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Abstract: Basic results concerning the behaviour of binary relations are well-known today. However, there is a question what properties to study for arities greater than two (see [3]). In this paper we establish relations as sets of mappings and investigate questions motivated by relational database theory. First we study homomorphisms and the process of update of a relation. Further we deal with the interaction of homomorphism and functional dependencies. The last part is devoted to the investigation of relationships between homomorphism and operations.

Key Words: relation, operation, homomorphism, update

Mathematics Subject Classification: 04A05, 08A02

1. Introduction

Let A, D be nonempty sets, A be finite, and $D_i; i \in A$ nonempty subsets of D such that $D = \cup_{i \in A} D_i$. By a *relation* we understand a subset R of the set D^A (where D^A is a set of mappings of A into D) such that for each $f \in R$ and $i \in A$ there holds $f(i) \in D_i$. Set A is called *relational scheme* of R and we will denote it by $rs(R)$; elements of the set A are *attributes*. We say that D, resp. D_i ($i \in A$), is *domain of* R, resp. *domain of* R *w.r.t.* i (the notation is $D = dom(R)$, and $D_i = dom_i(R)$).

Let us recall basic operations analogical to those in relational data model.

Let R be a relation and P a nonempty subset of $rs(R)$. *Projection of* R *onto* P is a relation $\pi_P(R) \subseteq (\cup_{i \in P} dom_i(R))^P$ such that $f \in \pi_P(R)$ whenever there exists $g \in R$ satisfying the equality $f = g|_P$. Let $dom_i(\pi_P(R)) = dom_i(R)$.

Suppose $i_0, j_0 \in rs(R)$ be arbitrary attributes and $\theta \subseteq dom_{i_0}(R) \times dom_{j_0}(R)$. By a θ*-selection of* R *with the restriction* $i_0 \theta j_0$ we understand a subrelation $\sigma_{i_0 \theta j_0}(R)$ of R such that $dom_i(\sigma_{i_0 \theta j_0}(R)) = dom_i(R)$ for all $i \in rs(R)$, $f \in \sigma_{i_0 \theta j_0}(R)$ whenever $f \in R$ and $(f(i_0), f(j_0)) \in \theta$ (further we will write $f(i_0)\theta f(j_0)$).

If c is in $dom_{i_0}(R)$ and θ' is a binary relation on $dom_{i_0}(R)$, θ'*-selection of* R *with the restriction* $i_0\theta' c$ is a subrelation $\sigma_{i_0\theta' c}(R)$ of R such that $dom_i(\sigma_{i_0\theta' c}(R)) = dom_i(R)$ for all $i \in rs(R)$ and $f \in \sigma_{i_0\theta' c}(R)$ whenever $f \in R$ and $f(i_0)\theta' c$.

Product of relations R and S is a relation $R{\cdot}S$ such that $rs(R{\cdot}S) = rs(R) \overset{dis}{\cup} rs(S)$ (where $\overset{dis}{\cup}$ is disjoint union);

$$dom_i(R \cdot S) = \begin{cases} dom_i(R) & \text{for } i \in rs(R), \\ dom_i(S) & \text{for } i \in rs(S); \end{cases}$$

$f \in R \cdot S$ whenever there exist $g \in R$ and $h \in S$ fulfilling

$$f(i) = \begin{cases} g(i) & \text{for } i \in rs(R), \\ h(i) & \text{for } i \in rs(S). \end{cases}$$

If R, S are relations, $i_0 \in rs(R)$, $j_0 \in rs(S)$ and $\theta \subseteq dom_{i_0}(R) \times dom_{j_0}(S)$, then by a *θ-join* of R and S with the restriction $i_0\theta j_0$ we understand a relation $R(i_0\theta j_0)S \subseteq (dom(R) \cup dom(S))^{rs(R) \overset{dis}{\cup} rs(S)}$ such that

$$dom_i(R(i_0\theta j_0)S) = \begin{cases} dom_i(R) & \text{for } i \in rs(R), \\ dom_i(S) & \text{for } i \in rs(S); \end{cases}$$

$f \in R(i_0\theta j_0)S$ if there exist elements $g \in R$, $h \in S$ fulfilling $f(i) = g(i)$ for all $i \in rs(R)$, $f(j) = h(j)$ for all $j \in rs(S)$ and $g(i_0)\theta h(j_0)$.

If R, S are relations and $\alpha : rs(R) \to rs(S)$ is a bijection, *effective union* of R,S w.r.t. α is a relation $R\overset{\alpha}{\cup}S \subseteq (dom(R) \cup dom(S))^{rs(R)}$ such that $dom_i(R\overset{\alpha}{\cup}S) = dom_i(R) \cup dom_{\alpha(i)}(S)$ for all $i \in rs(R)$ and $f \in R\overset{\alpha}{\cup}S$ whenever there exist $g \in R$ with property $f(i) = g(i)$ for all $i \in rs(R)$ or $h \in S$ with property $h(\alpha(i)) = f(i)$ for all $i \in rs(R)$.

Effective intersection of R,S w.r.t. α is a relation $R\overset{\alpha}{\cap}S \subseteq (dom(R) \cap dom(S))^{rs(R)}$ fulfilling $dom_i(R\overset{\alpha}{\cap}S) = dom_i(R) \cap dom_{\alpha(i)}(S)$ for $i \in rs(R)$ and $f \in R\overset{\alpha}{\cap}S$ whenever $f \in R$ as well as $f \circ \alpha^{-1} \in S$.

Effective difference of R,S w.r.t. α is a relation $R\overset{\alpha}{-}S \subseteq R$ with properties $dom_i(R\overset{\alpha}{-}S) = dom_i(R)$ for all $i \in rs(R)$ and $f \in R\overset{\alpha}{-}S$ whenever $f \in R$ as well as $f \circ \alpha^{-1} \notin S$.

The word "effective" in these operations requires the same cardinality of $rs(R)$, $rs(S)$.

If R, S are relations, $\alpha : rs(S) \to rs(R)$ an injective mapping, $dom_j(S) = dom_{\alpha(j)}(R)$ for $j \in rs(S)$, $i_0 \in rs(R) - \alpha(rs(S))$, then *a quotient* of R,S w.r.t. α and i_0 is a relation $R(\div, \alpha, i_0)S \subseteq (dom(R))^{rs(R)-\alpha(rs(S))}$ such that $dom_i(R(\div, \alpha, i_0)S) = dom_i(R)$ for all $i \in rs(R) - \alpha(rs(S))$, $f \in R(\div, \alpha, i_0)S$ whenever there exist $g \in R$, $h \in S$ with the following properties:

(i) $g|_{rs(R)-\alpha(rs(S))} = f$;

(ii) $h(j) = g(\alpha(j))$ for all $j \in rs(S)$;

(iii) For every $h' \in S$ there exists $g' \in R$ with property $g'(i_0) = g(i_0)$ and for all $j \in rs(S)$ there holds $h'(j) = g'(\alpha(j))$.

2. Homomorphism and update

Let R, S be relations and $\alpha : rs(R) \to rs(S)$ a bijection. By a *homomorphism of R into S w.r.t.* α we understand arbitrary mapping $\varphi : dom(R) \to dom(S)$ fulfilling:

1. For every $i \in A$ there holds $\varphi(dom_i(R)) \subseteq dom_{\alpha(i)}(S)$;
2. for every $f \in R$ there holds $\varphi \circ f \circ \alpha^{-1} \in S$.

Let R, S be relations and $\varphi : dom(R) \to dom(S)$ a homomorphism of R into S w.r.t. bijection $\alpha : rs(R) \to rs(S)$. By a symbol $\hat{\varphi}$ we will denote a mapping $\hat{\varphi} : R \to S$ given by $\hat{\varphi}(f) = \varphi \circ f \circ \alpha^{-1}$ for all $f \in R$.

Example 1. Let us consider relations R, S, T with the same relational scheme $A =$ $= \{\text{name, salary}\}$ and with the same domain D, $D = D_{\text{name}} \cup D_{\text{salary}}$ where D_{salary} is a set of natural numbers and D_{name} is a set of character strings (see tables 1, 2, 3). Relations S and T are examples of update of R.

Let us further define bijective mappings $p_1 : R \to S$, $p_2 : R \to T$ $p_1(\text{Valenta}, 10000) = (\text{Valenta}, 12000)$, $p_1(f) = f$ otherwise; $p_2(\text{Valenta}, 10000) =$ $= (\text{Valenta}, 12000)$, $p_2(\text{Bruce}, 11000) = (\text{Bruce}, 12000)$, $p_2(\text{Nehoda}, 11000) =$ $= (\text{Nehoda}, 13000)$, $p_2(\text{Brabenec}, 13500) = (\text{Brabenec}, 14000)$.

Table 1: Relation R from ex.1.

	name	salary
f_1	Valenta	10000
f_2	Bruce	11000
f_3	Nehoda	11000
f_4	Brabenec	13500

Table 2: Relation S from ex.1.

	name	salary
g_1	Valenta	12000
g_2	Bruce	11000
g_3	Nehoda	11000
g_4	Brabenec	13500

Table 3: Relation T from ex.1.

	name	salary
h_1	Valenta	12000
h_2	Bruce	12000
h_3	Nehoda	13000
h_4	Brabenec	14000

Now let us consider homomorphism $\varphi : dom(R) \to dom(S)$ of R into S w.r.t. identical mapping on rs(R) given by $\varphi(x) = 12000$ for $x = 10000$, $\varphi(x) = x$ otherwise. Then the mappings $\hat{\varphi}$ and p_1 are the same but it is not possible to define homomorphism ψ of R into T w.r.t. $id_{rs(R)}$ fulfilling $\hat{\psi} = p_2$ (indeed, element $11000 \in D_{\text{salary}}$ is to be mapped onto 12000 as well as onto 13000, which is not feasible). We may conclude that not every update can be represented by the means of homomorphism (by update we understand such a change of relation R that there exists a bijection between R and the new relation – the changes of R do not lead to a reduction of elements of R). We therefore introduce the change relation. ♠

Let R, T be relations with the same relational schemes and the same domains, and $p : R \to T$ a bijective mapping. *A change relation* of R into T w.r.t. p is a binary relation $N_p(R,T)$ on $dom(R)$ given by

$$(a,b) \in N_p(R,T) \iff \exists i \in rs(R), f \in R : a = f(i), b = p(f)(i).$$

In this paper we shall use homomorphisms, change relations, surjections and bijections to study updates.

2.1. Let $R, S \subseteq D^A$ be relations, $p : R \to S$ surjective mapping, $i_0, j_0 \in A$, $\theta \subseteq dom_{i_0}(R) \times dom_{j_0}(R)$, $\theta' \subseteq dom_{i_0}(S) \times dom_{j_0}(S)$. If for each $f \in R$

$$f(i_0)\theta f(j_0) \qquad \iff \qquad p(f)(i_0)\theta' p(f)(j_0), \tag{1}$$

then

$$p(\sigma_{i_0\theta j_0}(R)) = \sigma_{i_0\theta' j_0}(p(R)).$$

2.2. Let $R, R_1 \subseteq D^A$, $S, S_1 \subseteq E^B$ be relations and $p : R \to R_1$, $q : S \to S_1$ bijections. If z is a bijection of $R \cdot S$ onto $R_1 \cdot S_1$ defined for $f \in R \cdot S$ by

$$z(f) = f', \text{ where } f'|_A = p(f|_A), f'|_B = q(f|_B),$$

then

$$p(R) \cdot q(S) = z(R \cdot S).$$

3. Homomorphism and functional dependencies

Let R be a relation and $X, Y \subseteq rs(R)$. We say that R fulfils *functional dependency* $X \to Y$, if for arbitrary elements f, g of R there holds

$$f(i) = g(i) \text{ for all } i \in X \Longrightarrow f(i) = g(i) \text{ for all } i \in Y.$$

The following example is an illustration of the fact that an update can cancel the validity of a functional dependency.

Example 2. Let R, S be relations, $dom(R) = \{a, b, c, d\}$, $rs(R) = \{x_1, x_2, y\}$, $dom(S) = \{a', b', d'\}$, $rs(S) = \{x_1', x_2', y'\}$. Let $\alpha(x_1) = x_1'$, $\alpha(x_2) = x_2'$, $\alpha(y) = y'$ and φ be a homomorphism of R into S w.r.t. α : $\varphi(a) = a'$, $\varphi(b) = \varphi(c) = b'$, $\varphi(d) = d'$. Let R be represented by table 4.

Table 4: Relation R from ex.2.

	x_1	x_2	y
f_1	a	b	c
f_2	a	c	d

Table 5: Relation $\hat{\varphi}(R)$ from ex.2.

	x_1'	x_2'	y'
g_1	a'	b'	b'
g_2	a'	b'	d'

R clearly fulfils $\{x_1, x_2\} \to \{y\}$ but $\hat{\varphi}(R) \subseteq S$ does not fulfil $\alpha(\{x_1, x_2\}) \to$ $\to \alpha(\{y\})$ (see table 5). (Relation S is not defined here but it is sufficient to know the image of R w.r.t. $\hat{\varphi}$) ♠

The next assertion gives a sufficient condition for a homomorphism to preserve a functional dependency.

3.1. Let R, S be relations, φ a homomorphism of R into S w.r.t. bijection $\alpha : rs(R) \to rs(S)$ and R fulfil $X \to Y$. Then $\hat{\varphi}(R)$ fulfils $\alpha(X) \to \alpha(Y)$ if and only if for any $f, g \in R$ there holds

$$\varphi(f(i)) = \varphi(g(i)) \text{ for } i \in X \Longrightarrow \varphi(f(i)) = \varphi(g(i)) \text{ for } i \in Y. \quad (2)$$

3.2. Let R, S be relations, φ a homomorphism of R into S w.r.t. bijection $\alpha : rs(R) \to rs(S)$ and R fulfil $X \to Y$. If the mapping $\varphi|_{\bigcup_{i \in X} dom_i(R)}$ is injective, then for any $f, g \in R$ condition (2) is fulfilled.

4. Homomorphism and relational operations

By *a superkey* X of R we understand such a nonempty set X that $X \subseteq rs(R)$ and R fulfils functional dependency $X \to rs(R)$. *A key* of R is a minimum superkey of R w.r.t. inclusion, i.e. such a superkey of R whose no proper subset is a superkey.

4.1. Let R, S be relations, $\alpha : rs(R) \to rs(S)$ a bijection, $\varphi : dom(R) \to \to dom(S)$ a homomorphism of R into S w.r.t. $\alpha : rs(R) \to rs(S)$ and $P \subseteq rs(R)$ an arbitrary nonempty subset. Then φ is a homomorphism of $\pi_P(R)$ into $\pi_{\alpha(P)}(S)$ w.r.t. bijection $\alpha|_P : P \to \alpha(P)$ and

$$\hat{\varphi}(\pi_P(R)) = \pi_{\alpha(P)}(\hat{\varphi}(R)).$$

The equation in **4.1** says that the order of projection and update represented by a homomorphism can be reversed. **4.2** gives an answer to the question whether this can be done for arbitrary update.

4.2. Let $R, S \subseteq D^A$ be relations, $p : R \to S$ a bijection, $N_p(R, S)$ a change relation of R into S w.r.t. p, P a superkey of R. If we define a bijection $p' : \pi_P(R) \to \to \pi_P(S)$ by $p'(f) := p(f')|_P$ for arbitrary $f \in \pi_P(R)$ where $f' \in R$ is (the only) element fulfilling $f = f'|_P$,then

$$p'(\pi_P(R)) = \pi_P(p(R)).$$

Example 3. An analogy of 4.2 does not hold if P is not a superkey of R. Indeed, let $P = \{2, 3\}$, $rs(R) = \{1, 2, 3\}$, R and S be relations represented by tables 6, 7, $p : R \to S$ be a bijection given by $p(a, b, c) = (a, b, c)$, $p(d, b, c) = (d, e, c)$. Then one can define no bijection $p' : \pi_P(R) \to \pi_P(S)$. ♠

Table 6: Relation R from ex.3.

	1	2	3
f_1	a	b	c
f_2	d	b	c

Table 7: Relation S from ex.3.

	1	2	3
g_1	a	b	c
g_2	d	e	c

Let us give an example of the fact that for homomorphism φ of R into S w.r.t. bijection $\alpha : rs(R) \to rs(S)$ there does not generally hold

$$\hat{\varphi}(\sigma_{i_0\theta c}(R)) = \sigma_{\alpha(i_0)\theta'\varphi(c)}(\hat{\varphi}(R)). \tag{3}$$

Example 4. Let us consider relation R represented by table 8, relation S with the same relational scheme and domain, and a homomorphism φ of R into S w.r.t.

identical map $\alpha : rs(R) \to rs(R)$ given as follows: for strings φ is an identity; further let $\varphi(10000) = 10000$, $\varphi(11000) = 13000$, $\varphi(13000) = 14000$, $\varphi(13500) =$ $= 13000$. Now we shall show that for any definition of $\varphi(12000)$ there does not hold (3) with θ-selection $\sigma_{\text{salary} \leq 12000}$ and $i_0 = \text{salary}$, $c = 12000$, $\theta = \theta' = " \leq "$.

Indeed, let us discuss all possible cases of defining the value of $\varphi(12000)$.

A. For $\varphi(12000) \in (-\infty, 10000)$ we have $\sigma_{\text{salary} \leq \varphi(12000)}(\hat{\varphi}(R)) = \emptyset$.

B. For $\varphi(12000) \in < 10000, 13000)$ we get $\sigma_{\text{salary} \leq \varphi(12000)}(\hat{\varphi}(R))$ (table 9).

Table 8: Relation R from ex.4.

	name	salary
f_1	Valenta	10000
f_2	Bruce	11000
f_3	Nehoda	13000
f_4	Brabenec	13500

Table 9: Relation $\sigma_{\text{salary} \leq \varphi(12000)}(\hat{\varphi}(R))$ from ex. 4 — case B.

	name	salary
g_1	Valenta	10000

Table 10: Relation $\sigma_{\text{salary} \leq \varphi(12000)}(\hat{\varphi}(R))$ from ex.4 — case C.

	name	salary
h_1	Valenta	10000
h_2	Bruce	13000
h_4	Brabenec	13000

Table 11: Relation $\hat{\varphi}(R)$ from ex.4.

	name	salary
f_1	Valenta	10000
f_2	Bruce	13000
f_3	Nehoda	14000
f_4	Brabenec	13000

Table 12: Relation $\hat{\varphi}(\sigma_{\text{salary}\leq 12000}(R))$ from ex.4.

	name	salary
f_1	Valenta	10000
f_2	Bruce	13000

C. For $\varphi(12000) \in < 13000, 14000)$ we get $\sigma_{\text{salary}\leq\varphi(12000)}(\hat{\varphi}(R))$ (table 10).

D. And finally for $\varphi(12000) \geq 14000$ there holds $\sigma_{\text{salary}\leq\varphi(12000)}(\hat{\varphi}(R)) = \hat{\varphi}(R)$, (for relation $\hat{\varphi}(R)$ see table 11).

Equation (3) does not hold in any of cases A,B,C,D

(for relation $\hat{\varphi}(\sigma_{\text{salary}\leq 12000}(R))$ see table 12).

In cases A,B inclusion $\supset$ of (3) is valid, in cases C,D the opposite one. ♠

4.3. Let R, S be relations, $\alpha : rs(R) \to rs(S)$ a bijection, $\varphi : dom(R) \to dom(S)$ a homomorphism of R into S w.r.t. α; $i_0, j_0 \in rs(R)$, $\theta \subseteq dom_{i_0}(R) \times dom_{j_0}(R)$, $\theta' \subseteq dom_{\alpha(i_0)}(S) \times dom_{\alpha(j_0)}(S)$. If

$$a\theta b \iff \varphi(a)\theta'\varphi(b) \text{ for all } a \in dom_{i_0}(R), b \in dom_{j_0}(R), \tag{4}$$

then also

$$\hat{\varphi}(\sigma_{i_0\theta j_0}(R)) = \sigma_{\alpha(i_0)\theta'\alpha(j_0)}(\hat{\varphi}(R)). \tag{5}$$

4.4. Let R, S be relations, $\alpha : rs(R) \to rs(S)$ a bijection, $\varphi : dom(R) \to dom(S)$ a homomorphism of R into S w.r.t. α; $i_0 \in A$, θ a binary relation on $dom_{i_0}(R)$, θ' a binary relation on $dom_{\alpha(i_0)}(S)$; $c \in dom_{i_0}(R)$. If

$$a\theta c \iff \varphi(a)\theta'\varphi(c) \text{ for all } a \in dom_{i_0}(R), \tag{6}$$

then also

$$\hat{\varphi}(\sigma_{i_0\theta c}(R)) = \sigma_{\alpha(i_0)\theta'\varphi(c)}(\hat{\varphi}(R)). \tag{7}$$

4.5. Let R, S, R_1, S_1 be relations such that $dom(R) \cap dom(S) = \emptyset$, card$(rs(R)) =$ =card$(rs(R_1))$, card$(rs(S))$ =card$(rs(S_1))$, $\varphi : rs(R) \to rs(R_1)$ is a homomorphism of R into R_1 w.r.t. bijection $\alpha : rs(R) \to rs(R_1)$; $\psi : dom(S) \to dom(S_1)$ is a homomorphism of S into S_1 w.r.t. bijection $\beta : rs(S) \to rs(S_1)$. Then

$$\hat{\varphi}(R) \cdot \hat{\psi}(S) = \hat{\chi}(R \cdot S),$$

where $\chi : dom(R) \cup dom(S) \to dom(R_1) \cup dom(S_1)$ is a homomorphism of $R \cdot S$ into $R_1 \cdot S_1$ w.r.t. bijection $\delta := \alpha \cup \beta : rs(R) \overset{dis}{\cup} rs(S) \to rs(R_1) \overset{dis}{\cup} rs(S_1)$ (i.e. $\delta|_{rs(R)} = \alpha$, $\delta|_{rs(S)} = \beta$) such that $\chi|_{dom(R)} = \varphi$, $\chi|_{dom(S)} = \psi$.

4.6. Let the assumptions of 3.5 hold and moreover $i_0 \in rs(R)$, $j_0 \in rs(S)$, $\theta \subseteq dom_{i_0}(R) \times dom_{j_0}(S)$, $\theta' \subseteq dom_{\alpha(i_0)}(R_1) \times dom_{\beta(j_0)}(S_1)$. If

$$a\theta b \iff \varphi(a)\theta'\psi(b) \text{ for all } a \in dom_{i_0}(R), b \in dom_{j_0}(S),$$

then also

$$\hat{\chi}(R(i_0\theta j_0)S) = \hat{\varphi}(R)(\alpha(i_0)\theta'\beta(j_0))\hat{\psi}(S).$$

4.7. Let R, R_1, S, S_1 be relations, $\alpha : rs(R) \to rs(R_1)$, $\beta : rs(S) \to rs(S_1)$, $\gamma : rs(R) \to rs(S)$ be bijections, $\varphi : dom(R) \to dom(R_1)$ a homomophism of R into R_1 w.r.t.α; $\psi : dom(S) \to dom(S_1)$ a homomorphism of S into S_1 w.r.t. β, $dom(R) \cap dom(S) = \emptyset$; a mapping $\chi : dom(R) \cup dom(S) \to dom(R_1) \cup dom(S_1)$ is given by $\chi|_{dom(R)} = \varphi$, $\chi|_{dom(S)} = \psi$. Then χ is a homomorphism of $R\overset{\gamma}{\cup}S$ into $R_1 \overset{\beta\circ\gamma\circ\alpha^{-1}}{\cup} S_1$ w.r.t. α and

$$\hat{\chi}(R\overset{\gamma}{\cup}S) = \hat{\varphi}(R) \overset{\beta\circ\gamma\circ\alpha^{-1}}{\cup} \hat{\psi}(S).$$

Example 5. Let R, R_1, S, S_1 be relations, $\alpha_1 : rs(R) \to rs(R_1)$, $\beta_1 : rs(S) \to \to rs(S_1)$ bijections, $\alpha_0 : rs(S) \to rs(R)$ an injective mapping, $i_0 \in rs(R) - - \alpha_0(rs(S))$, $\varphi : dom(R) \to dom(R_1)$ a homomorphism of R into R_1 w.r.t. α_1, $dom_j(S) \subseteq dom_{\alpha_0(j)}(R)$ for any $j \in rs(S)$ (i.e. φ is also a homomorphism of S into S_1 w.r.t. β_1). Then an inclusion $\hat{\varphi}(R)(\div, \alpha_1 \circ \alpha_0 \circ \beta_1^{-1}, \alpha_1(i_0))\hat{\varphi}(S) \subseteq \subseteq \hat{\varphi}(R(\div, \alpha_0, i_0)S)$ does not hold in general.

Indeed, let relations R, S be given by tables 13, 14. If we define $\alpha_0(i_5) = i_1$, then relation $R(\div, \alpha_0, i_0)S$ can be represented by table 15. Let a homomorphism φ be given by: $\varphi(S_6) = S_5$, $\varphi(x) = x$ otherwise for $x \in dom(R)$; α_1 be an identity mapping on $\{i_0, i_1, i_2, i_3, i_4\}$, β_1 an identity mapping on $\{i_5\}$. Then relations $\hat{\varphi}(R)$, $\hat{\varphi}(S)$ are represented by 16, 17.

Table 13: Relation R from ex.5.

	i_0	i_1	i_2	i_3	i_4
f_1	D_1	S_2	5	100	20
f_2	D_1	S_6	12	10	600
f_3	D_4	S_2	5	100	15
f_4	D_4	S_5	15	5	300
f_5	D_4	S_6	10	5	350

Table 14: Relation S from ex.5.

	i_5
g_1	S_2
g_2	S_5
g_3	S_6

Table 15: Relation $R(\div, \alpha_0, i_0)S$ from ex.5. from ex.5.

	i_0	i_2	i_3	i_4
h_1	D_4	5	100	15
h_2	D_4	15	5	300
h_3	D_4	10	5	350

Table 16: Relation $\hat{\varphi}(R)$ from ex.5.

	i_0	i_1	i_2	i_3	i_4
k_1	D_1	S_2	5	100	20
k_2	D_1	S_5	12	10	600
k_3	D_4	S_2	5	100	15
k_4	D_4	S_5	15	5	300
k_5	D_4	S_5	10	5	350

Table 17: Relation $\hat{\varphi}(S)$ from ex.5.

	i_5
l_1	S_2
l_2	S_5

Now one can see that

$$\hat{\varphi}(R)(\div, \alpha_1 \circ \alpha_0 \circ \beta_1^{-1}, i_0)\hat{\varphi}(S) = \hat{\varphi}(R),$$

$$\hat{\varphi}(R(\div, \alpha_0, i_0)S) = R(\div, \alpha_0, i_0)S.$$

Therefore there holds inclusion

$$\hat{\varphi}(R)(\div, \alpha_1 \circ \alpha_0 \circ \beta_1^{-1}, \alpha_1(i_0))\hat{\varphi}(S) \supseteq \hat{\varphi}(R(\div, \alpha_0, i_0)S),$$

but the opposite inclusion is not fulfilled. ♠

Conclusion

There could be made effort to extend assertions **4.3** to **4.7** to arbitrary update (not just to an update represented by a homomorphism) using the concept of bijective mapping and change relation as it is done with **4.1** in **4.2**. Moreover, homomorphism might be a good tool to describe structures and processes different from updates – for example relationships between objects or methods of objects in object-oriented data model. That should be a part of further work in this area.

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