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ABSTRACT. Lower and upper bounds are presented on the number of parity-check digits required for a linear code that locates a single sub-block containing errors which are in the form of moderate-density CT burst.

1. Introduction

The search for practical coding techniques on error control in digital data transmission has concentrated in two areas: error detection and error correction. Wolf and Elspas [6] introduced a third coding technique called error-locating codes. This coding technique offers a soft compromise between error detecting codes and error correcting codes. In this technique, the block of message is divided into mutually exclusive sub-blocks and while decoding, it is possible to detect the error and simultaneously it is possible to identify the sub-block in which the error lies.

The use of error-locating codes offers a compromise between short and long block lengths by providing an additional design parameter. The amount of redundancy required for such codes is not excessive and error location provides an attractive alternative to conventional error detection in decision feed back communications. Dass [3] studied codes in which errors occur in the form of bursts and also in the form of low-density bursts [4]. The study of such codes have been made with respect to the open-loop bursts [5], c.f. Peterson and Weldon Jr. 1972), p. 109.

This paper presents a study of moderate-density burst error-locating codes keeping in view the weight constraint over the CT (Chien and Tang [1]) burst of length b (fixed), for a positive integer b. Lower and upper bounds on the necessary and sufficient number of parity-check digits required for the existence of such codes are given.

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In what follows, we shall consider a linear code to be a subspace of n-tuples over $GF(q)$ and by a burst of length b (fixed), we shall mean an n-tuple whose only nonzero components are confined to b consecutive positions, the first of which is nonzero and the number of its starting positions is the first $(n-b+1)$ components (refer Dass [2]). The weight of a vector shall be considered in the Hamming's sense, and by a moderate-density CT burst, we shall mean a burst of length b (fixed) with weight lying between $w1$ and $w2$ $(0 \leq w1 \leq w2 \leq b)$. The block of n digits, consisting of r check digits, and $k = n - r$ information digits is considered to be sub-divided into s mutually exclusive sub-blocks. Each sub-block contains $t = \frac{n}{s}$ digits.

2. BOUNDS ON THE NUMBER OF CHECK DIGITS

An error-locating code capable of detecting and locating a single sub-block containing errors that are bursts of length b (fixed) with weight lying between $w1$ and $w2$ $(0 \leq w1 \leq w2 \leq b)$ must satisfy the following conditions :

(a) The syndrome resulting from the occurrence of an error which is a burst of length b (fixed) with weight lying between $w1$ and $w2$ $(0 \leq w1 \leq w2 \leq b)$ within any one single sub-block must be distinct from the all-zeros syndrome.

(b) The syndrome resulting from the occurrence of a burst error of length b (fixed) with weight lying between $w1$ and $w2$ $(0 \leq w1 \leq w2 \leq b)$ within a single sub-block must be distinct from the syndrome resulting from any other burst of length b (fixed) with weight lying between $w1$ and $w2$ $(0 \leq w1 \leq w2 \leq b)$ within some other sub-block.

In the following we shall derive two results. The first results gives a lower bound whereas the second result gives an upper bound on the number of check digits required for the existence of a linear code over $GF(q)$ capable of detecting and locating a single sub-block containing errors that are bursts of length b (fixed) with weight lying between $w1$ and $w2$ $(0 \leq w1 \leq w2 \leq b)$.

Theorem 1. The number of check digits r required for an (n, k) linear code, sub-divided into s sub-sub-blocks of length t each, that locates a single corrupted sub-block containing errors that are bursts of length b (fixed) with weight lying between $w1$ and $w2$ $(0 \leq w1 \leq w2 \leq b)$ is bounded from below by

$$r \geq \log_q \left(1 + s(q^{[\frac{d}{2}]} - 1)\right), \quad w1 \leq d \leq w2 \leq b, \tag{1}$$

where $[\frac{d}{2}]$ denotes the largest integer less than or equal to $\frac{d}{2}$.

Proof. Out of the first b components of every sub-block of length t each, we fix any d nonzero positions where d lies between $w1$ and $w2$ $(0 \leq w1 \leq d \leq w2 \leq b)$ in such a manner that the first $[\frac{b}{2}]$ components contain $[\frac{d}{2}]$ nonzero positions and the remaining $[\frac{b}{2}]$ components, that is, $([\frac{b}{2}]+1)$-th to b-th, contains $[\frac{d}{2}]$ nonzero positions. Since the code detects all bursts of length b (fixed) with weight d $(w1 \leq d \leq w2)$ lying in any sub-block, syndromes produced by vectors of weight $[\frac{d}{2}]$ out of the first $[\frac{b}{2}]$ components must be distinct from syndromes likewise resulting from vectors of weight $[\frac{d}{2}]$ out of the remaining $[\frac{b}{2}]$ components that is, $([\frac{b}{2}]+1)$-th to b-th components in the same block, or else their difference will be a burst of length

b(fixed) with weight d ($w1 \leq d \leq w2$) in that block resulting in the zero syndrome [in violation of condition (a)].

Again, since the code locates a single sub-block containing errors that are bursts of length b (fixed) with weight d ($w1 \leq d \leq w2$), syndromes produced by burst errors of length b (fixed) with weight d ($w1 \leq d \leq w2$) in different sub-blocks must be distinct by condition (b). In view of this condition, the following two cases are required to be considered.

Case I. When $[\frac{d}{2}] \geq w1$.

In this case, condition (b) is applicable, in particular, for the syndromes of bursts of length $[\frac{b}{2}]$ with weight $[\frac{d}{2}]$ $(\geq w1)$. More precisely, the syndromes resulting from the occurrence of burst errors of length $[\frac{b}{2}]$ with weight $[\frac{d}{2}]$ of any sub-block whether in the same or in different sub-blocks, must be distinct. Since the number of possible errors in one blocks is $q^{[\frac{d}{2}]} - 1$, and there are s sub-blocks in all, therefore, total number of bursts errors in s sub-blocks is $s(q^{[\frac{d}{2}]} - 1)$.

Case II. When $[\frac{w_1}{2}] \leq [\frac{d}{2}] < w1$.

In this case also, the same arguments holds, and so will be the number of such bursts viz. $s(q^{[\frac{d}{2}]} - 1)$. It may be noted that when $[\frac{d}{2}] < [\frac{w_1}{2}]$, the condition (b) will not be applicable. Therefore, total number of distinct syndromes produced by burst errors of length $[\frac{b}{2}]$ with weight $[\frac{d}{2}]$ for all s sub-blocks including the all-zero syndrome is

$$\left(1 + s(q^{[\frac{d}{2}]} - 1)\right).$$

The maximum number of distinct syndromes available using r check digits is q^r. Hence we obtain

$$q^r \geq \left(1 + s(q^{[\frac{d}{2}]} - 1)\right) \tag{2}$$

from which the result follows by taking logarithms.

Remark

The result obtained in the above Theorem is independent of b and t. So the result in expr. (1) will hold so long as d is lying between $w1$ and $w2$ ($w1 \leq w2 \leq b \leq t$), and $n = st$.

In Theorem 2, we derive an upper bound on r, the number of check digits required. The proof involves relative modifications of the procedure used to establish the Varshamov-Gilbert-Sacks bound [5], c.f. Theorem 4.7.

Theorem 2. A code capable of detecting burst errors of length b (fixed) with weight lying between $w1$ and $w2$ ($0 \leq w1 \leq w2 \leq b$) occurring within a single sub-block, and of locating that sub-block, can always be constructed using r check digits where r is the smallest integer satisfying the inequality :

$$r > \log_q\left(1 + [\sum_{i=w_1-1}^{w_2-1} \binom{b-1}{i}(q-1)^i] \cdot \\ \cdot [1 + (s-1)(q-1)(t-b+1)\sum_{i=w_1-1}^{w_2-1} \binom{b-1}{i}(q-1)^i]\right) \tag{3}$$

Proof. We prove the existence of such a code by constructing a suitable parity check matrix H for the desired code. For the construction of the desired matrix H, we first construct a matrix H^1 and then H will be obtained by reversing altogether the columns of H^1. The procedure for the construction of the matrix H^1 is given as follows.

Suppose we select $(s-1)t$ columns corresponding to the first $(s-1)$ sub-blocks of H^1 and the first $(j-1)$ columns $h1, h2, \ldots, hj-1$ of the s-th sub-block.

For detection, the syndrome resulting from the occurrence of errors lying between $w1$ and $w2$ $(0 \leq w1 \leq w2 \leq b)$ out of b consecutive columns within any sub-block must be different from the all-zero r-tuple. So the j-th column h_j can be added to the s-th sub-block, provided that

(i) h_j is not a linear combination of any columns lying between $w1-1$ and $w2-1$ $(0 \leq w1 \leq w2 \leq b)$ columns out of the immediately preceding $(b-1)$ columns of the s-th sub-block. That is

$$h_j \neq \alpha_1 h_{j-b+1} + \alpha_2 h_{j-b+2} + \cdots + \alpha_{b-1} h_{j-1} \tag{4}$$

where $\alpha_i \in GF(q)$, and the number of nonzero $\alpha_i's$ lies between $w1-1$ and $w2-1$.

Therefore, the number of possible linear combinations which cannot be equal to h_j is

$$\sum_{i=w_1-1}^{w_2-1} \binom{b-1}{i} (q-1)^i. \tag{5}$$

For locating the corrupted sub-block, the syndrome of any burst of length b (fixed) with weight lying between $w1$ and $w2$ within a sub-block must be different from the syndrome resulting from any other burst of length b (fixed) with weight lying between $w1$ and $w2$. Therefore, the j-th column h_j to be added to the s-th sub-block should be such that

(ii) h_j is not a linear combination of columns lying between $w1-1$ and $w2-1$ columns out of the immediately preceding $(b-1)$ columns of the s-th sub-block, together with a linear combination of any columns lying between $w1$ and $w2$ out of any b consecutive columns from among any of the first $(s-1)$ sub-blocks. That is,

$$h_j \neq (\alpha_1 h_{j-b+1} + \alpha_2 h_{j-b+2} + \cdots + \alpha_{b-1} h_{j-1}) + (\beta_1 h_i + \beta_2 h_{i+1} + \cdots + \beta_b h_{i+b-1}) \tag{6}$$

where α_i, $\beta_i \in GF(q)$ are such that number of nonzero α_i's lies between $w1-1$ and $w2-1$, and number of nonzero β_i's lies between $w1$ and $w2$, and h_i's are b consecutive columns from amongst any one of the first $(s-1)$ sub-blocks.

Therefore, number of possible linear combinations which cannot be equal to h_j is

$$\left[\sum_{i=w_1-1}^{w_2-1} \binom{b-1}{i} (q-1)^i\right]\left[(s-1)(q-1)(j-b+1) \sum_{i=w_1-1}^{w_2-1} \binom{b-1}{i} (q-1)^i\right]. \tag{7}$$

At worst, all these linear combinations might yield a distinct sum. Therefore, h_j can always be added to H^1 provided that all the r-tuples are not exhausted by linear combinations computed in (5) and (7).

Therefore, the j-th column can be added to H^1 provided that

$$q^r > 1 + \text{Expr. (5)} + \text{Expr. (7)},$$

including the all zero pattern.

To obtain a code of length n, substituting t for j, the t-th column h_t can be added to the s-th sub-block provided that

$$q^r > 1 + [\sum_{i=w_1-1}^{w_2-1} \binom{b-1}{i}(q-1)^i][1+(s-1)(q-1)(t-b+1)\sum_{i=w_1-1}^{w_2-1} \binom{b-1}{i}(q-1)^i]$$

Thus, the matrix $H^1 = [h_i]$ (h_i denoting the i-th column of H_1) have been constructed from which we obtain the required parity check matrix $H = [H_i], [H_i$ denoting the i-th column of $H]$ by reversing its column altogether, that is

$$h_i \rightarrow H_{n-i+1}.$$

This leads to the sufficient condition stated in (3).

EXAMPLE

For an (10, 4) linear code over $GF(2)$, we consider the following 610 parity-check matrix which has been constructed by the synthesis procedure outlines in the proof of Theorem 2, by taking $s = 2,\ t = 5,\ b = 4,\ w_1 = 2$ and $w_2 = 3$.

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The null space of this matrix can be used as a code to locate a sub-block of length $t = 5$ containing burst errors of length $b = 4$ with weight 2 and 3. From the Error Pattern-Syndrome Table, we can observe that

(i) The syndromes of bursts of length 4 with weight 2 and 3 within any sub-blocks are nonzero.

(ii) The syndrome of a burst of length 4 with weight 2 and 3 within the first sub-block is different from the syndrome of a burst of length 4 with weight 2 and 3 within the other sub-block.

REMARK

We also can observe that the syndromes of some of the error patterns in the second sub-block coincides, viz. the error patterns (00000 10100) and (00000 01101) have the same syndrome 001101, and also the error patterns (00000 10010) and (00000 01011) have the same syndrome 110011. For the coding efficiency it is desired that the syndromes of some of the error patterns within any sub-block be identical whenever possible.

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Error Pattern	Syndrome
11000 00000	110000
01100 00000	011000
00110 00000	001100
00011 00000	000110
10100 00000	101000
01010 00000	010100
00101 00000	001010
10010 00000	100100
01001 00000	010010
11100 00000	111000
01110 00000	011100
00111 00000	001110
11010 00000	110100
01101 00000	011010
10110 00000	101100
01011 00000	010110
00000 11000	110001
00000 01100	111100
00000 10100	001101
00000 01010	000010
00000 10010	110011
00000 01001	000001
00000 11100	111101
00000 01110	001110
00000 11010	000011
00000 01101	001101
00000 10110	111111
00000 01011	110011

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