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ON A THIRD-ORDER THREE-POINT REGULAR BOUNDARY VALUE PROBLEM

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Abstract: This paper is concerned with the existence and uniquenass of solutions of the problem

$$u''' = f(t,u,u',u'')$$

$$u'(0) = u'(1) = u(\eta) = 0, \quad 0 \le \eta \le 1.$$

The existence is studied by means of topological degree methods.

Key words: Boundary value problems, Mawhin's continuation theorem, a priori bounds, uniqueness.

MS Classification : 34B10

1. Introduction. In this paper there are found some conditions for the existence and uniqueness of solutions of the problem

$$u''' = f(t,u,u',u''), \tag{1.1}$$

$$u'(0) = u'(1) = u(\eta) = 0, \quad 0 \le \eta \le 1. \tag{1.2}$$

This problem models the static deflection of a three-layered elastic beam. The proof of the main result is based on Mawhin's continuation theorem. The existence of solutions is related to the sign of f on certain subsets of $[0,1] \times R^3$. We shall prove an existence theorem without requiring a growth condition on the whole interval.

Multipoint boundary value problems /BVPs/ for differential equations of the n-th order have been studied by many authors (see References). For $n \geq 2$ and $2 \leq k \leq n$, the questions of existence and uniqueness of solutions of k-point BVPs Cauchy-Nicoletti, de la Valleé-Poussin or similar ones, in which the values of a solution or the values of its derivatives are given, have been solved f.e. in [10, 11, 12-15].

We consider equation (1.1) with three-point boundary conditions. In this case the Valleé-Poussin conditions have the form

$$u(a) = A, \quad u(c) = C, \quad u(b) = B, \tag{1.3}$$

where $-\infty < a < c < b < +\infty$, $A,B,C \in R$.

BVP (1.1), (1.3) has been investigated f.e. in [1, 2, 5, 18].

Replacing function values by their derivatives, we obtain

$$u'(a) = A, \quad u(c) = C, \quad u'(b) = B. \tag{1.4}$$

In [4], the subfunction method (see [3]) is used for the existence of solutions of BVP (1.1), (1.4), and in [16], the necessary and sufficient conditions for solvability of this problem are proved by means of lower and upper functions.

$$\text{BVP (1.1)}, \quad u(c) = 0, \quad u'(a) = u'(b), \quad u''(a) = u''(b) \tag{1.5}$$

where $-\infty < a \leq c \leq b < +\infty$, has been investigated in [17].

C.P.Gupta ([7]) studied the questions of the existence and uniqueness of solutions of the equation

$$-u''' - \pi^2 u' + g(x,u,u',u'') = e(x) \tag{1.6}$$

or

$$u''' + \pi^2 u' + g(x,u,u',u'') = e(x) \tag{1.7}$$

satisfying (1.2). The existence of a solution for the resonance problem (1.6), (1.2) was obtained when e was a Lebesguw-integrable function with

$$\int_0^1 e(x) \sin \pi x \, dx = 0$$

and g was a Caratheódory function, bounded on $[0, 1] \times B^2 \times R$ (for every bounded B of R) and

$$g(x,u,v,w)v \geq 0 , \quad x \in [0,1] , \quad u,v,w \in R .$$

For the existence of a solution for (1.7), (1.2) g, in addition, had to satisfy

$$\limsup_{|v| \to +\infty} \frac{g(x,u,v,w)}{v} = \beta < 3\pi^2 .$$

These results were proved by means of the method using second-order integro-differential boundary value problems and the Leray-Schauder continuation theorem.

2. Notations and definitions. In what follows we suppose that $C^i(a,b)$ is the set of all real functions having continuous i-th derivatives on $[a,b]$, $i = 0,1,2,3$;
$\|x\| = \max \{|x(t)| : a \leq t \leq b\}$, where $x \in C^0(a,b)$;
$\|x\|_1 = (\|x\|^2 + \|x'\|^2)^{1/2}$, where $x \in C^1(a,b)$;
$\|x\|_2 = (\|x\|^2 + \|x'\|^2 + \|x''\|^2)^{1/2}$, where $x \in C^2(a,b)$.
G is the Banach space of all functions from $C^2(0,1)$ satisfying (1.2) and having the norm $\|.\|_2$.
If $D \subset G$, then $\overline{D}$ and δD is the closure and the boundary of D in G, respectively.

Definition. A function $u \in C^3(0,1)$ which fulfils (1.1) for every $t \in [0,1]$ and satisfies (1.2) will be called a solution of the problem (1.1), (1.2).

3. Existence.

Lemma 1. Let $g \in C^0([0,1] \times R^3)$, r_1, $r_2 \in R$ and $r_1 < 0 < r_2$. Then for each solution $u \in G$ of the equation

$$u''' = g(t,u,u',u'') \tag{3.1}$$

satisfying

$$r_1 \leq u'(t) \leq r_2 \qquad \text{for any } t \in [0,1] \ , \tag{3.2}$$

the inequality

$$|u(t)| < M \qquad \text{for any } t \in [0,1] \ , \tag{3.3}$$

where $M = \max \{|r_1| \ , \ r_2\}$, is valid.

Proof. Let us suppose that there exists $t_0 \in [0,1]$ such that $|u(t_0)| = M$. If $t_0 = 0$, $\eta = 1$ or $\eta = 0$, $t_0 = 1$ and (3.2) is valid, then $|u'(t)| = M$ for every $t \in [0,1]$, which contradicts (1.2). If t_0, $\eta \in (0,1)$, then there exists $t_1 \in (0,1)$ such that $|u'(t_1)| > M$, which contradicts (3.2). Lemma is proved.

Lemma 2. Let there exist r_1, $r_2 \in R$, $r_1 < 0 < r_2$ and $g \in C^0([0,1] \times R^3)$ such that

$$g(t,x,r_1,0) < 0 \quad \text{and} \quad g(t,x,r_2,0) > 0 \tag{3.4}$$
$$\text{for any } t \in [0,1] \ , \ x \in (-M,M) \ .$$

Then each solution $u \in G$ of the equation (3.1) satisfying (3.2) fulfils

$$\max \{u'(t) : 0 \leq t \leq 1\} \neq r_2 \text{ and } \min \{u'(t) : 0 \leq t \leq 1\} \neq r_1 \ . \tag{3.5}$$

Proof. Let us suppose that $u \in G$ satisfies (3.1), (3.2) and $\max \{u'(t) : 0 \leq t \leq 1\} = r_2$. Then there exists $t_0 \in (0,1)$ such that $u'(t_0) = r_2$. Then $u''(t_0) = 0$ and $u'''(t_0) \leq 0$. According to (3.1), $g(t_0,u(t_0),r_2,0) \leq 0$, which contradicts (3.4).
We can obtain a similar contradiction for $\min\{u'(t) : 0 \leq t \leq 1\} = r_1$. Lemma is proved.

Lemma 3. Let there exist F, ε, r_1, r_2, c_1, $c_2 \in R$, $r_1 < 0 < r_2$, $c_1 < 0 < c_2$, $0 < \varepsilon \leq 1$ and $g \in C^0([0,1] \times R^3)$ such that

$$c_2 > \frac{2|r_1|}{\varepsilon} \ , \quad |c_1| > \frac{2r_2}{\varepsilon} \ , \quad F \leq \frac{\min \{|c_1|, c_2\}}{2\varepsilon}$$

and

$$|g(t,x,y,z)| \leq F \tag{3.6}$$

for $t \in I_{1,\varepsilon} = [1-\varepsilon, 1]$, $x \in (-M,M)$, $y \in [r_1, r_2]$, $z \in [c_1, c_2]$.
Further let

$$g(t,x,y,c_1) < 0 \quad \text{and} \quad g(t,x,y,c_2) > 0 \tag{3.7}$$

for $t \in [0,1)$, $x \in (-M,M)$, $y \in [r_1, r_2]$.
Then for each solution u G of the problem (3.1), (1.2) satisfying (3.2) and

$$c_1 \leq u''(t) \leq c_2 \qquad \text{for any} \quad t \in [0,1] \tag{3.8}$$

the inequalities

$$\begin{aligned} &\max \{u''(t) : 0 \leq t \leq 1\} \neq c_2 \quad \text{and} \\ &\min \{u''(t) : 0 \leq t \leq 1\} \neq c_1 \end{aligned} \tag{3.9}$$

are valid.

Proof. Let us suppose that u G satisfies (3.1), (1.2) and let $\max \{u''(t) : 0 \leq t \leq 1\} = c_2$. Then there exists $t_0 \in [0,1]$ such that $u''(t_0) = c_2$. If $t_0 \in (0,1)$, then $u'''(t_0) = 0$ and according to (3.1) $g(t_0, u(t_0), u'(t_0), c_2) = 0$, which contradicts (3.7). If $t_0 = 0$, then $u''(0) = c_2$ and $u'''(0) \leq 0$ and according to (3.1) $g(0, u(0), 0, c_2) \leq 0$, which contradicts (3.7). If $t_0 = 1$, then $u''(1) = c_2$. From (3.1) and (3.6) we obtain $|u'''(t)| \leq F$ for $t \in I_{1,\varepsilon}$. From the relation between F and c_2 follows $u''(t) \geq$ $\geq c_2 - F \cdot \varepsilon \geq \frac{c_2}{2}$ for $t \in I_{1,\varepsilon}$. From (1.2) and from the relation between c_2 and r_1 it follows that $u'(1-\varepsilon) < r_1$, which contradicts (3.2). We can obtain a similar contradiction for $\min \{u''(t) :$ $: 0 \leq t \leq 1\} = c_1$. Lemma is proved.

Lemma 4. Let there exist $k, F, r_1, r_2, c_1, c_2, \varepsilon, \lambda \in R$, $r_1 < 0 < r_2$, $c_1 < 0 < c_2$, $F \geq c_2 + |c_1|$, $0 < \varepsilon \leq 1$, $0 \leq \lambda \leq 1$ and $f \in C^0([0,1] \times R^3)$. Let the function $\tilde{f} : [0,1] \times R^3 \times [0,1] \longrightarrow R$ be defined by

$$\tilde{f}(t,x,y,z,\lambda) = \lambda f(t,x,y,z) + (1-\lambda)(ky + z),$$

where $0 < k < \dfrac{\min \{|c_1|, c_2\}}{\max \{|r_1|, r_2\}}$.

Let f fulfil (3.6) and

$$f(t,x,r_1,0) \le 0 \quad \text{and} \quad f(t,x,r_2,0) \ge 0 \qquad (3.10)$$
$$\text{for any} \quad t \in [0,1], \; x \in (-M,M)$$

and

$$f(t,x,y,c_1) \le 0 \quad \text{and} \quad f(t,x,y,c_2) \ge 0 \qquad (3.11)$$
$$\text{for any } t \in [0,1),\; x \in (-M,M),\; y \in [r_1,r_2] \,.$$

Then $\tilde{f}$ satisfies (3.4), (3.6) and (3.7) for any $\lambda \in (0,1)$.

Proof. Let $t \in [0,1]$, $x \in (-M,M)$, $\lambda \in (0,1)$ and f fulfil (3.10). Then

$$\tilde{f}(t,x,r_1,0,\lambda) = \lambda f(t,x,r_1,0) + (1-\lambda)(kr_1) < 0 \quad \text{and}$$

$$\tilde{f}(t,x,r_2,0,\lambda) = \lambda f(t,x,r_2,0) + (1-\lambda)(kr_2) > 0 \,.$$

Further let $t \in [0,1)$, $x \in (-M,M)$, $y \in [r_1,r_2]$, $\lambda \in (0,1)$ and f fulfil (3.11). Then

$$\tilde{f}(t,x,y,c_1) = \lambda f(t,x,y,c_1) + (1-\lambda)(ky + c_1) < 0 \quad \text{and}$$

$$\tilde{f}(t,x,y,c_2) = \lambda f(t,x,y,c_2) + (1-\lambda)(ky + c_2) > 0 \,.$$

Further let $t \in I_{1,\varepsilon}$, $x \in (-M,M)$, $y \in [r_1,r_2]$, $z \in [c_1,c_2]$ and f fulfil (3.6). Then

$$|\tilde{f}(t,x,y,z,\lambda)| = |\lambda f(t,x,y,z) + (1-\lambda)(ky+z)| <$$
$$< \lambda F + (1-\lambda)\left(\frac{\min\{|c_1|,c_2\}}{\max\{|r_1|,r_2\}} \max\{|r_1|,r_2\} + \max\{|c_1|,c_2\}\right) =$$
$$= \lambda F + (1-\lambda)(|c_1| + c_2) \le F \,.$$

Lemma is proved.

Lemma 5. Let $f \in C^0([0,1] \times R^3 \times [0,1])$ and let there exists an open bounded set $D \subset G$ such that for any $\lambda \in (0,1)$ each solution $u_\lambda \in G$ of the equation

$$u''' = \lambda \tilde{f}(t,u,u',u'',\lambda) \qquad (3.12)$$

satisfies

$$u_\lambda \notin \partial D \qquad (3.13)$$

and let $0 \in D$.

Then for any $\lambda \in [0,1]$ the equation (3.13) has at least one solution in $\bar{D}$.

Proof. Lemma follows from the Mawhin continuation theorem [6. Theorem IV.1, p.27].

Theorem 6. Let there exist $F, r_1, r_2, c_1, c_2, \mathcal{E} \in R$, $r_1 < 0 < r_2$, $c_1 < 0 < c_2$, $0 < \mathcal{E} \leq 1$ and $f \in C^0([0,1] \times R^3)$ such that

$$c_2 > \frac{2|r_1|}{\mathcal{E}}, \quad |c_1| > \frac{2r_2}{\mathcal{E}}, \quad |c_1| + c_2 \leq F \leq \frac{\min |c_1|, c_2}{2\mathcal{E}}$$

If f fulfils (3.6), (3.10) and (3.11), then the problem (1.1), (1.2) has a solution u satisfying

$$-M < u(t) < M, \quad r_1 \leq u'(t) \leq r_2, \quad c_1 \leq u''(t) \leq c_2 \qquad (3.14)$$

Proof. Put

$$D = \{x \in G : -M < x(t) < M,\ r_1 < x'(t) < r_2,\ c_1 < x''(t) < c_2, \text{ for } t \in [0,1]\}.$$

Then $x \in \partial D$ if

$$\{-M \leq x(t) \leq M,\ r_1 \leq x'(t) \leq r_2, \text{ for } t \in [0,1]\} \quad \text{and}$$
$$\{\max x''(t) = c_2 \quad \text{or} \quad \min x''(t) = c_1 \quad \text{on} \quad [0,1]\}$$

or

$$\{-M \leq x(t) \leq M,\ c_1 \leq x''(t) \leq c_2, \text{ for } t \in [0,1]\} \quad \text{and}$$
$$\{\max x'(t) = r_2 \quad \text{or} \quad \min x'(t) = r_1 \quad \text{on} \quad [0,1]\}$$

or

$$\{r_1 \leq x'(t) \leq r_2,\ c_1 \leq x''(t) \leq c_2, \text{ for } t \in [0,1]\} \quad \text{and}$$
$$\{\max x(t) = M \quad \text{or} \quad \min x(t) = -M \quad \text{on} \quad [0,1]\}.$$

Let $\tilde{f}$ be defined in the same way as in Lemma 4. Let $\lambda \in (0,1)$ and $u_\lambda \in G$ be a solution of (3.12). According to Lemma 4 $\tilde{f}$ satisfies (3.4), (3.6) and (3.7). If u_λ fulfils (3.2) and (3.8), then by Lemma 1 u_λ satisfies (3.3), by Lemma 2 u_λ satisfies (3.5) and by Lemma 3 u_λ satisfies (3.9). Thus we get $u_\lambda \notin \partial D$. Using Lemma 5, we obtain that for any $\lambda \in [0,1]$ the equation (3.12) has at least one solution in $\overline{D}$. From Lemma 1 it follows that the problem (1.1), (1.2) has a solution satisfying (3.14). Theorem is proved.

Note. Similarly it is possible to prove a theorem which is in a certain way symetric to the Theorem 6. It follows.

Theorem 6'. Let there exist $F, r_1, r_2, c_1, c_2, \mathcal{E} \in R$, $r_1 < 0 < r_2$,

$c_1 < 0 < c_2$, $0 < \varepsilon \leq 1$ and $f \in C^0([0,1] \times R^3)$ such that

$$c_2 > \frac{2r_2}{\varepsilon}, \quad |c_1| > \frac{2|r_1|}{\varepsilon}, \quad |c_1| + c_2 \leq F \leq \frac{\min\{|c_1|, c_2\}}{2\varepsilon}$$

If f fulfils (3.10),

$$|g(t,x,y,z)| \leq F \qquad (3.6)'$$

for $t \in I_{0,\varepsilon} = [0,\varepsilon]$, $x \in (-M,M)$, $y \in [r_1,r_2]$, $z \in [c_1,c_2]$

$$\text{and} \quad f(t,x,y,c_1) \geq 0 \quad \text{and} \quad f(t,x,y,c_2) \leq 0 \qquad (3.11)'$$

for $t \in (0,1]$, $x \in (-M,M)$, $y \in [r_1,r_2]$,

then the problem (1.1), (1.2) has a solution u satisfying (3.14).

Theorem 7. Let $f \in C^0([0,1] \times R^3)$. If f fulfils

$$f(t,x,y,z)y \geq 0 \qquad (3.15)$$

for any $t \in [0,1]$, $x,y,z \in R$,

then the problem (1.1), (1.2) has only a trivial solution.

Proof. Let u be a solution of (1.1), (1.2). Multiplying now the equation (1.1) by u and integrating on the interval $[0,1]$ we get

$$\int_0^1 u''' u' dt = \int_0^1 f(t,u,u',u'')u' dt \geq 0 .$$

Hence,

$$-\int_0^1 (u'')^2 \, dt \geq 0 ,$$

which implies that $u''(t) = 0$ and further, that $u(t) = 0$ for $t \in [0,1]$. The assumption (3.15) implies that

$$f(t,x,0,z) = 0 \qquad (3.16)$$

for any $t \in [0,1]$, $x,z \in R$.

From (3.16) it follows that $u(t) = 0$ for $t \in [0,1]$ is a solution of (1.1), (1.2). Theorem is proved.

Note. Let us set

$$f_1(t,x,y,z) = a(t)x^{2k}(y+d)^{2n+1} + z^{2m+1},$$
$$f_2(t,x,y,z) = a(t)x^{2k}(y+d)^{2n+1} + z,$$

$$\psi(t) = \begin{cases} 1 & \text{for } t \in [0, \frac{1}{2}] \\ 3-4t & \text{for } t \in (\frac{1}{2}, \frac{3}{4}) \\ 0 & \text{for } t \in [\frac{3}{4}, 1], \end{cases}$$

where $a \in C^0(0,1)$, $0 \leq a(t) \leq 1$ for $t \in [0,1]$, $d \in R$ and k, m, n are nonnegative integers. Then for example the function f_1, where $|d| < \frac{1}{10}$, satisfies the assumptions of Theorem 6 for $\mathcal{E} = \frac{1}{4}$, $r_{1,2} = \mp \frac{1}{10}$, $c_{1,2} = \mp 1$, $F = 2$. Functions f_2 and $\psi(t)f_1 + (1 - \psi(t))f_2$, where $|d| < \frac{1}{2}$, satisfy the assumptions of Theorem 6 for $\mathcal{E} = \frac{1}{4}$, $r_{1,2} = \mp \frac{1}{2}$, $c_{1,2} = \mp 5$, $F = 10$.

4. Uniqueness.

Theorem 8. Let $f \in C^0([0,1] \times R^3)$ and for any $t \in [0,1]$, $x_i, y_i, z_i \in R$, $i = 1,2$, the inequality

$$f(t,x_1,y_1,z_1) - f(t,x_2,y_2,z_2)(y_1 - y_2) \geq 0 \tag{4.1}$$

is valid. Then BVP (1.1), (1.2) has at most one solution.

Proof. Let u_1, u_2 be solutions of BVP (1.1), (1.2). We see by setting $v = u_1 - u_2$, that

$$-v''' + f(t,u_1,u_1',u_1'') - f(t,u_2,u_2',u_2'') = 0, \tag{4.2}$$

$$v'(0) = v'(1) = v(\eta) = 0. \tag{4.3}$$

Multiplying now the equation (4.2) by $v' = u_1' - u_2'$ and integrating on the interval $[0,1]$ we get

$$0 = -\int_0^1 v'''v'\,dt + \int_0^1 (f(t,u_1,u_1',u_1'') - f(t,u_2,u_2',u_2''))(u_1' - u_2')dt \geq$$

$$\geq -\int_0^1 v'''v'\,dt = \int_0^1 v''^2\,dt \geq 0.$$

Hence

$$\int_0^1 v''^2 \, dt = 0 ,$$

which implies that $v''(t) = 0$ for every $t \in [0,1]$ and by (4.3) we get $v(t) = 0$ for every $t \in [0,1]$. The uniqueness is proved.

Lemma 9. [8, Theorem 256, p.219]. If f is absolutely continuous on $[t_1, t_2]$, f is Lebesgue integrable on (t_1, t_2) and $f(t_o) = 0$, where $-\infty < t_1 \leq t_o \leq t_2 < +\infty$, then

$$\int_{t_1}^{t_2} f^2(t)dt \leq [2(t_2 - t_1)/\pi]^2 \int_{t_1}^{t_2} f'^2(t)dt .$$

Theorem 10. Let $f \in C^0([0,1] \times R^3)$ and let there exist possitive constans α , β , γ satisfying

$$\alpha \frac{4}{\pi^3} + \beta \frac{2}{\pi^2} + \gamma \frac{2}{\pi} < 1 \tag{4.4}$$

such, that for any $t \in [0,1]$, $x_i, y_i, z_i \in R$, $i = 1,2$ the inequality

$$|f(t,x_1,y_1,z_1) - f(t,x_2,y_2,z_2)| \leq \alpha|x_1 - x_2| + \\ + \beta|y_1 - y_2| + \gamma|z_1 - z_2| \tag{4.5}$$

is valid. Then BVP (1.1), (1.2) has at most one solution.

Proof. Let u_1, u_2 be solutions of BVP (1.1), (1.2). We see by setting $v = u_1 - u_2$, that

$$v'(0) = v'(1) = v(\eta) = 0 . \tag{4.6}$$

According to the last equation there exists $t_o \in [0,1]$ such that $v''(t_o) = 0$. Put $\rho = (\int_0^1 v'''^2(t)dt)^{1/2}$. Then by Lemma 9

$$(\int_0^1 v''^2(t)dt)^{1/2} \leq \frac{2}{\pi} \cdot \rho .$$

By the Wirtinger inequality [9, p.409] we get

$$\left(\int_0^1 v'^2(t)dt\right)^{1/2} \leq \frac{2}{\pi^2}\cdot \vartheta \quad .$$

Further by Lemma 9 we obtain

$$\left(\int_0^1 v^2(t)dt\right)^{1/2} \leq \frac{4}{\pi^3}\cdot \vartheta \quad .$$

From (4.5) we get

$$\vartheta \leq \varrho\left(\alpha\frac{4}{\pi^3} + \beta\frac{2}{\pi^2} + \gamma\frac{2}{\pi}\right) \; .$$

According to (4.4) we obtain $\vartheta = 0$ and by (4.6) $v = 0$. Uniqueness is proved.

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