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INDIVIDUALISED NORMATIVE LOGIC

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The systems of normative logic built on basis of first-order predicate calculus are not satisfactorily developed so far. Nevertheless such a way of normative logic development seems to have a lot of good reasons. First of all usage of maximal classical logic seems to be quite natural, because it fulfills all desirable methodological properties. Next - from the point of wiew of normative disciplines fully qualified - there is possibility to express all constituents of norms. In this case I mean the subject of a norm formalised by an individual variable or constant. Except, there is a possibility to investigate the structure of norms in extended meaning.

Principal question is what we do demand of such a formalisation. Tendency to build the systems of normative logic as special cases of systems of modal logic (see f.i. [1]) is rather misleading. If we build normative or modal logic on basis of propositional calculus, such conception looks fully acceptable and convincing. If we use predicate calculus as basis, some difficulties come into being. The relations between alethic modalities and quantifiers, which are postulated by modal logic, are out of questions. But their analogies in normative logic are not acceptable. The character and the sense of logical necessity and possibility and normative necessity and possibility are different. This difference is omitted in such considerations. In addition, the usage of existential quantifier in formulation of norms is problematical and it seems, that in this context it is redundant and that the language of purely normative considerations can exist without it. Examples with existential quantifier deal always with norms of type permission (see f.i. [2]) and they do not disprove this objection. The norms of this type can be formulated equivalently without existential quantifier. I tried in [6] to suggest a normative logic with quantifiers in such a way, so it could satisfy mentioned demands. I suppose it is necessary to revise even this system and I present a new conception here as follows.

Normative logic can be built at least by two ways, namely as monadic or dyadic, i.e. as formalisation of absolute or hypothetic norms. Because dyadic normative logic was created to eliminate paradoxes of monadic normative logic (caused by material implication - see f.i. [3]), we will accept the same way here. We use the symbols p, q for notation of propositions of arbitrary structure (it is possible to demand by [4] q to be a description of arbitrary state and p to be a description of the state the realization of which is within the bounds of human possibilities). The expression O(p/q) denotes "it is obliged, that in the situation described by proposition q was realized (by considerated subject) the state described by proposition p" or in short "it is obliged p in case q", F(p/q) - "it is forbidden p in case q", P(p/q) - "it is permitted p in case q" and I(p/q) - "it is (normatively) indifferent p in case q". Mentioned expressions are formalized notes of norms in the symbolics of dyadic normative logic. In these expressions let us call the symbol q the conditional component of norm and the symbol p the normative component of the norm. The conditional component of norm expresses the situation in which the norm is aplicated and the normative component describes regulated action.

Building the formal system of dyadic normative logic, we usually choose functor O as basic and other three are defined in such a way:

$$F(p/q) \;=\; \text{df } O(\neg\, p/q)$$

$$P(p/q) \;=\; \text{df } \neg F(p/q)$$

$$I(p/q) \;=\; \text{df } P(p/q)\ \&\ P(\neg\, p/q),$$

where symbol $\neg$ denotes the negation and & the conjunction. These definitions can be used only in the system, which fulfills the demand of normative consistency and normative completness, which can be expressed by formulas

$$\neg[O(p/q)\ \&\ O(\neg\, p/q)] \quad \text{or} \quad \neg[O(p/q)\ \&\ F(p/q)]$$

$$O(p/q) \vee F(p/q) \vee I(p/q)\ ,$$

respectively. These formulas must be deducable in this system.

In following text we will use this notation: x, y, ... - individual variables; a, b, ... - individual constants; m, n - - ∅-ary predicate constants; p, q, r, ... - predicate constants with arity greater than zero; $\neg$, & , v, ->, <-> are symbols for negation, conjunction, disjunction, material implication and equivalency resp.; ∀, ∃ - are universal and existential quantifier. Now let every normative functor be "individualised" so that it will be followed by list of individual variables or constants, which refer to individuals to which this function is adressed. Expression (of individualised normative logic)

$$O(x)[p(x)/m]$$

then expresses "it is obliged (to all) x to perform p in case m", expression

$$F(x)[q(x,y)/r(x,y)]$$

we would read "it is forbidden (to all) x to be in the relation q with y in the case that x is with y in the relation r", the expression

$$P(x,y)\forall z[s(x,y,z)/t(x,y,z)]$$

means "it is permitted (to all) x and (to all) y (for all) z to be with z in the relation s in the case, that they are with it in the relation t", etc.

Let us consider all these expressions to be "meaningfull". Such usage of operators O, F and P evidently bounds the variables appearing in the subjoined list by certain manner. Then the list evidently fulfills two functions - firstly it denotes and eventually differentiates the addressees of norms, secondly it enumerates the variables, which are normatively bounded by "universal quantifier" and by this way it must as well formally and informally handle with them. Other variables in expression (if there are any) are normatively free, i.e. proper normative function does not deal with them (it is not addressed to them). That is why there cannot be any objection to it, these variables be eventually bounded by existential quantifier, if it is necessary. Such a need would not appear for individuals of the type addressee of norm, because such situation in meaningless. Similarly for individuals normatively free which are in supporting role existential quantifier cannot be used meaningfully either before the parenthese or in it. The first case - f.i.

$$O(x)\exists y[p(x,y)/q(x,y)] \qquad (*)$$

would normatively set up (prescribe) an existency, what cannot be accepted. The second case would divide into these possibilities:

(A here denotes arbitrary atomic formula with free occurence of variable y and in this context eventually even with free occurence of x)

a) existential quantifier is in conditional compound, f.i.

$O(x)[p(x)/\exists yA]$,

which is the only acceptable and meaningful usage of existential quantifier. But this expression can be equivalently written down as

$O(x)\forall y[p(x)/A]$,

so that we can avoid existential quantifier; but we must ask the question, to which the variable y functions, when it does not occur in the normative compound;

b) existential quantifier is in the normative compound, f.i.

$O(x)[\exists yp(x,y)/m]$,

what is the case similar to formula (*);

c) existential quantifier is both in conditional and normative compound for the same variable, f.i.

$O(x)[\exists yp(x,y)/\exists yA]$,

which is either misunderstanding (if it is to identification of values of y the a) is the right note) or the expression is not adequate from reason b).

In examples mentined above we without a word presuppose, that

1) the measure of formalization corresponds to the need to express the subject - addressee of norms so that it could be identified (normative bounded variable or signified constant) and it could be possible eventually comprehend its relations to the other factors of the same "quality" (others variables or constants),

2) individual variables are used in both compounds of formalized norms functionally, i.e. each variable, which occurs in the conditional compound, must occur as well in the normative compound.

I hope, I managed to encounter all presuppositions on which I want to found formal system of individualised normative logic.

Formalization of the system INL - individualised normative logic we carry out on the basis of axiomatic system of first--order predicate logic. Language of INL contents in addition the symbol O with added list of individual variables in brackets. We denote by symbol O() the case, when the list contains arbitrary free variables of normative compound. Well formed formulas of INL will be defined as following:

1) if A and B are well formed formulas of predicate logic, when
   - a) formula B does not contain any quantifier
   - b) all (free) variables contained in formula B occur at the same time in formula A and formula A contains no quantifier,

   and if $x1, x2, \ldots, xn$ are all free variables occuring in formula A, then well formed formulas of INL are following expressions
   - i) $O(x1, \ldots, xn)[A/B]$,
   - ii) $O(xi1, \ldots, xik)[A/B]$, for any permutation of indexes $i1, \ldots, ik \in \{1, 2, \ldots, n\}$ and $\emptyset < k <= n$,
   - iii) $O(xi1, \ldots, xik)\forall xj1 \ldots \forall xjl[A/B]$, for any permutations of indexes $i1, i2, \ldots, ik \in \{1, 2, \ldots, n\}$, $j1, j2, \ldots, jl \in \{1, 2, \ldots, n\} \setminus \{i1, i2, \ldots, ik\}$, where $k > \emptyset$ and $k+l <= n$,
   - iv) $O(xi1, \ldots, xik)\forall xj1 \ldots \forall xjl[\forall zA/B]$, for any permutations of indexes $i1, i2, \ldots, ik \in \{1, 2, \ldots, n\}$, $j1, j2, \ldots, jl \in \{1, 2, \ldots, n\} \setminus \{i1, i2, \ldots, ik\}$, where $k > \emptyset$, $k+l < n$ and variable z which is free in A and differs from all variables xi and xj.

2) If the expressions A and B are well formed formulas of INL, then also the expressions $\neg A$, $A \& B$, $A \vee B$, $A \rightarrow B$ and $A <-> B$ are well formed formulas of INL.

The set of axioms of INL is formed of these formulas (by [5], [6] and [7]):

A1. $\neg\{O()[A/B] \& \neg O()[\neg A/B]\}$

A2. $O()[A \& B/C] <-> \{O()[A/C] \& O()[B/C]\}$

A3. $O()[A/B \vee C] \leftrightarrow \{O()[A/C] \;\&\; O()[A/C]\}$

A4. $O()[A/B] \;\&\; O()[B/C] \rightarrow O()[A/C]$

A5. $O()[A/B] \rightarrow \neg O()[A/\neg B]$

A6. $O()[A \;\&\; B/C] \rightarrow \neg O()[\neg A/B \;\&\; C]$

A7. $O()\forall x[A/B] \leftrightarrow O()[\forall x A/B]$.

A8. $O(x)[A(x)/B(x)] \rightarrow O(a)[A(x/a)/B(x/a)]$

Rules of inference are in the system of INL:

R1. modus ponens

R2. rule of generalization extended to O-expressions as follows: if x is free in $O()[A/B]$, then $O()\forall x[A/B]$

R3. rule of extensionality, with the exception - in O-expressions it is not possible, when replacing, to use the formula containing existential quantifier.

The definitions of functors will be next component of the system, which guarantees the completness of system:

D1. $F()[A/B] =_{df} O()[\neg A/B]$

D2. $P()[A/B] =_{df} \neg F()[A/B]$

D3. $I()[A/B] =_{df} P()[A/B] \;\&\; P()[\neg A/B]$.

The semantics of the system of normative logic is usually solved analogically as in the systems of alethic modal logic (see f.i. [1], [2], [8]). On the certain set of possible worlds W there is chosen at least reflexive relation R, which marks - in the interpretation of modal logic attainability - in the interpretation of normative logic alternativity of certain possible world regard to given. The truth of formula in given possible world is conditioned by the truth of the formula in the attainable or alternative possible worlds. Against such an interpretation I have following objections:

1) For me, it does not seem to be adequate to assign truth values to elementary formulas of normative logic regardless their form. Any elementary formula of normative logic is symbolic form of (elementary) norm and norms have not truth

values, there is no sense to speak about true or false norm. But to be able to create compound norms by the help of classical connectives we need truth values as well as for formulation of consistent set of formulas of INL. Either we must accept "true" and "false" norms or to choose other characteristics and to extend of its values the domain of connectives of classical logic.

2) The interpretation of elementary formulas of normative logic by means of the relation of alternativity is able to distinquish between real possible world and it alternative deontically perfect possible worlds. This I consider insufficient especially for the interpretation of dyadic formulas - conditional compound of these formulas would be interpreted in other relation structure than normative compound. The first of them I call real system, the second goal - ideal system (see [9]), both are over the same domain of interpretation and they can be eventually disjunctive.

Creating my own formulation of semantics of the system on INL I take into account these presuppositions.

Let D to be nonempty set - domain of interpretation. Let us form over this set D two relation structures R and G. Let R and G differ at least in one relation, i.e. one structure contains a relation, which is not contained in the other or the same relation is defined in each of structures in a different way. Atomic formula p(x1, x2, ..., xn) of predicate logic we interpret in R (G) as following. We choose a subset p' so that $p' \in D \times D \times \ldots \times D$ (n-times) and for d1, ..., dn $\in$ D is p(d1, ..., dn) true in R (G) if and only if it is valid (d1, ..., dn) $\in p'$.

We interpret atomic formulas of INL as following:

O()[A/B] means, that formula A is true in G for all assignments of values to variables for which formula B is true in R or in G;

F()[A/B] means, that formula $\neg A$ is true in G for all assignments values to variables for which formula B is true in R or in G;

P()[A/B] means, that formula A is true in G at least for one assignment of values to variables for which formula B is true in R and in G.

Such an "ideal" interpretation corresponds to state, which is the purpose of norms. To interpretating person it just shows the possibility of realization of interpretated norm. Let us call the atomic formula of INL, which suits mentioned condition, valid in the given interpretation, i.e. regard to the set D and choise R and G, else let us call it nonvalid.

For next considerations we must extend classical propositional connectives to this new values. This modification we make analogically as it is for truth values. To truth value true corresponds "normative" value validity, to truth value false corresponds "normative" value nonvalidity. If we go on in using introduced symbols for logical connectives, then their interpretation will not be total functions, because we cannot afford a confusion of values. So conditional and normative compound of formulas of INL are interpreted in truth values, formulas of INL are interpreted in the "normative" values.

Let us call arbitrary formula of INL universally valid, if it is valid in each interpretation (over arbitrary domain).

I suppose the system of INL is not only adequate, but also maximally possible formalization of normative logic. Eventual defects of this system can be found out by machine applications, which is the aim of my next investigations.

SUMMARY

In this paper the conception of normative logic based on the first order predicate calculus is built. In the suggested formalization there is not possible to use existential quan-

tifier, i.e. the basis is not fully included into created system. Function of quantifiers is fulfilled even by deontic functors - they bound the variables which represent the addressees of norms. Semantics of the system is founded on the interpretation of norms in the real and the ideal system.

Souhrn

INDIVIDUALIZOVANÁ NORMATIVNÍ LOGIKA

V článku je vyložena koncepce normativní logiky vybudované na bázi predikátového počtu 1.řádu. V předložené formalizaci nelze používat existenční kvantifikátor, tedy báze budovaného systému není zahrnuta v plném rozsahu. Roli kvantifikátorů však plní i deontické funktory - jsou jimi vázány ty proměnné, které reprezentují adresáty norem. Sémantika systému je založena na interpretaci norem v reálném a cílovém systému.

Р е з ю м е

ИНДИВИДУАЛИЗОВАННАЯ ЛОГИКА НОРМ

В стати обяснена концепция логики построенной на базе предикатного исчисления первого порядка. В показанной формализации невозможно пользоваться квантором существования, значит эта база невключена в построенной системе в польном объёме. Но деонтические функторы тоже испольняют роль кванторов - ими связаны эти переменные, которые представляют адресатов норм. Семантика логики норм основана на интерпретации норм в реальной и идеальной системе.

REFERENCES

[1] K o s t j u k, V.N.: Elementy modalnoj logiki. Kijev 1978.

[2] H i n t i k k a, J.: Some Main Problems of Deontic Logic. In Hilpinen R.(Ed.): Deontic Logic: Introductory and Systematic Readings, Dodrecht 1971.

[3] von W r i g h t, G.H.: An Essay in Modal Logic. Amsterdam 1951.

[4] I v i n, A.A.: Logika norm. Moskva 1973.

[5] von W r i g h t, G.H.: A New System of Deontic Logic. In Hilpinen R.(Ed.): Deontic Logic: Introductory and Systematic Readings, Dordrecht 1971.

[6] Š t ě p á n, J.: Normativní logika s kvantifikátory. Sborník prací filosofické fakulty brněnské university (SPFFBU) B 30, 1983.

[7] Š t ě p á n, J.: Modifikace normativní logiky. SPFFBU B 32, 1985.

[8] C e l i š č e v, V.V.: Filosofskije problemy semantiki vozmožnych mirov. Novosibirsk 1977.

[9] Š t ě p á n, J.:Normativní teorie a realita. SPFFBU B 28, 1981.

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