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ON THE PROPERTIES OF CENTRAL DISPERSIONS OF LINEAR SECOND ORDER DIFFERENTIAL EQUATIONS BEING OF FINITE TYPE–SPECIAL

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The present article proceeds from Borůvka's theory of central dispersions of linear homogeneous second order differential equations in Jacobian form

$$y'' = q(t)y \tag{q}$$

treated at length for bothsided oscillatory equations on their definition interval. With respect to the definitions and to the properties of the individual kinds of the central dispersions it is impossible to carry over these concepts into the theory of equations of finite type automatically for the analogy of their utilization be preserved. In [2] there was introduced a certain generalization of concepts relating to all four kinds of the central dispersions considered for the equation (q) of finite type $m \geq 2$ - special and this, by means of the definitions of the special central dispersions of the appropriate kinds. There were also discussed conditions and properties of that generalization which may be summed up in the following definitions. The letter Z refers to a set of integers.

Definition 1.

By a (zm+k)-th special central dispersion of the first kind for $z \in Z$, $k = 0,1,\dots,m-1$ of the equation (q) being 1-special of finite type m on its definition interval $j = (a,b)$, for $q(t) \in C^{(0)}(j)$ (hereafter briefly $(q^{(1)})$) we mean a function

$$\Phi_{zm+k}(t) = \begin{cases} \varphi_k(t) & \text{for } t \in (a, a^{(1)}_{m-k}) \\ \varphi_{-(m-k)}(t) & \text{for } t \in (a^{(1)}_{m-k}, b) \end{cases}$$

where $\varphi(t)$ is the first kind central dispersion in terms of the definition stated in [1], whereby the points $a^{(1)}_i$ are the zeros of the 1-fundamental solution of the equation $(q^{(1)})$, forming the 1-fundamental sequence of this equation in the following ordering

$$a < a^{(1)}_1 < a^{(1)}_2 < \dots < a^{(1)}_{m-1} < b$$

Definition 2.

By a (zm+k)-th special central dispersion of the second kind for $z \in Z$, $k = 0,1,\dots,m-1$, of the equation (q) being 2-special of type m on the interval j, for $q(t) \in C^{(0)}(j)$ (hereafter briefly $(q^{(2)})$) we mean a function

$$\Psi_{zm+k}(t) = \begin{cases} \psi_k(t) & \text{for } t \in (a, a^{(2)}_{m-k}) \\ \psi_{-(m-k)}(t) & \text{for } t \in (a^{(2)}_{m-k}, b) \end{cases}$$

where $\psi(t)$ is the second kind central dispersion in terms of the definition stated in [1], whereby the points $a^{(2)}_i$ are the zeros of the derivative of the 2-fundamental solution of the equation $(q^{(2)})$ forming the 2-fundamental sequence $(a^{(2)})$ in the following ordering

$$a < a_1^{(2)} < a_2^{(2)} < \dots < a_{m-1}^{(2)} < b$$

Definition 3.

By a (zm+k)-th for $z \geq 0$ and by a (zm+k-1)-st for $z < 0$, $z \in Z$, $k = 1,2,\dots,m$ special central dispersion of the third kind of the equation (q) being 1-special of type m and at the same time 2-special of type m for $q(t) < 0$, $q(t) \in C^{(0)}(j)$ (hereafter briefly $(q^{(1,2)})$) we mean respectively the functions

$$\chi_{zm+k}(t) \text{ and } \chi_{zm+k-1}(t) = \begin{cases} \chi_k(t) & \text{for } t \in (a, a_{m-k+1}^{(3)}) \\ \chi_{-(m-k+1)}(t) & \text{for } t \in (a_{m-k+1}^{(3)}, b) \end{cases},$$

where $\chi(t)$ is the third kind central dispersion in terms of the definition stated in [1], whereby the points $a_i^{(3)}$ are the zeros of the 2-fundamental solution of the equation $(q^{(1,2)})$ forming the 3-fundamental sequence $(a^{(3)})$ in the following ordering

$$a < a_1^{(3)} < a_2^{(3)} < \dots < a_m^{(3)} < b$$

Definition 4.

By a (zm+k)-th for $z \geq 0$ and (zm+k-1)-st for $z < 0$, $z \in Z$, $k = 1,2,\dots,m$ special central dispersion of the fourth kind of the equation $(q^{(1,2)})$ we mean respectively the functions

$$\Omega_{zm+k}(t) \text{ and } \Omega_{zm+k-1}(t) = \begin{cases} \omega_k(t) & \text{for } t \in (a, a_{m-k+1}^{(4)}) \\ \omega_{-(m-k+1)}(t) & \text{for } t \in (a_{m-k+1}^{(4)}, b) \end{cases}$$

where $\omega(t)$ is the fourth kind central dispersion in terms of the definition stated in [1], whereby the points $(a_i^{(4)})$ are the zeros of derivative of the 1-fundamental solution of the equation $(q^{(1,2)})$ forming the 4-fundamental sequence $(a^{(4)})$ in the following ordering

$$a < a_1^{(4)} < a_2^{(4)} < \dots < a_m^{(4)} < b$$

In [2] there is investigated the algebraic structure of a set Γ related to all special central dispersions of the equation $(q^{(1,2)})$ defined in the domain J, i.e. on the interval j with the exception of the points of all four fundamental sequences. The set $\Gamma = G^{(1)} \cup G^{(2)} \cup G^{(3)} \cup G^{(4)}$ is a union of sets related to the special central dispersions of the individual kinds, where $G^{(1)} = \{\Phi_0, \Phi_1, \dots, \Phi_{m-1}\}$ and $G^{(2)} = \{\Psi_0, \Psi_1, \dots, \Psi_{m-1}\}$ are finite cyclic groups of order m with the generators Φ_1 and Ψ_1, respectively. Both groups have the unit element in common $\Phi_0 = \Psi_0 = t$ for all $t \in J$. The sets $G^{(3)} = \{\mathcal{X}_1, \mathcal{X}_2, \dots, \mathcal{X}_m\}$ and $G^{(4)} = \{\Omega_1, \Omega_2, \dots, \Omega_m\}$ comprise precisely m different elements, whereby the elements of the set $G^{(3)}$ are to those of the set $G^{(4)}$ inverse to each other. In the algebraic structure of the set there thus exists in the final dimense a certain analogy with the set of the central dispersions of the oscillatory differential equations (q).

In the following text we will observe relations of the special central dispersions of the individual kinds and their derivatives to the polar coordinates of the appropriate equation, taken in terms of the definition stated in [1]. Relations between the central dispersions and the phases of the oscillatory equations (q) are described by the Abel functional equations. Analogous also some modifications of these equations may be derived for an interpretation of the relations between the special central dispersions and the phases of the equations (q) of finite type, special, which is the contents of the following

Theorem 1.

1) For an arbitrary first phase α of the equation $(q^{(1)})$

and an arbitrary special central dispersion of the first kind Φ_k of this equation, where k = 0,1,...,m-1, there is fulfilled the relation

$$\alpha[\Phi_k(x)] = \begin{cases} \alpha(x) + \varepsilon k \pi & \text{for } x \in (a, a^{(1)}_{m-k}) \\ \alpha(x) - \varepsilon(m-k)\pi & \text{for } x \in (a^{(1)}_{m-k}, b) \end{cases}, \quad (1)$$

where $\varepsilon = \operatorname{sign} \alpha'$, $a^{(1)}_k \in (a^{(1)})$, $a^{(1)}_m = b$.

2) For an arbitrary second phase β of the equation $(q^{(2)})$ and an arbitrary central dispersion of the second kind Ψ_k of this equation, where k = 0,1,...,m-1, there is fulfilled the relation

$$\beta[\Psi_k(x)] = \begin{cases} \beta(x) + \varepsilon k \pi & \text{for } x \in (a, a^{(2)}_{m-k}) \\ \beta(x) - \varepsilon(m-k)\pi & \text{for } x \in (a^{(2)}_{m-k}, b) \end{cases}, \quad (2)$$

where $\varepsilon = \operatorname{sign} \beta'$, $a^{(2)}_k \in (a^{(2)})$, $a^{(2)}_m = b$.

3) For every first phase α and for the second phase β of the same basis (u,v) of the equation $(q^{(1,2)})$ satisfying the relation $0 < \beta(x) - \alpha(x) < \pi$ or the relation $-\pi < \beta(x) - \alpha(x) < 0$ and for the k-th special central dispersion of the third find X_k of the same equation, where k = 1,2,...,m, there is respectively fulfilled:

$$\beta[X_k(x)] = \begin{cases} \alpha(x) + \frac{1}{2}\left[(2k-1)\varepsilon + 1\right]\pi & \text{for } x \in (a, a^{(3)}_{m-k+1}) \\ \alpha(x) + \frac{1}{2}\left[-(2m-2k+1)\varepsilon + 1\right]\pi & \text{for } x \in (a^{(3)}_{m-k+1}, b) \end{cases}, (3)$$

or

$$\beta[X_k(x)] = \begin{cases} \alpha(x) + \frac{1}{2}\left[(2k-1)\varepsilon - 1\right]\pi & \text{for } x \in (a, a^{(3)}_{m-k+1}) \\ \alpha(x) + \frac{1}{2}\left[-(2m-2k+1)\varepsilon - 1\right]\pi & \text{for } x \in (a^{(3)}_{m-k+1}, b) \end{cases}, (4)$$

where $\varepsilon = \operatorname{sign}\alpha' = \operatorname{sign}\beta'$, $a_k^{(3)} \in (a^{(3)})$.

4) For every first phase α and for the second phase β of the same basis (u,v) of the equation $(q^{(1,2)})$ satisfying the relation $0 < \beta(x) - \alpha(x) < \pi$ or the relation $-\pi < \beta(x) - \alpha(x) < 0$ and for the k-th special central dispersion of the fourth kind Ω_k of the same equation, where $k = 1,2,\dots,m$, there is respectively fulfilled:

$$\alpha[\Omega_k(x)] = \begin{cases} \beta(x) + \frac{1}{2}[(2k-1)\,\varepsilon - 1]\pi & \text{for } x \in (a, a^{(4)}_{m-k+1}) \\ \beta(x) + \frac{1}{2}[-(2m-2k+1)\,\varepsilon - 1]\pi & \text{for } x \in (a^{(4)}_{m-k+1}, b) \end{cases} , \qquad (5)$$

or

$$\alpha[\Omega_k(x)] = \begin{cases} \beta(x) + \frac{1}{2}[(2k-1)\,\varepsilon + 1]\pi & \text{for } x \in (a, a^{(4)}_{m-k+1}) \\ \beta(x) + \frac{1}{2}[-(2m-2k+1)\,\varepsilon + 1]\pi & \text{for } x \in (a^{(4)}_{m-k+1}, b) \end{cases} , \qquad (6)$$

where $\varepsilon = \operatorname{sign}\alpha' = \operatorname{sign}\beta'$, $a_k^{(4)} \in (a^{(4)})$.

Proof: 1) Let α be a first phase of the basis (u,v) of the equation $(q^{(1)})$, $x \in j$ an arbitrary point, $x \neq a^{(1)}_{m-k}$. Consider such a solution y of the equation $(q^{(1)})$ which has a zero in x. By relation (27) § 5 in [1] it is possible to express this solution in j as

$$y(t) = k\, r(t) \sin[\alpha(t) - \alpha(x)] , \qquad (7)$$

where $k \neq 0$ is a certain constant, r(t) is the first amplitude of the basis (u,v). Denoting $A(t) = \alpha(t) - \alpha(x)$ it holds $A(x) = 0$, and the function A(t) is everywhere increasing or everywhere decreasing according as $\varepsilon = 1$ or $\varepsilon = -1$. Consider next the point x_1 at which the function A(t) takes the values $\varepsilon k \pi$ or $-(m-k)\varepsilon\pi$ respectively. For $x \in (a, a^{(1)}_{m-k})$ and $x \in (a^{(1)}_{m-k}, b)$ such a point really exists, and it follows from (7) that this is the k-th and the (m-k)-th zero lying to the right and to the left of the solution y, respectively. So, it holds $x_1 = \Phi_k(x)$ and thus also the first part of the assertion given.

2) Let β be a second phase of the basis (u,v) of the equation $(q^{(2)})$, $x \in j$, $x \neq a^{(2)}_{m-k}$ an arbitrary point. Consider such a solution y of the equation $(q^{(2)})$, whose derivative has a zero in x. By relation (27) § 5 in [1] it is possible to express this derivative throughout the interval j as

$$y'(t) = \pm k\, s(t) \sin\left[\beta(t) - \beta(x)\right] , \tag{8}$$

where $k \neq 0$ is a certain constant, s(t) is the second amplitude of the basis (u,v). Denoting $B(t) = \beta(t) - \beta(x)$, yields $B(x) = 0$, and the function B(t) is everywhere increasing or everywhere decreassing according as $\varepsilon = 1$ or $\varepsilon = -1$. Consider next the point x_2 at which the function B(t) takes the values $\varepsilon k \pi$ or $-(m-k)\varepsilon\pi$ respectively. For $x \in (a, a^{(2)}_{m-k})$ and $x \in (a^{(2)}_{m-k}, b)$ such a point really exists, and it follows from (8) that this is the k-th and (m-k)-th zero lying on the right and on the left of the solution y, respectively. So, it holds $x_2 = \Psi_k(x)$ and thus also the second part of the assertion given.

3) Let α and β be the first and the second phase of a basis (u,v) of the equation $(q^{(1,2)})$ such that the condition $0 < \beta(x) - \alpha(x) < \pi$ is fulfilled. Let $x \in j$, $x \neq a^{(3)}_{m-k+1}$ be arbitrary. Consider such a solution y of the equation $(q^{(1,2)})$ which has a zero in x. By relation (27) § 5 in [1] it is possible to express the derivative of this solution as

$$y'(t) = \pm k\, s(t) \sin\left[\beta(t) - \alpha(t)\right], \tag{9}$$

where $k \neq 0$ is a certain constant. Denote now $C(t) = \beta(t) - \alpha(x)$. The function C(x) is either an increasing or a decreasing function everywhere in j, according as $\varepsilon = 1$ or $\varepsilon = -1$. Let $\varepsilon = 1$. Consider now the point x_3 at which the function C(t) takes the values $k\pi$ or $-(m-k)\pi$. For

$x \in (a, a_{m-k+1}^{(3)})$ resp. $x \in (a_{m-k+1}^{(3)}, b)$ such a point really exists and it becomes apparent from (9) that this is the k-th or the (m-k+1)-st zero of the function y lying to the right or to the left of x. So, it holds $x_3 = \chi_k(x)$ and thus also the third part of the assertion with the given condition. Analogous may be proved the validity of (3) for $\varepsilon = -1$ as well as that of (4) for $-\pi < \beta(x) - \alpha(x) < 0$.

4) Let α, β be the first and the second phase of the same basis (u,v) of the equation $(q^{(1,2)})$ satisfying the condition $0 < \beta(x) - \alpha(x) < \pi$. Let $x \in j$, $x \neq a_{m-k+1}^{(4)}$ be arbitrary. Consider such a solution y whose derivative has a zero in x. By (27) § 5 in [1] this solution may be expressed as

$$y(t) = k\, r(t) \sin[\alpha(t) - \beta(x)] , \tag{10}$$

where $k \neq 0$ is a certain constant. Denote now $D(t) = \alpha(t) - \beta(x)$. The function D(t) is either an increasing or a decreasing function everywhere in j, according as $\varepsilon = 1$ or $\varepsilon = -1$. Let $\varepsilon = 1$. Consider next the point x_4 at which the function D(t) takes the values $(k-1)\pi$ or $-(m-k+1)\pi$, respectively. For $x \in (a, a_{m-k+1}^{(4)})$ or $x \in (a_{m-k+1}^{(4)}, b)$ such a point really exists and it becomes apparent from (10) that this is the k-th or (m-k+1)-st zero of the solution y lying to the right or to the left of x, respectively. So, it holds $x_4 = \Omega_k(x)$ and thus also (5) in the fourth part of the Theorem proved. Analogous may also be proved the validity of (5) for $\varepsilon = -1$ or that of (6) for $-\pi < \beta(x) - \alpha(x) < 0$.

In [2] there is stated that the special central dispersions of the first kind of $(q^{(1)})$ represent the functions of $C^{(3)}$ on both definition intervals; the special central dispersions of the second kind or of the third and fourth kinds of $(q^{(2)})$ or $(q^{(1,2)})$, respectively, represent the functions of $C^{(1)}$ on both definition intervals. The following Theorem discusses the expressions of the first derivative of these functions.

Theorem 2

1) The first derivative of the first kind special central dispersion $\Phi_k(t)$ of the equation $(q^{(1)})$ has for all $t \in j$, $t \neq a^{(1)}_{m-k}$ the following expressions

$$\text{a)}\quad \Phi'_k(t) = -\frac{u'(t)\, v[\Phi_k(t)] - u[\Phi_k(t)]\, v'(t)}{u(t)\, v'[\Phi_k(t)] - u'[\Phi_k(t)]\, v(t)}\,, \tag{11}$$

where u,v are two arbitrary independent solutions of the equation $(q^{(1)})$;

$$\text{b)}\quad \Phi'_k(t) = \begin{cases} \dfrac{u^2[\Phi_k(t)]}{u^2(t)} & \text{for}\quad u(t) \neq 0 \\[2ex] \dfrac{u'^2(t)}{u'^2[\Phi_k(t)]} & \text{for}\quad u(t) = 0 \end{cases} \tag{12}$$

where u(t) is an arbitrary solution of the equation $(q^{(1)})$;

$$\text{c)}\quad \Phi'_k(t) = \frac{\alpha'(t)}{\alpha'[\Phi_k(t)]}\,, \tag{13}$$

where α is an arbitrary first phase of the equation $q^{(1)}$.

2) The first derivative of the second order special central dispersion $\Psi_k(t)$ of the equation $(q^{(2)})$ has for all $t \in j$, $t \neq a^{(2)}_{m-k}$, $k \in \{0,1,\dots,m-1\}$ the following expressions:

$$\text{a)}\quad \Psi'_k(t) = -\frac{q(t)}{q[\Psi_k(t)]}\,\frac{u(t)\, v'[\Psi_k(t)] - u'[\Psi_k(t)]\, v(t)}{u'(t)\, v[\Psi_k(t)] - u[\Psi_k(t)]\, v'(t)} \tag{14}$$

where u,v are two arbitrary independent solutions of the equation $(q^{(2)})$;

b) $$\psi_k'(t) = \begin{cases} \dfrac{q(t)}{q[\psi_k(t)]} \dfrac{u'^2[\psi_k(t)]}{u'^2(t)} & \text{for } u'(t) \neq 0 \\[2ex] \dfrac{q(t)}{q[\psi_k(t)]} \dfrac{u^2(t)}{u^2[\psi_k(t)]} & \text{for } u'(t) = 0 \end{cases} \tag{15}$$

where u is an arbitrary solution of the equation $q^{(2)}$;

c) $$\psi_k'(t) = \frac{\beta'(t)}{\beta'[\chi_k(t)]} \quad , \tag{16}$$

where β is an arbitrary second phase of the equation $(q^{(2)})$.

3) The first derivative of the third kind special central dispersion $\chi_k(t)$ of the equation $(q^{(1,2)})$ has for all $t \in j$, $t \neq a^{(3)}_{m-k+1}$, $k \in \{1,2,\dots,m\}$ the following expressions:

a) $$\chi_k'(t) = - \frac{1}{q[\chi_k(t)]} \frac{u'(t)\, v'[\chi_k(t)] - u'[\chi_k(t)]\, v'(t)}{u(t)\, v[\chi_k(t)] - u[\chi_k(t)]\, v(t)} \tag{17}$$

where u,v are two arbitrary independent solutions of the equation $(q^{(1,2)})$;

b) $$\chi_k'(t) = \begin{cases} - \dfrac{1}{q[\chi_k(t)]} \dfrac{u'^2[\chi_k(t)]}{u^2(t)} & \text{for } u(t) \neq 0 \\[2ex] - \dfrac{1}{q[\chi_k(t)]} \dfrac{u'^2(t)}{u^2[\chi_k(t)]} & \text{for } u(t) = 0 \end{cases} \quad , \tag{18}$$

where u is an arbitrary solution of $(q^{(1,2)})$;

c) $$\chi_k'(t) = \frac{\alpha'(t)}{\beta'[\chi_k(t)]} \quad , \tag{19}$$

where α, β are first and second phases of the same basis of the equation $(q^{(1,2)})$.

4) The first derivative of the fourth kind special central dispersion $\Omega_k(t)$ of the equation $(q^{(1,2)})$ has for all $t \in j$, $t \neq a^{(4)}_{m-k+1}$, $k \in \{1,2,\dots,m\}$ the following expressions:

$$\text{a)} \quad \Omega_k'(t) = -\,q(t)\,\frac{u(t)\,v[\Omega_k(t)] - u[\Omega_k(t)]\,v(t)}{u'(t)v'[\Omega_k(t)] - u'[\Omega_k(t)]\,v'(t)}\,, \tag{20}$$

where u,v are two arbitrary independent solutions of the $(q^{(1,2)})$

$$\text{b)} \quad \Omega_k'(t) = \frac{\beta'(t)}{\alpha'[\Omega_k(t)]}\,, \tag{21}$$

where α, β are the first and second phases of the same basis of the equation $(q^{(1,2)})$;

$$\text{c)} \quad \Omega_k'(t) = \begin{cases} -\,q(t)\,\dfrac{u^2[\Omega_k(t)]}{u'^2(t)} & \text{for } u'(t) \neq 0 \\[2ex] -\,q(t)\,\dfrac{u^2(t)}{u'^2[\Omega_k(t)]} & \text{for } u'(t) = 0 \end{cases} \tag{22}$$

where u is an arbitrary solution of $(q^{(1,2)})$.

Proof. In proving the validity of the individual assertions a), b) of Theorem 2 we may proceed from the fact the relations here were deduced fully in [1] § 13 for the central dispersions with an arbitrary integral index of the appropriate kinds of equations (q) being oscillatory, whereby the oscillatority condition of the equation was adopted only for the existence of the relative dispersions. In the eigen proofs was tooked advantage only of the ideas saying that the points t, $\varphi(t)$ are 1-conjugate, the points t, $\psi(t)$ are 2-conjugate, the point $\chi(t)$ is 3-conjugate to the point t and the point $\omega(t)$ is 4-conjugate to the point t.

Thus, with respect to the definitions of the special central dispersions, we see that the validity of all above mentioned relations on the corresponding definition intervals may be carried over directly in an appropriate analogy. The validity of the functional relations (13), (16), (19), (21) follows directly from the statement of Theorem 1.

Souhrn

K VLASTNOSTEM CENTRÁLNÍCH DISPERZÍ LINEÁRNÍCH DIFERENCIÁLNÍCH ROVNIC 2.ŘÁDU KONEČNÉHO TYPU - SPECIÁLNÍCH

Text článku vychází z Borůvkovy teorie centrálních disperzí lineárních diferenciálních rovnic 2.řádu $y'' = q(t)y$ podrobně rozpracované pro rovnice oboustranně oscilatorické. V článku [2] zavedla autorka jisté zobecnění pojmů centrálních disperzí všech uvažovaných druhů pro rovnice konečného typu $m \geq 2$ - speciální a diskutovala podmínky a vlastnosti takovýchto zobecnění. Zde předkládaný text je věnován vyšetření vztahů ve [2] definovaných speciálních centrálních disperzí všech čtyř druhů a jejich derivací k polárním souřadnicím rovnice. Jsou odvozeny modifikace Abelových funkcionálních rovnic.

Р е з ю м е

О СВОЙСТВАХ ЦЕНТРАЛЬНЫХ ДИСПЕРСИЙ ДЛИНЕЙНЫХ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ 2-ого ПОРЯДКА, КОНЕЧНОГО ТИПА-СПЕЦИАЛЬНЫХ

Текст этой статьи всходит из теории центральных дисперсий для линейных дифференциальных уравнений 2-ого норядка $y'' = q(t)y$, основаной на литературе [1] для уравнений с осцилирующими решениями. В статье [2] введено одно обобщение понятий центральных дисперсий отдельных родов для уравнения конечного типа $m \geq 2$, специального и исследованы условия и свойства этих обобщений. Целью этой статьи расследование отношений на литературе [2] определенных специальных центральных дисперсий отдельных родов и их производных к полярным координатам уравнения. Здесь выведены модификации функциональных уравнений Абеля.

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