considered and denoted by superscript on the left. The unit step input signal carries the superscript 0 and unit impulse input signal is denoted by superscript 1.

In this case we can write the function (2.3) in the form

(2.3′) $${}^s\bar{x}(t) = \sum_i \bar{a}_i \, v^{(l)}(t)$$

where

(2.4′) $$\mathscr{L}\{v^{(l)}(t)\} = \frac{p^l}{N^2(p)}.$$

The suitability of the form (2.3′) of the substitute function $\bar{x}(t)$ follows from relations (for $\bar{n} = n$):

for unit impulse input signal ($l = i$)

$$(2.5) \qquad \mathscr{L}\{{}^1\bar{x}(t)\}\big|_{\bar{a}_i = a_i} = \frac{\sum_{i=0}^{n} a_i p^i}{N^2(p)} = \frac{N(p)}{N^2(p)} = \frac{1}{N(p)} = \mathscr{L}\{{}^1x(t)\}$$

for unit step input signal ($l = i - 1$)

$$(2.6) \qquad \mathscr{L}\{{}^0\bar{x}(t)\}\big|_{\bar{a}_i = a_i} = \frac{a_0}{p}\left[\frac{1}{N^2(p)} - \frac{1}{a_0^2}\right] + \frac{\sum_{i=1}^{n} a_i p^{i-1}}{N^2(p)} =$$

$$= \frac{\sum_{i=0}^{n} a_i p^i}{pN^2(p)} - \frac{1}{a_0 p} = \frac{1}{p}\left[\frac{1}{N(p)} - \frac{1}{a_0}\right] = \mathscr{L}\{{}^0x(t)\}\,.$$

Functions $v^{(l)}(t)$ were derived in reference paper [10] in connection with deriving the linearized sensitivity functionals of the estimates of coefficients $\bar{a}_i$.

Let us assume that functions $v^{(l)}(t)$ are known and that these functions do not depend on the estimates of the coefficients $\bar{a}_i$. Let us start from the procedure of identification on the basis of the linear least squares estimator [4] for the limit case of the discrete alternative – for the single response $\tilde{x}(t)$ composed from the points of q measured responses $\tilde{x}_j(t)$. We shall consider only independent, stationary, ergodic Gaussian noise. In this case we must start from r points of each response $\tilde{x}_j(t)$ with minimal distance $\Delta\tau$. In limit case for one point of each response $\tilde{x}_j(t)$ we can write the estimate of the variance $\bar{\sigma}^2$ in the form

$$(2.7) \qquad \bar{\sigma}^2 = \frac{1}{q}\sum_{j=0}^{q}[\tilde{x}_j(j\,\Delta t) - x(j\,\Delta t)]^2\frac{\Delta t}{\Delta t}\,.$$

Hence for $t \in (0, \tau)$

$$\bar{\sigma}^2 q\,\Delta t = \bar{\sigma}^2\tau = \bar{\delta}^2 = \sum_{j=0}^{q}[\tilde{x}_j(j\,\Delta t) - x(j\,\Delta t)]^2\,\Delta t \doteq \int_0^\tau[\tilde{x}(t) - x(t)]^2\,\mathrm{d}t$$

where $\bar{\delta}^2$ is the estimate of the measurement error. We consider the resulting response as one point with the variance $\bar{\sigma}^2$.

The vector of the estimates of coefficients $\bar{a}_i$ is determined by relation

$$(2.8) \qquad \boldsymbol{a} = \left[\int_0^\tau(\boldsymbol{G}\boldsymbol{G}^{\mathrm{T}})\,\mathrm{d}t\right]^{-1}\int_0^\tau(\boldsymbol{G}\,{}^1\tilde{x})\,\mathrm{d}t$$

where

$$\boldsymbol{a}^{\mathrm{T}} = (a_0 a_1 \ldots a_n)\,,$$

$$\boldsymbol{G}^{\mathrm{T}} = (v^{(0)}(t)\,v^{(1)}(t)\ldots v^{(n)}(t))\,.$$

The pertinent covariance matrix can be written in the form

(2.9)
$$\boldsymbol{\Psi} = \delta^2 \left[\int_0^\tau (\mathbf{G}\mathbf{G}^\mathrm{T}) \, \mathrm{d}t \right]^{-1} = \delta^2 \mathbf{A}^{-1} ,$$

where the elements α^{hi} of the inverse matrix $\mathbf{A}^{-1}$ can be calculated from

(2.10)
$$\alpha^{hi} = \frac{|A_{ih}|}{|A|} .$$

$|A_{ih}|$ is the complement of element α_{ih} in determinant $|A|$. $|A|$ is the determinant of matrix $\mathbf{A}$. Elements α_{ih} of the matrix $\boldsymbol{A}$ are determined by relations

(2.11)
$$\alpha_{ih} = (v^{(i)}, v^{(h)}) = \int_0^\tau v^{(i)}(t) \, v^{(h)}(t) \, \mathrm{d}t ,$$
$$\alpha_{ii} = (v^{(i)}, v^{(i)}) = \|v^{(i)}\|^2 .$$

For the unit impulse input responses $h, i = 0, 1, 2, \ldots, n$. The lenght of the time interval τ will be chosen so that $\int_\tau^\infty v^{(i)}(t) \, v^{(h)}(t) \, dt$ is negligibly small.

The utilization of the given relations for practical identification of the systems would encounter some problems. It is not the objective of the present paper to design a practical identification method. The subject matter of this paper is the problem of the accuracy of the estimates of coefficients $\bar{a}_i$. Thus we can start from the most favourable characteristics of the estimates $\bar{a}_i$ given by relations

(2.12)
$$\mathrm{E}(\bar{a}_i) = a_i ,$$
$$\mathrm{D}(\bar{a}_i) = \delta^2 \alpha^{ii} .$$

The dependence of the variances of the estimates $\bar{a}_i$ on the type of function describing the system is determined in the considered theoretical case by the scalar products (2.11).

The kernel of useful signals of considered types lies in region for $t < 2a_1$. Therefore the integration interval of the noise will be chosen

(2.13)
$$T = 2a_1 < \tau .$$

The measurement error is

(2.14)
$$\delta^2 = T\sigma^2 .$$

We consider so the favourable alternative.

The dispersion σ^2 or the standard deviation σ gives a tolerance strip, in which the measured responses will lie, e.g. the measured response $x(t)$ for $0 \geqq t \geqq T$ will lie with the probability $P = 95\%$ inside a tolerance strip of the breadth of $1{,}96\sigma \approx 2\sigma$ i.e.

(2.15)
$$x(t) - 2\sigma \leqq \tilde{x}(t) \leqq x(t) + 2\sigma .$$

3. INFLUENCE OF THE FORM OF THE INPUT SIGNALS UPON THE RESULTS OF THE IDENTIFICATION

In this section we shall analyse the influence of the form of the input signal upon the accuracy of the estimates of the coefficients of the transfer functions and we shall mainly try to show the incorrectness of the statement, often quoted in the literature, regarding the highest possible information of the responses to the input signal in the form of an impulse. We shall limit this analysis to a step and to a Dirac impuls input signals, i.e. to extreme cases of the range of input signals most frequently employed in theory. For both alternatives the measuring error will be the same.

The suitability of the considered input signals can be judged in the best way according to the ratio of the respective variances. For the ratio of the variances of the coefficient $\bar{a}_n$ of the substitute transfer function (2.2) for $\bar{a}_0 = a_0$; $\bar{a}_1 = a_1$; $\bar{a}_2 = a_2; \ldots; \bar{a}_{n-1} = a_{n-1}$ we can write on the basis of the relations (2.10) and (2.12)

$$(3.1) \qquad R_n = \frac{{}^1\mathrm{D}(\bar{a}_n)}{{}^0\mathrm{D}(\bar{a}_n)} = \frac{\dfrac{\delta^2}{\|{}_n v^{(n)}\|^2}}{\dfrac{\delta^2}{\|{}_n v^{(n-1)}\|^2}} = \frac{\|{}_n v^{(n-1)}\|^2}{\|{}_n v^{(n)}\|^2}\,.$$

The lower index n denotes the order of the transfer function (2.1) pertinent to the norms $\|v^{(l)}\|$.

The ratio of the variances R_n is given only by the properties of the function describing the system.

For the norms $\|v^{(l)}\|$ pertinent to the transfer functions of the type (2.1) we can write the Schwarz-Bunjakovski inequalities (for $l = 0, 1, 2, \ldots, n$ and for $\tau = \infty$)

$$(3.2) \qquad \|v^{(l)}\|\;\|v^{(l+2)}\| \geqq |(v^{(l)}\, v^{(l+2)})| = \|v^{(l+1)}\|^2\,.$$

Hence holds

$$(3.3) \qquad \frac{\|v^{(0)}\|}{\|v^{(1)}\|} \geqq \frac{\|v^{(1)}\|}{\|v^{(2)}\|} \geqq \ldots \geqq \frac{\|v^{(n-1)}\|}{\|v^{(n)}\|}\,.$$

The inequalities (3.3) show that the responses to an impulse or to input signals near an impulse, are mainly suited in the identification of the coefficients at the highest powers of the complex variable.

Now let us show that for the considered systems does generally not hold

$$(3.4) \qquad R_n \leqq 1\,.$$

We start with the first order systems. According to the relation (3.1) we get

$$(3.5) \qquad R_1 = \frac{{}^1\mathrm{D}(\bar{a}_1)}{{}^0\mathrm{D}(\bar{a}_1)} = \frac{\|{}_1 v^{(0)}\|^2}{\|{}_1 v^{(1)}\|^2}\,.$$

The norms $\|{}_1v^{(0)}\|$ and $\|{}_1v^{(1)}\|$ in relation (3.5) can be calculated numerically by means of the Routh-Schur algorithm for the stability test [7]. This algoritm is given in Appendix A. According to the relations (A.7) and (A.8) we get for the transfer function

(3.6) $$\mathscr{L}\{{}_1v^{(l)}(t)\} = \frac{p^l}{(a_0 + a_1 p)^2} = \frac{p^l}{c_0 + c_1 p + c_2 p^2},$$

(3.7) $$\|{}_1v^{(0)}\|^2 = \frac{1}{2c_0c_1} = \frac{1}{4a_0^3 a_1},$$

(3.8) $$\|{}_1v^{(1)}\|^2 = \frac{1}{2c_1c_2} = \frac{1}{2\,.\,2a_0a_1\,.\,a_1^2} = \frac{1}{4}\frac{1}{a_0a_1^3}.$$

Hence

(3.9) $$R_1 = \frac{a_1^2}{a_0^2}.$$

We can assume $a_0 = 1$. The ratio of the variances according to the relation (3.5) depends therefore on the magnitude of the coefficient a_1. The influence of this coefficient is judged best according to the magnitude of the norms of the corresponding responses. The norms relating to the responses to the input step signal ($s = 0$ and ${}^0x(t)$ shifted so that ${}^0x(\infty) = 0$) or to the impulse input signal ($s = 1$) are defined by the relations

(3.10) $$\|{}^sx\| = \left\{\int_0^\infty {}^sx^2(t)\,\mathrm{d}t\right\}^{1/2}$$

and can be calculated numerically by means of the algorithm given in Appendix A.

Both these norms are connected by the Schwarz-Bunjakovski inequality. For the transfer function (2.1) we can write

(3.11) $$\|{}^0x\|\,\|{}^1x\| \geqq |({}^0x,\,{}^1x)| = \frac{1}{2}\frac{1}{a_0^2}.$$

The sign of equality in the relation (3.11) holds for norms pertinent to the systems of first order. The dependence of the norms $\|{}^0x\|$, $\|{}^1x\|$ and the ratio of variances R_1 according to the relation (3.5) on the magnitude of the coefficient a_1 is given by the following extreme cases (for $a_0 = 1$)

(3.12) $$a_1 = 1\,; \quad \|{}^0x\| = \|{}^1x\| = \frac{1}{\sqrt{2}}\,; \quad R_1 = 1\,;$$

$$a_1 \to 0\,; \quad \|{}^0x\| \to 0\,; \quad \|{}^1x\| \to \infty\,; \quad R_1 \to 0\,;$$

$$a_1 \to \infty\,; \quad \|{}^0x\| \to \infty\,; \quad \|{}^1x\| \to 0\,; \quad R_1 \to \infty\,.$$

In practice we are always limited in the range of measurable amplitudes, so that mostly we cannot utilize the magnitudes of the responses to the impulse input signal for $a_1 < 1$ (regardless of the realizability of the Dirac impulse). For systems of the first order thus holds on the mentioned assumption and for the responses, which can be realized with the maximum unit amplitude

$$R_1 \geqq 1 \,. \tag{3.13}$$

The relations (3.5), (3.9), (3.12) hold only for an independent estimate of the coefficient $\bar{a}_1$ i.e. for $\bar{a}_0 = a_0$. Now let us calculate the ratio of variances $\hat{R}_1$ for the case that the estimate of the coefficient $\bar{a}_1$ depends on the estimate of the coefficient $\bar{a}_0$. According to the relations (2.9), (2.10) and (2.12) we get

$$\hat{R}_1 = \frac{{}^1\hat{\mathrm{D}}(\bar{a}_1)}{{}^0\hat{\mathrm{D}}(\bar{a}_1)} = \frac{\|{}_1v^{(0)}\|^2}{\|{}_1v^{(1)}\|^2}\left[1 - \frac{({}_1v^{(-1)}, {}_1v^{(0)})^2}{\|{}_1v^{(-1)}\|^2\|{}_1v^{(0)}\|^2}\right] . \tag{3.14}$$

By means of the algorithm in Appendix A we can calculate

$$\hat{R}_1 = 0{\cdot}2\,\frac{a_1^2}{a_0^2} \,. \tag{3.14'}$$

For the assessment of the influence of the form of the input signals it is necessary to derive the relations for the ratio of the estimate-variances of the coefficient $\bar{a}_0$. According to the relations in Appendix A we get for dependent estimate $\bar{a}_0$

$$\frac{{}^1\hat{\mathrm{D}}(\bar{a}_0)}{{}^0\hat{\mathrm{D}}(\bar{a}_0)} = \frac{\|{}_1v^{(-1)}\|^2}{\|{}_1v^{(0)}\|^2}\left[1 - \frac{({}_1v^{(-1)}, {}_1v^{(0)})^2}{\|{}_1v^{(-1)}\|^2\,\|{}_1v^{(0)}\|^2}\right] = a_0^2 a_1^2 \tag{3.15}$$

and for independent estimate $\bar{a}_0$

$$\frac{{}^1\mathrm{D}(\bar{a}_0)}{{}^0\mathrm{D}(\bar{a}_0)} = 5a_0^2 a_1^2 \,. \tag{3.16}$$

The relations (3.15) and (3.16) show, that it is advantageous to identify the coefficients a_0 and a_1 separately. The relation (3.5) garantees then better the conditions for the assessment of the influence of the input signals.

The situation is analogous for systems of higher order. For the ratios of the variances of the estimates of the coefficients $\bar{a}_n$ of the second and third order systems of the type (2.1) we can derive general relatively simple relations. We start from the transfer function

$$F_s(p) = \frac{1}{1 + \sum_{i=1}^{3} a_i p^i} = \frac{1}{1 + a_1 p + \varkappa a_1^2 p + \varkappa \varrho a_1^3 p} \tag{3.17}$$

where $\varkappa = a_2/a_1^2$ and $\varrho = a_3/a_1 a_2$.

For the second order systems ($\varrho = 0$) we get

$$(3.18) \qquad R_2 = \frac{{}^1\mathrm{D}(\bar{a}_2)}{{}^0\mathrm{D}(\bar{a}_2)} = \frac{\|{}_2v^{(1)}\|^2}{\|{}_2v^{(2)}\|^2} = a_1^2\varkappa = a_2 \, .$$

The calculation of the norms $\|{}_2v^{(1)}\|$ and $\|{}_2v^{(2)}\|$ is given as example in Appendix A.

Similarly we get for the third order systems

$$(3.19) \qquad R_3 = \frac{{}^1\mathrm{D}(\bar{a}_3)}{{}^0\mathrm{D}(\bar{a}_3)} = \frac{\|{}_3v^{(2)}\|^2}{\|{}_3v^{(3)}\|^2} = a_1^2 \frac{\varkappa(\varkappa + \varrho)}{1 + \varkappa} \, .$$

The values of the dimensionless parameters $\varkappa$ and ϱ vary in a narrow range for the majority of practical control systems ($\varkappa \in (0{\cdot}04;\ 0{\cdot}4)$; $\varrho \in (0{\cdot}02;\ 0{\cdot}2)$). The ratios of the variances according to the relations (3.18) and (3.19) depend therefore mainly on the magnitude of the coefficient a_1. The influence of this coefficient can be judged according to the Schwarz-Bunjakovski inequality (3.11). For considered systems there holds a weak inequality in this relation. For $a_1 > 1$ however in the case of responses to an impulse input signals it can be better utilized the zone of measurable amplitudes. Since we are limited even in these cases to systems with the coefficient a_1 in a certain narrow zone, the responses to the impulse input signals or to the signals near impulse signal have the limited significance for practical identification.

4. SYSTEM IDENTIFICATION IN OPEN CONTROL LOOP

Under the term "system identification in open control loop" we shall understand identification on the basis of the responses of systems in distinction to identification on the basis of responses of closed control loops. In this section we limit our consideration only to stepinput signals (see sec. 3).

First we define the relative standard deviation of the estimate of the coefficient $\bar{a}_i$ [11]

$$(4.1) \qquad {}^0s_i = \frac{[{}^0\mathrm{D}(\bar{a}_i)]^{1/2}}{a_i} \, .$$

For Gaussian noise the values ${}^0s_i$ indicate limits round the correct value of the coefficient a_i (e.g. $a_i \pm 0{\cdot}4a_i$ for ${}^0s_i = 0{\cdot}4$) within which the estimate of the respective coefficient will lie with the probability $P = 68\%$.

First we shall analyse the third order system of the type (3.17). The relative standard deviation of the estimate of the coefficient $\bar{a}_3$ (for $\bar{a}_1 = a_1$; $\bar{a}_2 = a_2$) is given by the relation ($\delta = \sigma \sqrt{(2a_1)}$)

$$(4.2) \qquad {}^0s_3 = \frac{\delta}{a_3\|v^{(2)}\|} = \frac{\sigma\sqrt{(2a_1)}}{a_3\|v^{(2)}\|} \, .$$

In following sections we shall consider only the substitute transfer functions of the third order, therefore we omit the index on the left. By means of the algorithm in

Appendix A we can determine a general expression for the norm $\|v^{(2)}\|$. The relation (4.2) can then be written in the form

$$^{0}s_3 = \frac{2\sqrt{(2)}\,(1-\varrho)^{3/2}\,\sigma}{\varrho(\varkappa+\varrho)^{1/2}}\,. \tag{4.3}$$

The dimensionless parameters $\varkappa$ and ϱ pertinent to third-order systems of the type (3.17) with real poles lie in the intervals $\varkappa \in (0; 0{\cdot}33)$; $\varrho \in (0; 0{\cdot}11)$. This is shown by the Euler inequalities

$$a_j^2 \geqq a_{j-1}a_{j+1}\left(1+\frac{1}{j}\right)\left(1+\frac{1}{n-j}\right), \tag{4.4}$$

$$a_1^2 \geqq 3a_2 \Rightarrow \frac{a_2}{a_1^2} = \varkappa \leqq 0{\cdot}33\,, \tag{4.5}$$

$$a_2^2 \geqq 3a_1a_3 \Rightarrow \frac{a_2}{a_1^2} = \varkappa \geqq 3\,\frac{a_3}{a_1a_2} = 3\varrho\,, \tag{4.6}$$

$$\varrho \leqq \tfrac{1}{9} = 0{\cdot}11\,.$$

The relative standard deviation of the estimate $\bar{a}_3$ diminishes with the rising parameters $\varkappa$ and ϱ. Hence we can put for third-order systems with real poles

$$^{0}s_3^* \geqq \left.\frac{2\sqrt{(2)}\,(1-\varrho)^{3/2}\,\sigma}{\varrho(\varkappa+\varrho)^{1/2}}\right|_{\substack{\varkappa=0\cdot33\\ \varrho=0\cdot11}} = 32\sigma\,. \tag{4.7}$$

Similarly, we can determine the relative minimum standard deviation of the estimate of the coefficient $\bar{a}_3$ of systems with real poles, without limiting the order, with the transfer functions of the type (2.1) (for $a_0 = 1$), which can be approximated by third order systems. We start from the unfavourable case – from the minorants of the systems of the type (2.1) with real poles, which are given by systems with transfer functions of the type [10; 11; 13]

$$F_s(p) = \frac{e^{-T_d p}}{1+T_1p} = \frac{1}{(1+T_1p)\left(1+T_dp+\dfrac{T_d^2}{2!}p^2+\ldots\right)} \tag{4.8}$$

where T_d is the transport lag. For the coefficients of the polynomial with real poles the following inequality holds [6]

$$4\left[a_{i-1}^2-\left(1+\frac{1}{n-i+1}\right)\left(1+\frac{1}{i-1}\right)a_ia_{i-2}\right]\cdot \tag{4.9}$$
$$\cdot\left[a_{i-2}^2-\left(1+\frac{1}{n-i+2}\right)\left(1+\frac{1}{i-2}\right)a_{i-3}a_{i-1}\right] \geqq$$
$$\geqq \left[a_{i-1}a_{i-2}-\left(1+\frac{2}{n-i+1}\right)\left(1+\frac{2}{i-2}\right)a_{i-3}a_i\right]^2.$$

The sign of equality holds for the coefficients of the polynomials pertinent to the minorants with the transfer function (4.8). We obtain for $a_1 = T_1 + T_d$; $a_2 = T_d(T_1 + \frac{1}{2}T_d)$; $a_3 = \frac{1}{2}T_d^2(T_1 + \frac{1}{3}T_d)$

$$(4.10) \qquad 4(a_2^2 - 1{\cdot}5a_3a_1)(a_1^2 - 2a_2) = (a_2a_1 - 3a_3)^2 .$$

The relation (4.10) can be rewritten for dimensionless parameters $\varkappa$ and ϱ

$$(4.10') \qquad 4\left(1 - 1{\cdot}5\frac{\varrho}{\varkappa}\right)(1 - 2\varkappa) = (1 - 3\varrho)^2 .$$

For the interval ϱ $(0 < \varrho < 0{\cdot}11)$ we calculate according to the relation (4.10′) the interval of the parameter $\varkappa$ $(0 < \varkappa < 0{\cdot}22)$. For these intervals of $\varkappa$ and ϱ the relation (4.7) shows also the identifiability of the coefficient a_3 of transfer functions (2.1) with real poles without limiting the order.

The limited possibilities of system identification in open control loops are still better demonstrated by the number of distinguishable systems of the type (2.1) with

Table 1.

Standard deviation σ	Number N of distinguishable	
	third-order systems	systems of unlimited order
10^{-2}	15	20
$5 \ . 10^{-3}$	60	80
$2{\cdot}5 \ . 10^{-3}$	240	320
$1{\cdot}25 . 10^{-3}$	960	1280
10^{-3}	1500	2000

real poles listed in Table 1. The number of distinguishable systems in view of the estimate of the coefficient $\bar{a}_i$ is given by the expression [12]:

$$(4.11) \qquad N_{ik} = \frac{M_k(\bar{a}_i)}{[{}^0\mathrm{D}_s(\bar{a}_i)]^{1/2}} ,$$

where $M_k(\bar{a}_i)$ is the considered interval of the estimates of the coefficient $\bar{a}_i$, and $\mathrm{D}_s(\bar{a}_i)$ is the mean variance of the estimates of $\bar{a}_i$ across this interval. The values in Table 1 were calculated by dividing the region of the coefficients a_2 and a_3 into p triangles $(p = 6)$, for which $N_k = \frac{1}{2}N_{2k}N_{3k}$ was calculated. The total number of distinguishable systems is $N = \sum_{k=1}^{p} N_k$. The calculation sets out on the following assumptions:

a) We know the estimate of the coefficient $\bar{a}_1$ $(\bar{a}_1 = a_1;\ \bar{a}_0 = a_0 = 1)$,

b) the interval of the coefficients a_2 and a_3 is given by the Euler inequalities ($\varkappa \in \in (0{\cdot}05;\ 0{\cdot}33)$; $\varrho \in (0{\cdot}01;\ 0{\cdot}11)$),

c) for systems of unlimited order we assume the approximation by third-order systems.

In practical problems we can expect the measurement accuracy with standard deviation near $\sigma = 10^{-2}$. The measurement accuracy characterized by standard deviations near $\sigma = 10^{-3}$ can be reached in special cases (e.g. by analog computers).

The listed results apply to systems, which are described by transfer functions of the type (2.1) with real poles or, may be, to systems which are near to them; these systems have at least two real dominant poles. Dominant are poles having the smallest absolute value.

From the considerations presented in this paragraph we can arrive at the following conclusions:

a) Systems of the third order can be regarded as the touchstone of identification methods (Table 1).

b) Identification of systems of the considered types in open control loops can for the greater part not result in sufficiently accurate estimates nor in coefficient $\bar{a}_3$.

5. PERSPECTIVES OF IDENTIFICATION METHODS

From the conclusions of section 4 it follows that for the accuracy of measurement normally attained in practice a solution of identification does not exist even for the most simple systems. Now, what are the perspectives of identification in the light of this assertion? According to relation (2.12) there are two possibilities of improving identification results.

First is the reduction of the measurement error δ. This way of improving identification results will be troublesome from the technical as well as primarily from the economic point of view.

The second possibility is the reduction of the element α^{ii} i.e. changing norms $\|v^{(i)}\|$. The change of norms $\|v^{(i)}\|$ means the change of the corresponding sensitivity functionals. The linearized sensitivity functional of the estimate of the coefficient $\bar{a}_i$ is given by relation [10] (for the step input signal)

$$\frac{\partial E}{\partial \bar{a}_i} = 2(\bar{a}_i - a_i)\, \|v^{(i-1)}\|^2 \tag{5.1}$$

where

$$E = \|{}^0\bar{x} - {}^0x\|^2 \,.$$

It was shown in reference [10] that functionals of sensitivity vary under the influence of feedback. Hence the way to an improvement of identification results is system identification in closed control loop.

For the sake of simplicity let us demonstrate the possibilities of identification in closed control loop by a practical problem. Consider a system of the third order with transfer function

$$F_s(p) = \frac{1}{10p^3 + 21p^2 + 12p + 1} = \frac{1}{(10p + 1)(p + 1)^2}. \tag{5.2}$$

In the feedback the simples controller will be considered – an ideal proportional controller. The transfer function of the control loop input variable – change in command variable will have the form of

$$F_w(p) = \frac{K}{10p^3 + 21p^2 + 12p + 1 + K}. \tag{5.3}$$

The improvement of the measuring accuracy is mainly limited by the systematic errors of the measuring instruments. From the view-point of systematic errors we can take into consideration the same measuring error in the identification in an open

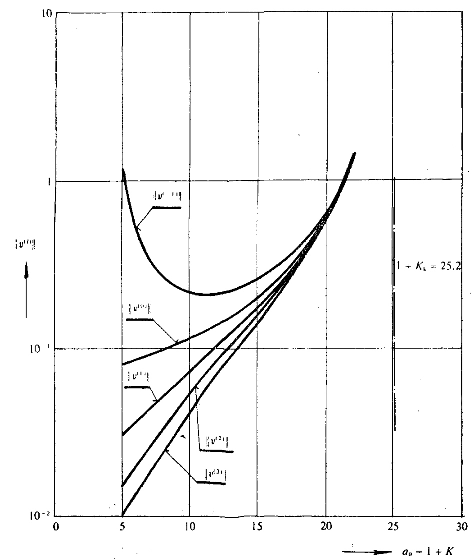

Fig. 1. The dependence of the norms $\|v^{(l)}\|$ pertinent to transfer function (5.3) on the gain coefficient.

control loop and in a closed control loop. Thus we assume that the use of the relations (2.12), which hold only for a Gaussian noise, leads to errors that exert no essential influence on the analysis of the variances from the view point of the functions describing the systems.

Fig. 1 shows the dependence of the norms $\|v^{(l)}\|$ pertinent according to the relation (2.4) to the transfer function (5.3) on the gain coefficient K. In Table 2 the norms $\|v^{(l)}\|$ pertinent to the transfer function (5.2) and (5.3) (for $K = 19$) are given.

Table 2.

The norms $\|v^{(l)}\|$ pertinent to	$\|v^{(-1)}\|$	$\|v^{(0)}\|$	$\|v^{(1)}\|$	$\|v^{(2)}\|$	$\|v^{(3)}\|$
transfer function (5.2)	4·05	$1{\cdot}55 \,.\, 10^{-1}$	$1{\cdot}28 \,.\, 10^{-2}$	$3{\cdot}14 \,.\, 10^{-3}$	$1{\cdot}7 \,.\, 10^{-3}$
transfer function (5.3) for $K = 19$	$6{\cdot}13 \,.\, 10^{-1}$	$6{\cdot}06 \,.\, 10^{-1}$	$6{\cdot}0 \,.\, 10^{-1}$	$5{\cdot}98 \,.\, 10^{-1}$	$6{\cdot}0 \,.\, 10^{-1}$

The norm $\|v^{(2)}\|$, which is decisive for relative standard deviation ${}^0s_3$ of the estimate of the coefficient $\bar{a}_3$, increases according to Table 2 190 times. We consider in this case the independent estimate of coefficient $\bar{a}_3$. We thus assume that we know all other coefficients and that the diference $|\bar{a}_3 - a_3|$ is sufficiently small [10; 11].

For large difference $|\bar{a}_3 - a_3|$ it is necessary to start from the "limit" approximation error [11] i.e. from the error between the ideal system of the n-th order and the substitute system of the $(n - 1)$-th order (for $\bar{a}_0 = a_0$; $\bar{a}_1 = a_1, \ldots, \bar{a}_{n-1} = a_{n-1}$)

$$(5.4) \qquad S_{jn}^2 = \|{}^0x_{jn}(t) - {}^0x_{j,(n-1)}(t)\|^2 =$$

$$= \int_0^\tau \left[\mathscr{L}^{-1} \left\{ \frac{a_n p^{n-1}}{N_j(p)\, \bar{N}_j(p)} \right\} \right]^2 \mathrm{d}t = a_n^2 \|\bar{v}^{(n-1)}\|^2 .$$

The index j denotes the responses pertinent to the transfer function (5.3) for the j-th value of the gain coefficient K.

For the considered third order system we obtain

$$(5.5) \qquad S_{j3} = \|{}^0x_{j3}(t) - {}^0\bar{x}_{j2}(t)\| = a_3 \|\bar{v}_j^{(2)}\| .$$

According to the algorithm in Appendix A we can calculate for $\tau = \infty$ the norms $\|\bar{v}^{(2)}\|$ and the norms $\|{}^0x_j\|$ of the useful output signals for different values of the gain coefficient K. The identifiability of the coefficient a_3 can be assessed according to the ratio of the norm of the useful signal and the limit approximation error

$$Q_j = \frac{\|{}^0x_{jn}\|}{S_{jn}}. \tag{5.6}$$

It is always necessary to consider the command variable as input variable. Only in this case it holds $\|{}^0x_j\| \approx$ const for $K \in (1; 0{\cdot}9K_k)$ (K_k is critical gain coefficient).

The norms $\|{}^0x_j\|$, the limit approximation errors S_{j3} and the ratios Q_j for considered example are given for different values of the gain coefficient K in Table 3.

Table 3 shows, that by identification of systems in closed control loop we can essentially improve the accuracy of the estimate of the coefficient $\bar{a}_3$.

Table 3.

$a_0 = 1 + K$	1	5	10	15	20
$\|{}^0x_j\|$	2·63	1·2	1·3	1·5	2
S_{j3}	$2{\cdot}85 \,.\, 10^{-2}$	$1{\cdot}3 \,.\, 10^{-1}$	$3{\cdot}55 \,.\, 10^{-1}$	$7{\cdot}1 \,.\, 10^{-1}$	1·45
Q_j	92	9·2	3·6	2·1	1·4

Now let us calculate the relative standard deviations of the dependent estimates of the coefficients $\bar{a}_i$ according to the relations (2.10), (2.12), and (4.1). E.g. for relative standard deviation of the estimate of the coefficient $\bar{a}_3$ we can write in the case, when we start from response to impulse input signal

$$ {}^1s_{3j} = \frac{[{}^1\mathrm{D}_j(\bar{a}_3)]^{1/2}}{a_3} = \frac{\delta}{a_3}(\alpha_j^{33})^{1/2} = \frac{\delta}{a_3\|v_j^{(3)}\|} \,. \\ . \frac{1}{\left(1 - \frac{\|v_j^{(2)}\|^4}{\|v_j^{(1)}\|^2\,\|v_j^{(3)}\|^2}\right)}. \tag{5.7}$$

For the step input signal we obtain

$$ {}^0s_{3j} = \frac{[{}^0\mathrm{D}_j(\bar{a}_3)]^{1/2}}{a_3} = \frac{\delta}{a_3\|v_j^{(2)}\|} \,. \tag{5.8}$$

$$. \frac{1}{1 - \frac{\|v_j^{(1)}\|^2}{\|v_j^{(2)}\|^2} \; \frac{\|v_j^{(-1)}\|^2\,\|v_j^{(1)}\|^4 - \|v_j^{(0)}\|^4\,\|v_j^{(1)}\|^2}{\|v_j^{(-1)}\|^2\,\|v_j^{(0)}\|^2\,\|v_j^{(1)}\|^2 - \|v_j^{(0)}\|^6 - (v_j^{(-1)}, v_j^{(0)})^2\,\|v_j^{(1)}\|^2}} \,.$$

In Fig. 2a (for the step input signal) and in Fig. 2b (for the impulse input signal), are given the relative standard deviations of the estimates of the coefficients of the transfer function (5.3) in dependence on the gain coefficient. Interval T, for which noise is beeing considered, was chosen as the double of the inverse value of the real partion of the smallest root.

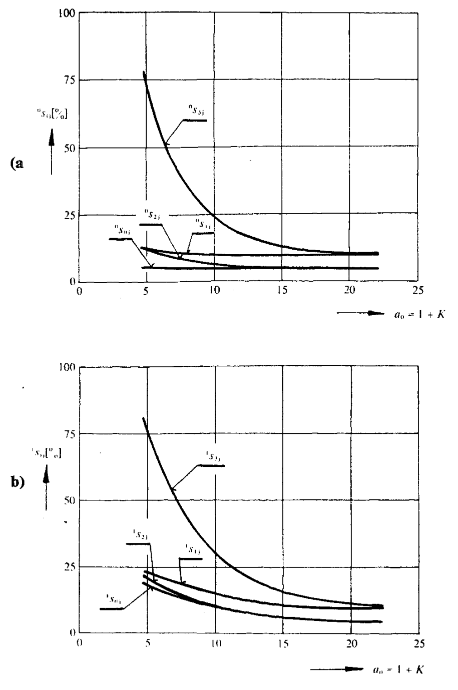

Fig. 2. The dependence of relative standard deviations ${}^0s_{ij}$ (Fig. 2a) and ${}^1s_{ij}$ (Fig. 2b) pertinent to transfer function (5.3) on the gain coefficient.

Table 4 shows the difference between the identification in open and in closed control loop in the case of dependent estimates $\bar{a}_i$. In this table the influence of the restriction of the output signals on the relative standard deviations ${}^0s_{ij}$ is demonstrated. In practical tasks amplitudes of the output signals are always limited. Let us

Table 4.

		${}^0s_{0j}$	${}^0s_{1j}$	${}^0s_{2j}$	${}^0s_{3j}$
The transfer function (5·2)		0·045	0·09	0·68	2·4
The transfer function (5·3) for $K_j = 19$	$m_{K=19} = 1$	0·05	0·11	0·05	0·12
	$m_{K=19} = 0{\cdot}59$	0·089	0·19	0·085	0·2

assume that output signals can be realized with the maximum unit amplitud

(5.9) $$m_j[|{}^0x_j(0)| + |{}^0x_j(t_{\max})|] \approx 1$$

or for unshifted responses $({}^0x_j^*(\infty) = K/1 + K)$

(5.10) $$m_j|{}^0x_j^*(t_{\max})| \approx 1\,,$$

where ${}^0x_j^*(t_{\max})$ is the maximum deviation of the step response pertinent to transfer functions

(5.11) $$F_w(p) = \frac{K}{p(K + 1 + \sum_{i=1}^{n} a_i p^i)}\,.$$

The coefficient m_j is then given by the relation

(5.12) $$m_j = \frac{1}{|{}^0x_j^*(t_{\max})|}\,.$$

For the case considered in Table 4 we get $m_{K=19} = 0{\cdot}59$. The maximum deviation can be calculated from the component of the dominant roots [13]. Especially at the limit of stability the overshoot $|{}^0x_j^*(t_{\max}) - {}^0x_j^*(\infty)|$ pertinent to the systems of the type (5.3) is given by simple relation

(5.13) $$|{}^0x_j^*(t_{\max}) - {}^0x_j^*(\infty)| = \frac{K_k}{a_3\omega_k^2\left[\left(\frac{a_2}{a_3}\right)^2 + \frac{a_1}{a_3}\right]^{1/2}} =$$

$$= \frac{K_k}{\left[(K_k + 1)^2 + \frac{1}{\varkappa}(K_k + 1)\right]^{1/2}}$$

where ω_k is the critical frequency and K_k is the critical gain coefficient.

Hence for the coefficient m_k we obtain

$$(5.14) \qquad m_k = \frac{(K_k + 1)\left[1 + \frac{1}{\varkappa(K_k + 1)}\right]}{K_k\left[2 + \frac{1}{\varkappa(K_k + 1)}\right]} .$$

The minimal coefficient $m_k = 0{\cdot}5$ we get for $K_k = \infty$ i.e. for the second order transfer functions.

Table 4 shows, that by identification of systems in closed control loop we can improve the accuracy of the estimates of coefficients $\bar{a}_2$ and $\bar{a}_3$.

Fig. 2a and 2b show, that the relative standard deviations of dependent estimates of the odd and even coefficients for the gain coefficient $K_j \to K_k$ are equal

$$(5.15) \qquad {}^0s_{1k} = {}^0s_{3k} \, ; \quad {}^0s_{0k} = {}^0s_{2k}$$

$$ {}^1s_{1k} = {}^1s_{3k} \, ; \quad {}^1s_{0k} = {}^1s_{2k} \, ,$$

when index k denotes the standard deviations pertinent to the transfer function (5.3) for the gain coefficient $K_j \to K_k$. Let us analyse the relations (5.15). In Appendix B it is shown that for the gain coefficient $K_j \to K_k$ the signs of equality hold in relations (3.2), (3.3) or (B. 2). One can write according to the relation (B. 6)

$$(5.16) \qquad \frac{{}^0s_{3k}}{{}^0s_{1k}} = \frac{a_1 \|v_k^{(0)}\|}{a_3 \|v_k^{(2)}\|} = \frac{a_1}{a_3}\, a_1^2 \varkappa \varrho = 1 \, .$$

An analogous relation holds for relative standard deviations of the estimates of the coefficients $\bar{a}_{0k}$ and $\bar{a}_2$.

The identification in closed control loop for the gain coefficient near to the critical has one serious short-comming. Because the norms $\|v_k^{(i)}\|$ are linear dependent (Appendix B), it is possible to identify only one even and one odd coefficient.

From the results given in sec. 4 and 5 it follows that the "good" estimates of all coefficients of the third order transfer functions can be obtained only by combination of the procedures in open and closed control loops.

6. CONCLUSIONS

Let us summarize the results of this paper with regard to the practical applicability of identification methods.

a) The influence of the form of the input signals upon the results of the identification is in literature overestimated. For a limited channel of the output signals it is in most cases more advantageous to operate with step input signals (sec. 2). A step is easy to apply and is sufficiently drastic from the viewpoint of the pertinent sensitivity functionals. The use of the input signals near impulse is restricted to special cases.

b) On transfer functions of the third order and of transfer functions of higher orders which can be approximated by those of third order it was shown in sec. 3 that the practical possibilities of identification in open control loop are limited. For measurement accuracy normally encountered in practice the identification procedures in open control loop can not result in sufficiently accurate estimates nor in coefficients $\bar{a}_3$ (relation (4.7), Table 1). Relations presented in sec. 3 and Table 1 explain the failure of most identification methods in the solution of practical problems.

c) On example of the third order system it was demonstrated (Fig. 2; Tables 3, 4), that the accuracy of the estimates of coefficients $\bar{a}_2$ and $\bar{a}_3$ can be essentially improved by identification procedures in closed control loop. For gain coefficient $K_j \rightarrow K_k$ only the vector of even and odd coefficients can be identified (relations (5.15), (5,16) and (B.6). From this it follows that only a combination of identification procedures in open and closed control loops can lead to "good" estimates of all coefficients of considered transfer functions. This conclusion is in agreement with reference [10] containing the statement, that the identification of systems must start with functions describing control processes for various values of the constants of controllers in feedback. The "good" identification method must then be iterative. The results of sec. 5 show, that the use of a dual control (i.e. the simultaneous solution of identification and optimization problems) will be possible only in special cases.

d) Identification methods published in literature normally overestimate the formal apparatus of statistics. Contrariwise, they underestimate or by pass the fact that measurement accuracy is always limited in practice. No respect is normally paid to the properties of functions describing the investigated systems.

APPENDIX A

CALCULATION OF THE NORMS $\|v^{(l)}\|$

The scalar products of time functions $f_1(t)$ and $f_2(t)$ of the type (2.11) can be calculated according to the relation

$$(f_1, f_2) = \int_0^\infty f_1(t) f_2(t) \, dt = \frac{1}{2\pi j} \int_{-j\omega}^{+j\omega} F_1(p) F_2(-p) \, dp \tag{A.1}$$

where $F_1(p)$ and $F_2(p)$ are the Laplace transforms of the functions $f_1(t)$ and $f_2(t)$. For $f_1 = f_2 = f$ we get

$$\|f\|^2 = \int_0^\infty f^2(t) \, dt = \frac{1}{2\pi j} \int_{-j\omega}^{+j\omega} F(p) F(-p) \, dp \, . \tag{A.2}$$

Integrals (A.1) and (A.2) could be generally evaluated as the sum of residues of the roots p_i pertinent to the denominator of transfer function $F_1(p)$ or $F(p)$. In paper [7] it is derived the algorithm permitting the calculation of the norms $\|f\|^2$ without the knowledge of roots p_i.

We start from the general transfer function pertinent to the functions $v^{(l)}(t)$

$$(A.3)\qquad \mathscr{L}\{v^{(l)}(t)\} = \frac{M(p)}{N^2(p)}\, p^l = \frac{\sum_{h=0}^{r-1} b_h p^h}{\sum_{m=0}^{r} c_m p^m} = \frac{D_{r-1}(p)}{V_r(p)}\,.$$

By the right-side reduction of the polynomial $V_r(p)$ of the denominator of the transfer function (A.3) we obtain the polynomial $V_{r-1}(p)$ lower by one degree

$$(A.4)\qquad V_{r-1}(p) = V_r(p) - \frac{c_r}{c_{r-1}}\, p\, \overline{V}_r(p)\,,$$

where $\overline{V}_r(p)$ is an even or an odd part of the polynomial $V_r(p)$, depending on the circumstance whether r is odd or even. For the g-th reduction step we can write

$$(A.5)\qquad V_{r-g}(p) = V_{r-g+1}(p) - \frac{{}^{g-1}c_{r-g+1}}{{}^{g-1}c_{r-g}}\, p\overline{V}_{r-g+1}(p)\,.$$

Simirarly, we must reduce the numerator of the transfer function (A.3)

$$(A.6)\qquad D_{r-g-1}(p) = D_{r-g}(p) - \frac{{}^{g-1}b_{r-g}}{{}^{g-1}c_{r-g}}\, \overline{V}_{r-g+1}(p)\,.$$

Some coefficients b_h may be equal to zero. The square of the norm $\|v^{(l)}\|^2$ changes in each reduction by

$$(A.7)\qquad \Delta P_g = \frac{1}{2}\, \frac{({}^{g-1}b_{r-g})^2}{{}^{g-1}c_{r-g+1}\,{}^{g-1}c_{r-g}}\,.$$

The square of the norm $\|v^{(l)}\|$ then is

$$(A.8)\qquad \|v^{(l)}\|^2 = \sum_{g=1}^{r} \Delta P_g\,.$$

As example we calculate the square of the norm $\|{}_2v^{(2)}\|^2$ pertinent to the transfer function (3.17) for $\varrho = 0$

$$(A.9)\qquad \mathscr{L}\{{}_2v^{(2)}(t)\} = \frac{p^2}{(1 + a_1 p + \varkappa a_1^2 p^2)^2} =$$

$$= \frac{p^2}{1 + 2a_1 p + (1 + 2\varkappa)\, a_1^2 p^2 + 2\varkappa a_1^3 p^3 + \varkappa^2 a_1^4 p^4} = \frac{b_2 p^2}{\sum_{m=0}^{4} c_m p^m}\,.$$

For the right-side reduction of the polynomial $\sum_{m=0}^{4} c_m p^m$ we use the algorithm given in paper [7]

(A.10)

$$
\begin{array}{ccccc|c}
c_0 & c_1 & c_2 & c_3 & c_4 & \dfrac{c_4}{c_3} \\
 & \searrow & -c_1 \dfrac{c_4}{c_3} & \searrow & -c_4 & \\
\hline
{}^1c_0 & {}^1c_1 & {}^1c_2 & {}^1c_3 & 0 & \dfrac{{}^1c_3}{{}^1c_2} \\
\searrow & -{}^1c_0 \dfrac{{}^1c_3}{{}^1c_2} & \searrow & -{}^1c_3 & & \\
\hline
{}^2c_0 & {}^{2\prime}c_1 & {}^2c_2 & 0 & &
\end{array}
$$

For the general form of numerator

$$D_{r-1}(p) = \sum_{h=0}^{3} b_h p^h .$$

the right side reduction can be written in the form [7]

(A.11)

$$
\begin{array}{cccc|c}
\underline{b_0} & b_1 & \underline{b_2} & b_3 & \dfrac{b_3}{c_3} \\
 & -c_1 \dfrac{b_3}{c_3} & & -b_3 & \\
\hline
{}^1b_0 & \underline{{}^1b_1} & {}^1b_2 & 0 & \dfrac{{}^1b_2}{{}^1c_2} \\
-{}^1c_0 \dfrac{{}^1b_2}{{}^1c_2} & & -{}^1b_2 & & \\
\hline
{}^2b_0 & {}^2b_1 & 0 & &
\end{array}
$$

According to the relations (A.7) and (A.8) we obtain (${}^3b_0 = {}^2b_0$; ${}^3c_0 = {}^2c_0$; ${}^3c_1 = {}^2c_1$)

(A.12)
$$\|{}_2v^{(2)}\|^2 = \frac{1}{2}\left\{\frac{(b_3)^2}{c_3c_4} + \frac{({}^1b_2)^2}{{}^1c_2{}^1c_3} + \frac{({}^2b_1)^2}{{}^2c_1{}^2c_2} + \frac{({}^3b_0)^2}{{}^3c_0{}^3c_1}\right\}.$$

By substitution of the coefficient a_1 and dimensionless parameter $\varkappa$ according to the transfer function (A.9) we obtain ($b_3 = {}^2b_1 = 0$)

$$\text{(A.13)} \qquad \|{}_2v^{(2)}\|^2 = \frac{1}{2}\left\{\frac{1}{{}^1c_2{}^1c_3} + \frac{\left(\frac{{}^1c_0}{{}^1c_2}\right)^2}{{}^2c_1{}^2c_2}\right\} =$$

$$= \frac{1}{2}\left\{\frac{1}{2a_1^5(1+\varkappa)\varkappa} + \frac{1}{2a_1^5(1+\varkappa)}\right\} = \frac{1}{4a_1^5\varkappa}.$$

By means of the algorithms (A.10) and (A.11) we can calculate the other norms $\|{}_2v^{(l)}\|$ too, e.g. for the square of the norm $\|{}_2v^{(1)}\|^2$ we obtain (${}^3b_0 = {}^1b_2 = b_3 = 0$)

$$\text{(A.14)} \qquad \|{}_2v^{(1)}\|^2 = \frac{1}{2}\frac{({}^2b_1)^2}{{}^2c_1{}^2c_2} = \frac{1}{4a_1^3}.$$

APPENDIX B

THE SCHWARZ-BUNJAKOVSKI INEQUALITY FOR THE NORMS $\|v_j^{(l)}\|$

For the norms $\|v_j^{(l)}\|$ pertinent according to the relation (2.4) to transfer functions of the type

$$\text{(B.1)} \qquad F_w(p) = \frac{K_j}{K_j + \sum_{i=0}^{n} a_i p^i}$$

we can write the Schwarz-Bunjakovski inequality (for $l = 0, 1, 2, \ldots, n$ and for $\tau = \infty$ in the relation (2,11))

$$\text{(B.2)} \qquad \|v_j^{(l)}\|\,\|v_j^{(l+2)}\| \geqq |(v_j^{(l)}, v_j^{(l+2)})| = \|v^{(l+1)}\|^2.$$

Hence holds

$$\text{(B.3)} \qquad \frac{\|v_j^{(0)}\|}{\|v_j^{(1)}\|} \geqq \frac{\|v_j^{(1)}\|}{\|v_j^{(2)}\|} \geqq \ldots \geqq \frac{\|v_j^{(n-1)}\|}{\|v_j^{(n)}\|}.$$

Let us show, that the signs of equality in the relations (B.2) and (B.3) hold for $K_j \to K_k$ (K_k is the critical gain coefficient). First let us consider the ratios of the norms $\|v_k^{(l)}\|/\|v_k^{(l+1)}\|$ pertinent to the third order transfer function (B.1) for $K_j \to K_k$. According to the algorithm in Appendix A we can write ($a_{0k}\varrho = 1$); $a_{0k} = a_0 + K_k$)

$$\text{(B.4)} \qquad \frac{\|v_k^{(0)}\|^2}{\|v_k^{(1)}\|^2} = \frac{\frac{1}{2}\frac{1}{c_0{}^4c_1}}{\frac{1}{2}\frac{1}{{}^4c_1{}^3c_2}} = \frac{{}^3c_2}{c_0} = \frac{1}{c_0}\left({}^1c_2 - {}^1c_4\frac{c_1{}^1c_4 - c_0c_5}{c_3{}^1c_4 - {}^1c_2c_5}\right).$$

Substitution of ${}^1c_2 = 2a_{0k}\varkappa a_1^2$; ${}^1c_4 = \varkappa a_1^4$; $c_0 = a_{0k}^2$; $c_1 = 2a_{0k}a_1$; $c_3 = 2\varkappa a_1^3(1 + a_{0k}\varrho)$; $c_5 = 2\varkappa^2\varrho a_1^5$ into the relation (B.4) leads to

$$\text{(B.5)} \qquad \frac{\|v_k^{(0)}\|^2}{\|v_k^{(1)}\|^2} = \frac{a_1^2\varkappa}{a_{0k}} = a_1^2\varkappa\varrho = \frac{a_3}{a_1} = \frac{1}{\omega_k^2},$$

where ω_k is the critical frequency.

With analogous procedure we can obtain the relations for other ratios of norms $\|v_k^{(l)}\|$.

Hence holds

$$\text{(B.6)} \qquad \frac{\|v_k^{(0)}\|}{\|v_k^{(1)}\|} = \frac{\|v_k^{(1)}\|}{\|v_k^{(2)}\|} = \frac{\|v_k^{(2)}\|}{\|v_k^{(3)}\|} = \left(\frac{a_3}{a_1}\right)^{1/2} = \frac{1}{\omega_k}.$$

The relation (B.6) follows directly from the definition (2.4) of functions $v_k^{(l)}(t)$. In this case the component of the dominant roots is decisive for the ratios $\|v_k^{(l)}\|/\|v_k^{l+1}\|$. Hence the analogous relations hold for the ratios of the norms $\|v_k^{(l)}\|$ pertinent to high order systems.

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Některé problémy identifikace soustav

JAROMÍR ŠTĚPÁN

V práci je nejdříve analyzován vliv tvaru vstupního signálu na přesnost výsledků identifikace soustav. Je ukázáno, že signály ve tvaru skoku jsou dostatečně vybuzující z hlediska funkcionálů citlivosti (odst. 3).

V odst. 4 jsou diskutovány omezené možnosti identifikace z odezev otevřených regulačních obvodů. Pro soustavy třetího řádu popřípadě pro soustavy vyšších řádů, které lze aproximovat soustavami třetího řádu, byly odvozeny vztahy, které ukazují, že nelze získat dostatečně přesné odhady koeficientů a_3 (vztah (4.7), tabulka 1).

V odst. 5 jsou analyzovány perspektivy identifikace soustav. Na základě numerického příkladu soustavy třetího řádu bylo demonstrováno, že přesnost odhadů koeficientů a_2 a a_3 lze podstatně zvýšit identifikačními metodami, které vycházejí z odezev uzavřeného regulačního obvodu. Se zvětšujícím se zesílením proporcionálního regulátoru ve zpětné vazbě se však postupně zvětšuje závislost koeficientů. Identifikačními metodami založenými na odezvách uzavřených regulačních obvodů lze proto zpravidla zjistit v jednom iteračním kroku jen jeden lichý a jeden sudý koeficient. Z toho plyne, že pouze kombinace identifikačních postupů vycházejících jednak z odezev soustav a jednak z odezev uzavřených regulačních obvodů může vést k dostatečně přesným odhadům co největšího počtu koeficientů přenosů typu (2.1).

Ing. Jaromír Štěpán, CSc., Ústav teorie informace a automatizace ČSAV (Institute of Information Theory and Automation — Czechoslovak Academy of Sciences), Vyšehradská 49, Praha 2.