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# USING THE POSSIBILITY THEORY IN FUZZY TEMPORAL REASONING

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All in the world is situated in the space and exists in the time. The time is an independent physical quantity influencing truth categories of real logical structures. Time data penetrate into logical formulas and condition their truth values to a significant extent. Temporalized premises force adequate inferential mechanisms.

## 1. A CLASSIFICATION OF TEMPORAL DATA

In temporal statements time data can be in
– an explicit form, e. g. an exact date, a well-defined relation,
– an implicit form, e. g. a presentation of the time in linguistic terms such as a new car, a modern flat, an up-to-date car.

Other classification of time data can be
– absolute data, if the meaning of a time date is constant in all its applications, e. g. an exact date, the relation "before",
– relative data, if the meaning of a time date changes in its different applications, e. g. the time relation "a little later".

The most important classification of time values is classified in a partition of time data into three disjunctive classes:
– the past, as the set of all time values before a referential time value,
– the present, as the set of time values which may be identified with a referential time value,
– the future, as the set of time values which shall be after a referential time value.

From the certainty point of view, any set of time data can be partitioned into two subsets, for
– certain (exact, complete, well-defined) data,
– uncertain (inexact, incomplete, fuzzy) data.

In this short paper we are only dealing with uncertain time data of fuzzy type from the past, present and future time.

Temporal data which occur in real applications of artificial intelligence are usually connected with non-temporal expressions describing particular facts or situations in a problem domain. Therefore, it is suitable to accept the model that a situation represented by a statement containing time consists of two components, namely a non-temporal one expressing the content item of the statement, and a temporal item expressing a time date which may be of a point, interval or combined time type. We admit for both components to contain, in general, uncertainty, especially fuzziness. Thus, we have to do with temporal statements of the type:

"John will be rich in a few years",

where both the content component "John will be rich" and the temporal component "in a few years" contain uncertainty of the fuzzy type.

Note that generally both the components of a temporal statement need not be mutually separable, but, for simplicity, we will deal only with the case of separable components such as in the above example.

## 2. A FORMALIZATION OF FUZZY TEMPORAL FORMULAS

Let $B = (A, \wedge, \vee, \neg)$ be Boolean algebra of fuzzy logical formulas, where A is a non-empty set of atomic fuzzy formulas of the form $p(x_1, \dots x_n), p$ being an $n$-ary fuzzy predicate (symbol) and $x_1, \dots, x_n$ being terms. Let $U \neq 0$ be a universe of discourse in which formulas of the set $B$ are interpreted in a common way. The set of formulas interpreted in such a way is denoted $B^*$.

By an interpretation, predicates shall be involved into formulas to which fuzzy relations correspond unambigously in the universe $U$. Fuzzy predicates are often expressed as values of linguistic variables, and the whole formula represents a reasonable, grammatically correct sentence (of a natural language).

From the other hand, by a point time structure we will understand an ordered pair $\langle T, R \rangle$, where $T$ is a non-empty set of point values of a time variable $t$, and $R$ is a precedence relation which is an unbounded, dense and connected strict linear ordering in the set $T$ (cf. [1], [2]).

Let $T^*$ be the set of all fuzzy subsets $\tau$ of the time set $T$ in a structure $\langle T, R \rangle$. Make an agreement that temporal adverbs, predicates and other time containing linguistic constructions which are often occurring in approximate reasoning, will be commonly called fuzzy temporal expressions (shortly $ft$-expressions) and denoted $e(\tau)$, in order to distinguish $ft$-expressions defined over the same fuzzy set $\tau$. The set of all admissible $ft$-expressions at a given time structure $\langle T, R \rangle$ is denoted $E^T$.

Now, produce the Cartesian product $B \times E^T$ and write its elements as pairs $(\beta, e)$, $\forall \beta \in B$, $\forall e \in E^T$. From the set $A \times E^T \subseteq B \times E^T$ select a subset $\mathcal{A}$ and call it the set of atomic fuzzy temporal formulas. Analogously, from the set $B \times E^T$ select a set $\mathcal{B}$ such that

1. $\mathcal{A} \subseteq \mathcal{B}$
2. if $\alpha \in \mathcal{B}$ then $\neg\alpha \in \mathcal{B}$
3. if $\alpha \in \mathcal{B}$, $\beta \in \mathcal{B}$ then $\alpha \wedge \beta \in \mathcal{B}$, $\alpha \vee \beta \in \mathcal{B}$

where

$$\neg(\alpha, e) = (\neg\alpha, e), \ \forall \alpha \in \mathcal{B}, \ \forall e \in E^T.$$

The smallest set $\mathcal{B} \subseteq B \times E^T$ having the above properties is called the set of fuzzy temporal formulas (shortly $ft$-formulas). Similarly, the smallest set $\mathcal{B}^* \subseteq B^* \times E^T$ defined analogously is called the set of fuzzy temporal statements (shortly $ft$-statements).

In the set $\mathcal{B}^*$ we define a truth function

$$v : \mathcal{B}^* \to [0,1]$$

which assigns for an $ft$-statement $(\varphi, e) \in \mathcal{B}^*$ the number $v[(\varphi, e)] = v(\varphi, e)$ from the interval [0,1] as its truth value such that for all statements $(\varphi, e) \in \mathcal{B}^*$, $(\psi, d) \in \mathcal{B}^*$

1. $v(\neg\varphi, e) = 1 - v(\varphi, e)$
2. $v((\varphi, e) \wedge (\psi, d)) = v(\varphi, e) * v(\psi, d)$
3. $v((\varphi, e) \vee (\psi, d)) = v(\varphi, e) \perp v(\psi, d)$,

where $*$ is a triangular norm representing a conjunction operator, and $\perp$ is a dual conorm for disjunction operator. The duality of the norm $*$ and conorm $\perp$ is expressed by [3]

$$a \perp b = 1 - (1 - a) * (1 - b).$$

## 3. USING POSSIBILITY DISTRIBUTIONS TO DEFINE TRUTH VALUES OF FUZZY TEMPORAL STATEMENTS

From a mathematical point of view, $ft$-expressions can be regarded as fuzzy Boolean variables taking the value 1 or 0 in the terms of possibility theory, according to an adequate possibility distribution $\Pi$. Therefore, it is useful to use some tools of possibility theory for determining truth evaluations of $ft$-statements.

For example, the $ft$-expression "a little later" can be represented, with a respect to a referential time point by the following possibility distribution

$$\begin{aligned} &\Pi_{a-\text{little}-\text{later}}(t_0) = \\ =\ &(0|t_0, 0.6|t_0 + 0.5\Delta t, 1|t_0 + \Delta t, 0.8|t_0 + 2\Delta t, 0.5|t_0 + 3\Delta t, 0.2|t_0 + 4\Delta t, 0|t_0 + 5\Delta t), \end{aligned}$$

where the relativity of the expression has been included into the quantity $\Delta t$. For instance, the degree of possibility that the $ft$-expression "a little later" takes the value 1 is 0.8 at the time point $t_0 + 2\Delta t$.

More generally, if $e = e(\tau)$ is an $ft$-expression represented by the fuzzy set $E = E(\tau) \in E^T$ with the membership function $\mu_E$, and $\Pi_E$ is a possibility distribution its values express the corresponding degrees of possibility that the variable $e$ takes the truth value 1 at the time point $t$, then we define

$$\Pi_e(t) = \mu_E(t),\ \forall\, t \in T.$$

From the other hand, let $p$ be a fuzzy predicate defined by a fuzzy subset $P \subseteq U^n$, where $n \geq 1$, with a membership function $\mu_P$. For all $u \in U^n$, let the value $\mu_P(u)$ expresses the degree of possibility that the truth value of the predicate $p$ is 1 (or true) at the point $u$. Then the predicate $p$ can be represented by a possibility distribution $\Pi_P$, its values being identical to the corresponding values of the membership function $\mu_P$,

$$\Pi_P(u) = \mu_P(u),\ \forall\, u \in U^n.$$

Generally, we define

$$\Pi_\varphi(u) = \mu_\Phi(u), \ \forall\, u \in U^n$$

for an $ft$-statement $\varphi \in B^*$, where $\mu_\Phi$ is the membership function of the fuzzy set $\Phi$ corresponding to the statement $\varphi$.

Note that the set $\mathcal{B}^*$ has a structure of Cartesian product for which at beginning we have supposed the non-interactivity of its components. This property entails that the components $\varphi, e$ in any $ft$-statement $(\varphi, e) \in \mathcal{B}^*$ are mutually independent and separable. Thus, their joint possibility distribution $\Pi_{\varphi,e}$ can be written in the form [4]

$$\Pi_{\varphi,e}(u,t) = \min(\Pi_\varphi(u), \Pi_e(t)), \forall\,(u,t) \in B^* \times E^T$$

where $\Pi_\varphi$ and $\Pi_e$ are marginal possibility distributions, they can be gained from the joint distribution $\Pi_{\varphi,e}$ by the projection operation,

$$\Pi_\varphi(u) = \sup_t \Pi_{\varphi,e}(u,t)$$

$$\Pi_e(t) = \sup_u \Pi_{\varphi,e}(u,t).$$

If the membership function of the fuzzy set corresponding to the joint possibility distribution $\Pi_{\varphi,e}$ is denoted $\mu_{\varphi,e}$ then from the above terms follows

$$\mu_{\varphi,e}(u,t) = \min(\mu_\varphi(u), \mu_e(t)), \ \forall\, u \in U, \ \forall\, t \in T.$$

The value $\mu_{\varphi,e}(u,t)$ represents the degree of possibility that the two-dimensional Boolean variable corresponding to an $ft$-statement $(\varphi, e)$ takes the value 1. Thus, it is intuitively quite natural to approximate the truth evaluation of the $ft$-statement $(\varphi, e)$ by elements of the matrix $Z$ defined by

$$z_{i,j} = \min(\mu_\varphi(u_i), \mu_e(t_j)),$$

where $(u_1, \ldots, u_n), (t_1, \ldots, t_m)$ are values of the variables $u$, $t$, respectively, for which the values of the membership functions $\mu_\varphi, \mu_e$ are well-defined. Unfortunately, the space available for this paper is exhausted. We will continue the research of fuzzy temporal reasoning in next papers.

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