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Alena Vanžurová

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CONSTRUCTION OF ONE EPIMORPHISM OF PROJECTIVE PLANES OVER CARTESIAN GROUPS

ALENA VANŽUROVÁ

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In this paper we shall construct a non-trivial example of an epimorphism for projective planes of a very general type.

In [3], generalized formal Laurent series were investigated. It was proved that a set of all Laurent series with coefficients in a given commutative Cartesian group T and exponents in an ordered loop L endowed with suitable addition and multiplication is again a commutative Cartesian group. We shall show here that there is a place of D(T,L) onto an original Cartesian group T. According to the general theory, this place induces an epimorphism of corresponding non-desarquesian planes over Cartesian groups.

Let T be a Cartesian group with commutative addition. Let $(L, \leq)$ be an ordered loop with unit element denoted by e. Let us recall that D(T,L) consists of all mappings $f: L \longrightarrow T$

with support spt(f) well ordered by the linear ordering $\leq$ of L. This set forms a commutative Cartesian group provided we define

$$(f+g)(x) = f(x) + g(x),$$

$$(f.g)(x) = \sum_{y+z=x} f(y).g(z)$$

for all $x \in L$. Now let us define a mapping $\varphi : D(T,L) \rightarrow T \cup \{\infty\}$ where $\infty \notin T$, in the following way:

$\varphi(f) = f(e)$ if $f(x) = 0$ for all $x < e, x \in L$,

$\varphi(f) = \infty$ if there exists $x_0 < e, x_0 \in L$ with $f(x_0) \neq 0$.

Proposition. The mapping φ is a place [1] of commutative Cartesian groups.

P r o o f. Let. $\varphi(f) \neq \infty$, $\varphi(g) \neq \infty$. Then $f(x) = g(x) = 0$ for all $x < e$. Thus $(f-g)(x) = 0$ for all $x < e$. Hence $\varphi(f-g) \neq \infty$. This gives $\varphi(f-g) = (f-g)(0) = f(0) - g(0) = \varphi(f) - \varphi(g)$. Since $(f.g)(x) = \sum_{\xi+\eta=x} f(\xi).g(\eta) = 0$ for every $x < e$, it must be $\varphi(f.g) \neq \infty$. A simple computation shows that $\varphi(f.g) = \varphi(f).\varphi(g)$.

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1) A place of a Cartesian group T onto a Cartesian group $\bar{T}$ is a mapping $\varphi : T \rightarrow \bar{T} \cup \{\infty\}$, $\infty \notin \bar{T}$, such that

(1) If $\varphi(f), \varphi(g) \neq \infty$ then $\varphi(f-g) = \varphi(f) - \varphi(g)$, $\varphi(f.g) = \varphi(f).\varphi(g)$.

(2) If $\varphi(f) \neq 0$ and $\varphi(g) = \infty$ then $\varphi(f.g) = \varphi(g.f) = \infty$.

(3) If $\varphi(-f.g+h.g) \neq \infty$ and $\varphi(g) = \infty$ then $\varphi(f) = \varphi(h)$.

(4) If $\varphi(f.g-f.k) \neq \infty$ and $\varphi(f) = \infty$ then $\varphi(g) = \varphi(k)$.

(5) If $h.g+f.k = f.g$ and $\varphi(f) = \varphi(g) = \varphi(h.g) = \varphi(f.k) = \infty$ then either $\varphi(h) = \infty$ or $\varphi(k) = \infty$.

Now suppose $\varphi(f) \neq 0$, $\varphi(g) = \infty$. Denote $x_1 = \min \operatorname{spt}(f)$ and $x_2 = \min \operatorname{spt}(g)$. In our case, $x_1 < e$, $x_2 < e$. Since $\min \operatorname{spt}(f.g) = \min \operatorname{spt}(f) + \min \operatorname{spt}(g) = x_1 + x_2 < e$ and similarly for g.f, we obtain $\varphi(f.g) = \varphi(g.f) = \infty$.

Assume $\varphi(-f.g+h.g) \neq \infty$ and $\varphi(g) = \infty$. Let x_1 and x_2 be as above. Then $x_2 < e$. First let $\varphi(f) = \infty$. We have $x_1 < e$ and $x_1 + x_2 < e$ because of $\varphi(f.g) = \infty$. Further, $0 = (-f.g+h.g)(x_1+x_2) = -f(x_1).g(x_2) + \sum h(\xi).g(\eta)$, where $\xi \leq x_1$; $x_2 \leq \eta$ and $\xi + \eta = x_1+x_2$. This yields $\min \operatorname{spt}(h) \leq \leq x_1$, and $\varphi(h) = \infty$. Similarly, $\varphi(h) = \infty$ implies $\varphi(f) = = \infty$. Consequently, $\varphi(f) = \infty$ if and only if $\varphi(h) = \infty$. Now assume $\varphi(f) \neq \infty$. If $\varphi(f) = f(e) \neq 0$, that is, $x_1 = e$, then $0 = (-f.g+h.g)(x_2) = -f(e).g(x_2) + h(e).g(x_2)$, where $\xi + \eta = x_2$, $\xi \leq e$ and $x_2 \leq \eta$. From $g(x_2) \neq 0$ it follows $f(e) = h(e) = 0$, $\varphi(f) = \varphi(h)$. Thus the equality holds in both cases.

Let $h.g+f.k = f.g$ and let $\varphi(f) = \varphi(g) = \varphi(h.g) = = \varphi(f.k) = \infty$. Let x_1 and x_2 be as above and $x_3 = \min \operatorname{spt}(h)$, $x_4 = \min \operatorname{spt}(k)$. It holds $\min(x_3+x_2, x_1+x_4) \leq x_1+x_2$. Let us suppose that $e \leq x_3$ and $e \leq x_4$. Then the unequalities $x_1 < e \leq x_3$ and $x_2 < e \leq x_4$ imply $x_1+x_2 < x_3+x_2$, $x_1+x_2 < x_1+x_4$. This gives $x_1+x_2 < \min(x_3+x_2, x_1+x_4)$, a contradiction. Hence either $x_3 < e$ or $x_4 < e$, that is, either $\varphi(h) = \infty$ or $\varphi(k) = \infty$. This proves that φ is a place of commutative Cartesian groups.

<u>Corollary</u>. Let T be a commutative Cartesian group but not a skew-field. Then φ induces an epimorphism of the corresponding non-desarguesian plane over D(T,L) onto the non-desarguesian projective plane over T.

<u>Remark</u>. Let T be a (non-associative) ring without divisors of zero. Then D(T,L) is a ring with the same properties and the above mapping φ is a place of a ring D(T,L) to T. Moreover, a function $V: D(T,L) - \{0\} \longrightarrow L$ given by

$V(f) = \min spt(f)$

is a valuation on $D(T,L)$ with a value loop L.

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KONSTRUKCE JEDNOHO EPIMORFISMU PROJEKTIVNÍCH ROVIN NAD KARTÉZSKÝMI GRUPAMI

Souhrn

V článku je použito komutativní kartézské grupy formálních mocninných (Laurentových) řad s koeficienty v libovolné komutativní kartézské grupě a exponenty v uspořádané lupě ke konstrukci netriviálních epimorfismů projektivních rovin velmi obecného typu.

КОНСТРУКЦИЯ ОДНОГО ЭПИМОРФИЗМА ПРОЕКТИВНЫХ ПЛОСКОСТЕЙ НАД КАРТЕЗИАНСКИМИ **ГРУППАМИ**

Резюме

В статье применяется конструкция обобщенных степенных рядов с экспонентами в упорядоченной лупе и коэффициентами в коммутативной картезианской группе к построению нетривиального эпиморфизма проективных плоскостей над картезианскими группами.

Author's address:
RNDr. Alena Vanžurová, CSc.
přírodovědecká fakulta
Univerzity Palackého
Leninova 26
771 46 Olomouc
ČSSR /Czechoslovakia/

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