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Kybernetika, Vol. 30 (1994), No. 4, 425--432

Persistent URL: `http://dml.cz/dmlcz/124163`

KYBERNETIKA — VOLUME 30 (1994), NUMBER 4, PAGES 425–432

ON-LINE IDENTIFICATION AND CONTROL OF CHAOS IN A REAL CHUA'S CIRCUIT[1]

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This paper presents results of a study of identification and control of chaos in a real electronic circuit. In all experiments we used an op-amp implementation of Chua's circuit. An application-specific software package has been developed. This package consists of programs for handling automatic measurements in the electronic system, characterisation of the system on the basis of measured experimental time series (identification), generation of the control signals for stabilisation of chosen periodic state. The software is fully interactive – the user takes the decisions at each step of the procedure (e.g. length of the measured interval, choice of orbits to compute, goal of the control etc.) We have performed a number of experiments of controlling Chua's circuit. We were able to stabilize several low-period unstable orbits found earlier using the software package.

1. INTRODUCTION

Since the discovery of chaotic motion in Chua's circuit [12] and its confirmation in laboratory experiments [16] considerable efforts concentrated on studying properties of this extremely simple third-order circuit. This is due to two facts:

– First, this circuit offers unprecedented possibilities for experimental (both laboratory and numerical) work – it can be easily implemented using general purpose electronic elements, also existing equipment (oscilloscopes, spectrum analysers etc.) and general purpose circuit simulation programs (e.g. SPICE) could be used in the studies;

– Second, it has been shown [4, 5] that almost all known to date bifurcation phenomena and low dimensional chaotic attractors could be observed form this circuit for suitable parameter choices. Thus Chua's circuit can be considered as a universal tool for studying chaos and bifurcation phenomena — it has become a true paradigm for chaos.

In this paper we present some results concerning possibilities of controlling chaos in a real implementation of Chua's circuit. We developed a laboratory setup consisting of an op-amp implementation of Chua's circuit, computer-assisted measurement/data acquisition system, software package for identification and characteris-

[1] Presented at the IFAC Workshop on System Structure and Control held in Prague on September 3–5, 1992.

ation of the system using measured time-series, programs realising the control task and finally implementation of the control in the circuit via a digitally controlled resistor or capacitor. The priciples of the chaos control procedure follow the guidelines given by [7].

2. CHUA'S CIRCUIT

Chua's circuit is probably the simplest known to date electronic circuit (third-order, autonomous, reciprocal) exhibiting a variety of bifurcation phenomena and chaotic attractors. It consists of two linear capacitors, one linear inductor, one linear resistor and a single nonlinear (piecewise-linear) resistor — interconnected as shown in Fig. 1.

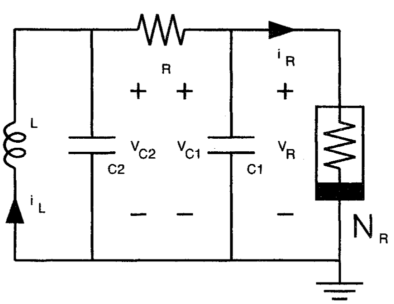

Fig. 1. Chua's circuit diagram.

Various practical implementations of the nonlinear resistor have been described in the literature - realisation of the circuit and laboratory experiments are relatively easy to make. In our study we used the circuitry proposed by Kennedy [9]. When varying one of the circuit parameters (R, C or L) circuit behavior changes qualitatively. For a wide range of parameter values Chua's circuit displays chaotic behavior e. g. the double scroll attractor could be observed [4]. Various kinds of behavior were described by Chua and Lin [5].

The dynamics of Chua's circuit can be described in a dimensionless form by a third order state equation:

$$\frac{dx}{dt} = \alpha\{y - (b+1)x - \frac{1}{2}(a-b)(|x+1| - |x-1|)\}, \quad (1)$$

$$\frac{dy}{dt} = x - y + z, \quad (2)$$

$$\frac{dz}{dt} = -\beta y, \quad (3)$$

where: $\alpha = \frac{C_2}{C_1}$, $\beta = \frac{C_2}{LG^2}$.
In our experiments we have modified the above circuit by adding a linear, voltage-

controlled resistor in parallel with the nonlinear one. Varying this resistance we have controlled the slopes of the nonlinear resistor. In fact one can choose any arbitrary circuit parameter as a control one. We have found the control using grounded resistor the easiest to implement.

In the procedure we will assume that the equations of the system can be written in the simplified form:

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{f}(\mathbf{x}, p), \tag{4}$$

where p is the chosen control parameter.

3. CHAOS CONTROL STRATEGY

We adopt the control strategy proposed by Ott, Grebogi and Yorke [13, 14] and Romeiras et al. [15]. The procedure consists of three steps.

3.1. Procedure for determination of periodic orbits

We assume that a sufficiently long section of system trajectory $\{x_i\}$, $i = 1, ..., N$ has been measured by our data acquisition procedure and stored in a file. Using the measured data we would like to find some of the unstable periodic orbits inherently embedded in the attractor. For this purpose we choose some $\varepsilon > 0$ and some point x_n from the time series. We follow the succesive points x_{n+1}, x_{n+2} etc. until we find the smallest k such that $||x_{n+k} - x_n|| < \varepsilon$. We repeat the calculations for all points from the time series. We claim that the orbits found using this procedure are very near the unstable (saddle) periodic orbits embedded within the attractor (comp. also [11, 3, 6]).

3.2. Procedure for determining the eigenvalues of chosen periodic orbit

This procedure follows the guidelines given by Lathrop and Kostelich [11], and also described by Ott et al. [13, 14]. First let us choose a Poincaré section for the trajectories of our three dimensional continuous system. We must choose a plane which intersects the periodic orbit taken into consideration. For simplicity we describe the following methods for a period-one orbit, which corresponds to a fixed point in the Poincaré map. We assume that the dynamics in the small neighbourhood of the fixed point is nearly linear. We use the linear approximation of the Poincaré map near the fixed point x_F and the nominal value of control parameter p_0.

$$x_{i+1} - x_F = A(x_i - x_F) + B(p_i - p_0) \tag{5}$$

A and B can be found using e. g. least squares method. Matrix A gives an approximation of the Jacobian matrix of the periodic orbit. For the evaluation of B repetition of the whole procedure for slightly changed p is needed. The eigenvalues of the Jacobian matrix can be further calculated using standard methods. Here, one of the eigenvalus is always unstable.

3.3. Controlling the periodic orbit

Let e_u, e_s be the unstable and stable eigenvectors of matrix A, and λ_u, λ_s the corresponding eigenvalues. Let f_u, f_s be the contravariant basis vactors defined by: $f_s e_u = f_u e_s = 0$ and $f_s e_s = f_u e_u = 1$. In order to control the system we observe the trajectory until it comes close to the chosen periodic orbit and then modify the control parameter in such a way as to push the trajectory onto the local stable manifold of the periodic point. From that we can derive the condition for p_{i+1}:

$$p_{i+1} = p_i + \frac{\lambda_u}{f_u B} f_u(x_i - x_F). \tag{6}$$

The control signal p_i is applied in the circuit only at the very moments when the actual chaotic trajectory passes near the chosen periodic point. Since by assumption the system is ergodic the trajectory will inevitably pass arbitrarily close to the chosen point. At such moments we can say that the trajectory is pushed onto the local stable manifold of the periodic point and should remain on it, at least for some time.

4. LABORATORY SETUP

The block diagram of the laboratory set-up used in our experiments is shown in Fig. 2. The central block is the 486 compatible computer equipped with the Advantech PCL818 high performance data aquisition card. The lab-card is capable of sampling the output from Chua's circuit at a rate of 100kHz which is high enough to obtain reliable results. The resolution is 12 bits. The system has the possibility of using analog or digital output which is very convenient for the control application — we have implemented the controlling parameter by means of an voltage-controlled resistor built on purpose.

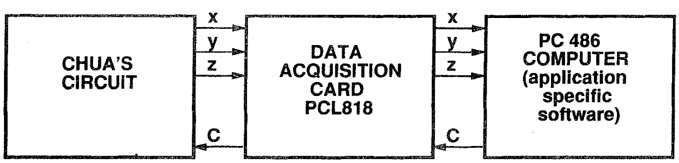

Fig. 2. Block diagram of the laboratory setup using PC486 and Advantech PCL818 A/D - D/A card.

Very important part of the system, apart from the hardware, is the specific software developed for the purpose of controlling chaotic circuits.

5. THE SOFTWARE PACKAGE

The software package consists of several blocks designated to perform separate tasks: 1) Data acquisition; 2) Identification of periodic orbits and linear approximation of system dynamics near chosen periodic orbit; 3) Control — calculation using standard methods.

As a first step of the identification/control procedure signals form the real Chua's circuit are transformed into a time-series (using the A/D conversion card) and stored in a file. This is achieved using standard software supplied with the card and its Turbo Pascal interface. The length of the time interval, the sampling rate can be specified by the user. Once the data are acquired the measurements are suspended (but the process is running without interruption) and the second part of the procedure starts.

The second part of the procedure is the identification in terms of unstable periodic orbits. Using the procedure proposed by Lathrop and Kostelich [11] and Kostelich and Yorke [10] the unstable periodic orbits embedded in the chaotic attractor are calculated. The user can specify further characteristics of the signal to be computed. These include reconstruction of attractor from a single time series, attractor dimension, Lyapunov exponents etc. Typical chaotic attractor (double scroll) observed in Chua's circuit and several periodic orbits found using our program are shown in Fig. 3. The first three orbits shown in Fig. 3 are nonsymmetrical. They correspond to a fixed point and a period 2 and 3 orbits in the Poincaré section. The last shown periodic orbit is symmetric with respect the origin.

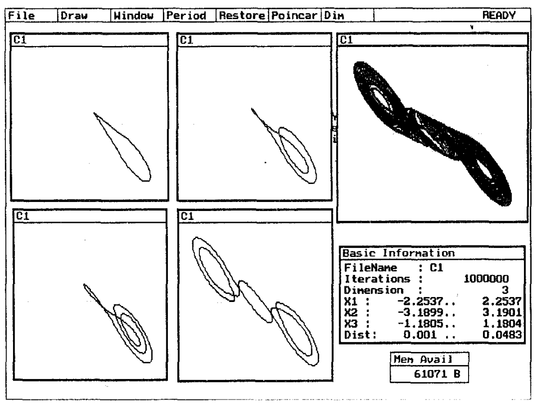

Fig. 3. Typical chaotic trajectory from Chua's circuit and four of the periodic orbits found within. Picture obtained using the developed software package.

When the unstable periodic orbits are found the user chooses the one which will be controlled/stabilised in the next step. Once the goal of the control is fixed the eigenvalues of the chosen orbit are being computed and the control signal is found following the simplest rules for pole placement in the linear system (e. g. Ackermann's method). The computed control signal can be only applied at specific moments when the trajectory of the system passes in the vicinity of the specified periodic orbit (thus we can claim that the linear approximation holds). To achieve this on-line measurements are resumed and the system is waiting for the proper moment to apply the control. This is done by means of a voltage controlled resistor (controlled by the analog output from the data acquisition card).

6. EXPERIMENTAL RESULTS

We have investigated the Chua's circuit for the following parameter set: L=18mH, R=1.6kΩ, C_1=10nF, C_2=100nF, the slopes of the nonlinear resistor are μ_1=-0.76 in the inner region and μ_2=-0.4 in the outer regions. For this set of parameters the circuit exhibits chaotic behavior. The strange attractor is called the double scroll.

In the first step we have recorded the system trajectory using PCL818 data acquisition card. We have sampled three variables of the system, namely v_{C1}, v_{C2}, i_L at a sample rate of 90kHz. After acquiring 200000 samples we have stored them in a file. Then using our software package we have produced the Poincaré section of the trajectory. Performing calculation on the Poincaré section is computationally efficient as it reduces significantly the time of the calculations. Then we have determined periodic points embedded within the Poincaré section of the attractor. In our experiments we have been searching for periodic orbits with a period smaller than five. We have chosen two unstable periodic orbits (period one and two) for stabilisation. Both of them are shown in Fig. 3. We have determined their eigenvalues. We have repeated these calculations for a slightly changed value of control parameter p, and finally we have obtained the parameters needed for control. During all the calculations the process was running without interruption.
Then we could start the control. We have monitored the trajectory of Chua's circuit, until it comes close to the desired periodic orbit. Every time when the trajectory intersected the Poincaré plane we calculated the new control parameter and applied it to the circuit. The results of the control are shown in Fig. 4. One can notice that after the transient the trajectory falls into the unstable periodic orbit and remains on it.

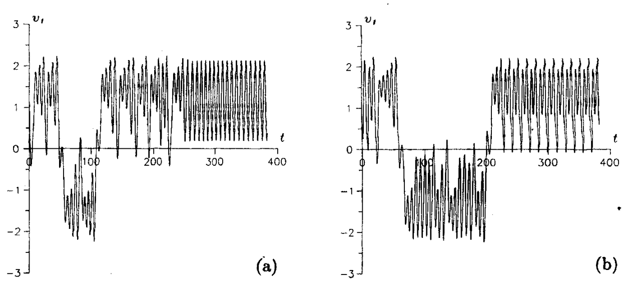

Fig. 4. Time responses of the controlled circuit. Chaotic transient before achieving control is clearly visible. (a) stabilising period-one orbit. (b) stabilising period-two orbit.

7. CONCLUSIONS

The results obtained so far are promissing – the identification part works correctly. We were able to calculate several periodic orbits from the experimental data and make detailed characterisation of the measured signals. Many different chaotic signals form Chua's circuit have been measured including the ones corresponding to Rössler's attractor, the double scroll, the double hook etc. In all cases fundamental periodic orbits could be calculated in very short time — in most cases 1000 cycles sample is sufficient for carrying out the calculations.

The laboratory setup dedicated to identification and control of chaotic regimes at this moment is undergoing thorough test of the control part. The procedure seems to be simple and straightforward however when implemented specific problems appear. Among these are the speed/accuracy tradeoff and application of the control signal in real time at the proper moment. In many tested cases we obtained stabilisation of a periodic orbit which was not the chosen one – this is due to computational and timing errors, i.e. not precise enough value of the generated control signal or application of this signal with a delay after the very moment when the chaotic trajectory passed in the desired region near chosen periodic orbit.

We expect that this laboratory setup can be used in other experiments with electronic circuits for characterising and identifying chaos in terms of unstable periodic orbits. The control part probably would require some further modifications.

ACKNOWLEDGEMENT

This research has been supported by the research grant from the University of Mining and Metallurgy. This paper has been prepared in part during M. Ogorzalek's stay at the Nonlinear Circuit Laboratory, University of California, Berkeley. We also acknowledge the computational support from Sun Microsystems.

(Received March 1, 1993.)

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