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INCREASE OF ACCURACY IN MODELLING THE QUOTIENT OF SMALL VALUES BY COMPUTERS MEDA 40TA, MEDA 41TC AND ADT 3000

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Modelling the quotient of two functions $z(x) = \frac{f(x)}{g(x)}$ is carried out by solving implicate functions $F(z(x), f(x), g(x)) \equiv f(x) - z(x)\,g(x) = 0$ (see figure 1). It must hold $g(x) < 0$ for the computing network to be stable. In modelling the quotient

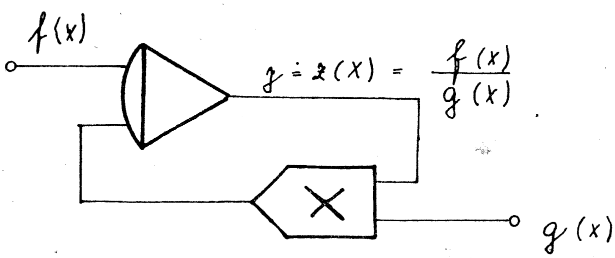

Fig. 1

by a diode (quadrator) multiplier arises a certain inaccuracy mainly given by the inaccuracy of this multiplier, for it forms the product on the basis of the relation $uv = \left(\frac{u+v}{2}\right)^2 - \left(\frac{u-v}{2}\right)^2$, where the quadratic dependences are approximatid by the linear dependences. A way of increasing the accuracy of the quotient of small values on multiplying out the quotient by a convenient coefficient in cases of equidistant distributions of break points of a broken line approximating the quadratic dependences at a diode multiplier was described in [1]. There was shown that the greatest errors occur when both quadrators are working in the same linear sections. The equidistant distribution of break points approximating broken lines in the approximation of quadratic dependences complies with the requirement for the best uniform approximation, where the maximal absolute values of errors in all sections are the same. Sonce multiplier do not apply the best uniform approximation of quadratic dependences (but, for instance, approximations close to the uniform approximation) as it is the case with the computers MEDA 40TA, MEDA 41TC and ADT 3000, when the break points are generally distributed. (See (16), (17), (18)).

Let us assume for simplicity that $f(x) = mx^\nu$, $g(x) = x^\nu$, $m =$ const. > 0, $\nu > 0$, or $f(x) = -mx^\nu$, $g(x) = -x^\nu$, for $g(x) \in \langle -1, 0)$ must hold for $g(x)$ to ensure the stability of the computing network and not to exceed the machine unit. For the output value y of the dividing circuit then

$$y \doteq z = \frac{mx^\nu}{x^\nu} = m \tag{1}$$

holds due to the inaccuracy of the dividing circuit. Both quadrators can work in thesame sections from certains values x, for

$$\lim_{x \to 0} \frac{|g(x) + y|}{2} = \lim_{x \to 0} \frac{|g(x) - y|}{2} = \frac{y}{2} \doteq \frac{m}{2}, \tag{2}$$

whereby the value of the expression $\frac{|g(x) + y|}{2}$ is approaching the value $\frac{m}{2}$ from above. (We assume $g(x) < 0$, $f(x) < 0$, $y \doteq m > 0$, $|g(x)| < |y|$.) From this value x, at which both quadrators work in the same section, marked errors arise in the output value. The values x will be determined for our cases ($g(x) \in \langle -1; 0)$, $y \doteq m > 0$, $|g(x)| < |y|$) by figure 2 as follows: The value of the expression $\frac{|g(x) + m|}{2}$ is approaching the value $\frac{m}{2}$ from balow, the value of the expression $\frac{|g(x) - m|}{2}$ from above, so that the following relations must be valid for both quadrators to work in the same sections:

$$\frac{|g(x) + m|}{2} \geqq x_j,$$

$$\frac{|g(x) - m|}{2} \leqq x_{j+1}, \tag{3}$$

where x_j, x_{j+1} are the break points of the broken line approximating the quadrators dependences (see figure 2).

Relations (3) may be written for our case $g(x) < 0$, $m > 0$, $|g(x)| < m$ in the form

$$\frac{g(x) + m}{2} \geqq x_j,$$
$$\frac{-g(x) + m}{2} \leqq x_{j+1}. \tag{4}$$

With some modification we get

$$g(x) \geqq 2x_j - m, \tag{5}$$
$$g(x) \geqq m - 2x_{j+1},$$

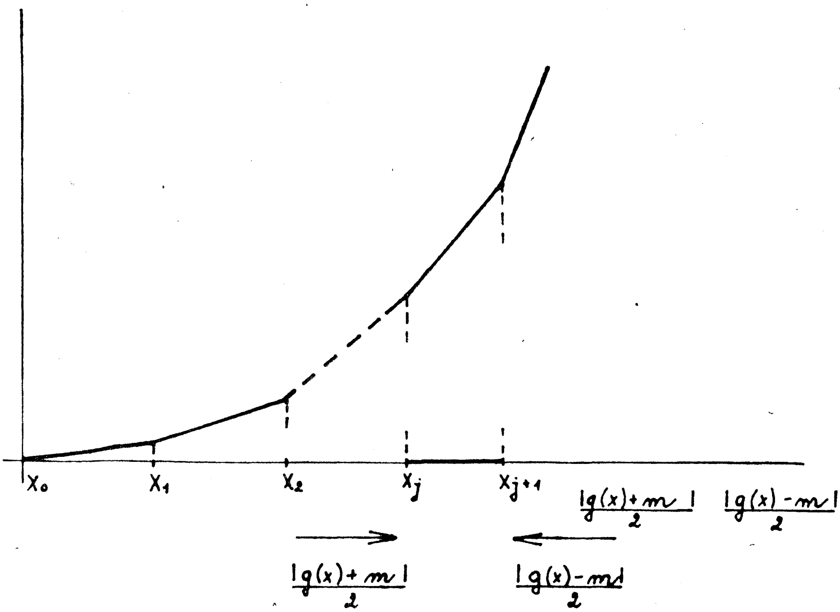

Fig. 2

Most adverse cases occur when the relations of (5) are fulfilled for the minimal possible $g(x)$ ($g(x) < 0$) and they occur if

$$2x_j - m = m - 2x_{j+1}, \tag{7}$$

i.e. in the values m given by the relation

$$m = x_j + x_{j+1}. \tag{8}$$

With values m by (5) and (8) we get

$$g(x) \geqq 2x_j - m = 2x_j - x_j - x_{j+1} = x_j - x_{j+1},$$
$$g(x) \geqq m - 2x_{j+1} = x_j + x_{j+1} - 2x_{j+1} = x_j - x_{j+1},$$

i.e. both quadrators are working in the same sections just from the value of the denominator

$$g(x) = x_j - x_{j+1}. \tag{9}$$

In the function $f(x)$ and $g(x)$ are in the form $f(x) = -m\,|\,x\,|^{\nu}$, $g(x) = -|\,x\,|^{\nu}$, the both quadrators are working for the values m given by (8) on the basis of (9) in the same section for the values x given by the relation

$$|\,x\,| \leqq \sqrt[\nu]{|\,x_j - x_{j+1}\,|}. \tag{10}$$

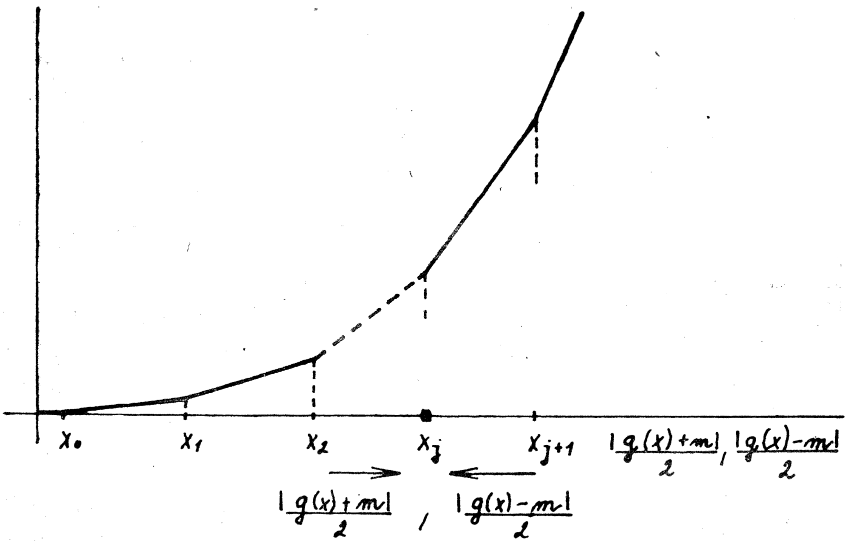

Fig. 3

These cases are especially adverse, because $0 < |\,x_j - x_{j+1}\,| < 1$ and with ν being increasing, the value of (10) increases. Ideal cases arise if in modelling the quotient both quadrators begin to work in the same sections not before $g(x) = 0$. By figure 3 they arise if

$$\lim_{x \to 0} \frac{|\,g(x) + m\,|}{2} = \lim_{x \to 0} \frac{|\,g(x) - m\,|}{2} = \frac{m}{2} = x_j \tag{11}$$

i.e. with the values

$$m = 2x_j. \tag{12}$$

Figure 4 illustrates an elementary diode eliminator of the quadrator of computers MEDA 40TA, MEDA 41TC and ADT 3000. The diode begin to release the current at the tension $u_a = u_{\text{sat}}$, where

$$u_a = \frac{(u + v)\,r_j - 10R_j}{r_j}. \tag{13}$$

The quantity of input values u and v at which the diode begins to release (the co-ordinates of the break points) will be determined from (13)

$$u + v = \frac{u_a(R_j + r_j) + 10R_j}{r_j},$$

i.e.

$$\frac{u + v}{2} = \frac{u_a(R_j + r_j) + 10R_j}{2r_j}. \tag{14}$$

If the quantities are repressed in the machine units, then

$$\frac{u + v}{2} = \frac{0{,}1u_a(R_j + r_j) + R_j}{2r_j} = x_{j-1}, \tag{15}$$

The computer MEDA 40TA (quadrator TDQ-1) has the values for resistance $r_1 = \infty$, $r_2 = 2617$ k, $r_3 = 770$ k, $r_4 = 463$ k, $r_5 = 330$ k, $r_6 = 256$ k, $r_7 = 210$ k,

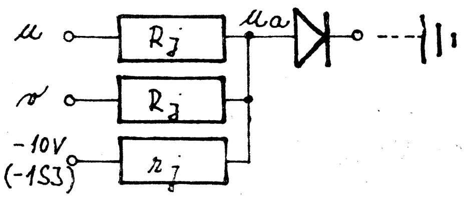

Fig. 4

$r_8 = 178$ k, $r_9 = 153.9$ k, $r_{10} = 136.2$ k, $R_1 = 366$ k, $R_2 = 2443$ k, $R_3 = R_4 = \ldots = R_{10} = 220$ k. If we apply the diodes KA 50, where $u_{sat} = 0.4$ V, then the first break points (assuming $m \leqq 1$ – see (8) and (12)) are

$$\begin{aligned} x_0 &= 0.020, & x_4 &= 0.366, \\ x_1 &= 0.068, & x_5 &= 0.466, \\ x_2 &= 0.168, & x_6 &= 0.564, \\ x_3 &= 0.267, \end{aligned} \tag{16}$$

Using the quadrator TDQ-2, where other values of resistance r_j and R_j are, we get

$$\begin{aligned} x_0 &= 0.020, & x_3 &= 0.312, \\ x_1 &= 0.118, & x_4 &= 0.409, \\ x_2 &= 0.218, & x_5 &= 0.503. \end{aligned} \tag{16a}$$

Analogous can be determined the coordinates of break points for the quadrator of the computer MEDA 41TC, where $r_1 = \infty$, $r_2 = 1157$ k, $r_3 = 561$ k1, $r_4 = 363$ k, $r_5 = 271$ k6, $r_6 = 217$ k, ..., $R_1 = 257$ k8, $R_2 = 220$ k, $R_3 = 220$ k, $R_4 = 220$ k, $R_5 = 220$ k, $R_6 = 220$ k ... and using here the diodes KA 50 then the first break points are

$$x_0 = 0.020, \quad x_3 = 0.335,$$
$$x_1 = 0.118, \quad x_4 = 0.440, \tag{17}$$
$$x_2 = 0.223, \quad x_5 = 0.547$$

and for the computer ADT 3000, using the diodes KA 502, that is $u_a \doteq 0$ and the values of resistance $r_1 = 700$ k53, $r_2 = 276$ k39, $r_3 = 174$ k01, $r_4 = 125$ k22, $r_5 = 98$ k86, $r_6 = 81$ k93, ..., $R_1 = 115$ k04, $R_2 = 97$ k56, $R_3 = 97$ k90, $R_4 = 96$ k68, $R_5 = 97$ k03, $R_6 = 97$ k88, ..., we get

$$x_0 = 0.082, \quad x_3 = 0.386,$$
$$x_1 = 0.176, \quad x_4 = 0.490, \tag{18}$$
$$x_2 = 0.281, \quad x_5 = 0.597.$$

In modelling the quotient $\frac{mx^\nu}{x^\nu}$ there occur favourable cases on the computer MEDA 40TA by (12), (16) and (16a) if

$$m = 0.136; 0.336; 0.534; 0.732; 0.932 \text{ (TDQ-1)}, \tag{19}$$
$$m = 0.236; 0.436; 0.624; 0.818; 1.006 \text{ (TDQ-2)}, \tag{19a}$$

with the computer MEDA 41TC by (12) and (17) if

$$m = 0.236; 0.446; 0.670; 0.880 \tag{20}$$

and with the computer ADT 3000 if

$$m = 0.164; 0.352; 0.562; 0.772; 0.980, \tag{21}$$

where m_{max} is chosen $m_{max} = 1$ for not exceeding the machine unit.

Most unfavourable cases occur by (8), (16), (16a), (17) and (18) with the computer MEDA 40TA if

$$m = 0.088; 0.236; 0.435; 0.633; 0.832; 1.030 \text{ (TDQ-1)}, \tag{22}$$
$$m = 0.138; 0.336; 0.530; 0.721; 0.912, \text{ (TDQ-2)} \tag{22a}$$

with the computer MEDA 41TC if

$$m = 0.138; 0.341; 0.558; 0.776; 0.987 \tag{23}$$

and with the computer ADT 3000 if

$$m = 0.258; 0.457; 0.667; 0.876. \tag{24}$$

For the different characteristics of the diodes and for the inaccuracy of resistances it is neccessary experimentally precise the theoretical values m determined. Figure 5, 6 and 7 illustrate the cases of quotients $\frac{mx}{x}$ for some values m by the computers

MEDA TA TDQ-2 (figure 5), MEDA TC (figure 6) and ADT (figure 7), respectively. The courses of the input values in general correspond to the theoretical conclusions and to relations of (8) and (12).

We meet with the cases of modelling the quotient for small values or of expressions $\frac{mx^{\nu}}{x^{\nu}}$ in programming differential equations with singularities of the type $\frac{0}{0}$.
The accuracy of modelling the expression $z(x) = \frac{f(x)}{g(x)}$, where $f(x) = ax^{\nu}$, $g(x) = bx^{\nu}$ holds for $x - 0$, that is

$$\frac{f(x)}{g(x)} = \frac{ax^{\nu}}{bx^{\nu}} = \frac{cx^{\nu}}{x^{\nu}}$$

may be increased if, instead of the function $z(x)$, we are modelling a suitable multiple of this function, that is

$$\frac{qf(x)}{g(x)} = \frac{qcx^{\nu}}{x^{\nu}},$$

where q is chosen such that $qc = m$ and m satisfies the relation of (12).

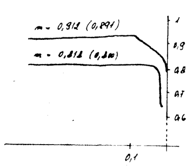

Fig. 5

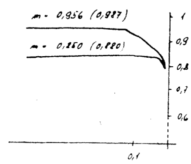

Fig. 6

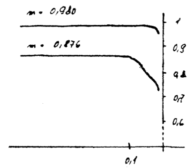

Fig. 7

For instance, in solving the equation

$$t^2y' - y = 2ct^3 - ct^2, \tag{25}$$

with the initial condition $y(0) = 0$, where solution is the function $y = ct^2$, we proceed as follows: Equation (25) is programmed in the form

$$y' - \frac{y}{t^2} = 2ct - c \tag{25a}$$

by the programme diagram in figure 8, $\lim_{t \to 0} \frac{y}{t^2} = c$. The expression $z = \frac{y}{t^2}$ is programmed in the form $qz = \frac{-q(y + 0{,}020ce^{-10t})}{-(t^2 + 0{,}020e^{-10t})}$, where qc satisfies the relation of (12). If we choose q such that the relation qc satisfies the relation of (8), we get the solution of the worst accuracy. For meeting further requirements on the accuracy of analog computation, we choose the greatest possible coefficient. For instance using the computer MEDA 41TC with $c = 1$ we get with $q = 0.956$ (the theoretical value by (23) is $q = 0.987$) due to the inaccuracy of modelling the quotient of small values a very innacurate solution ($\delta(y) = -0.730$); a relatively accurate solution is obtained with $q = 0.850$ (the theoretical value by (20) is $q = 0.880$) ($\delta(y) =$ $= -0.034$).

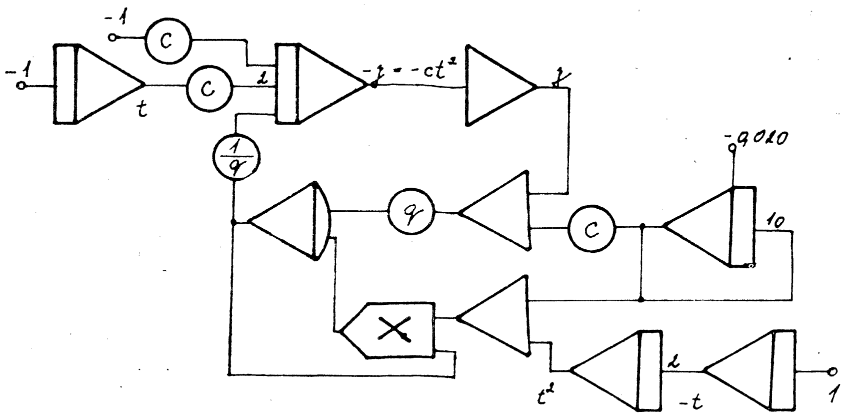

Fig. 8

ПОВЫШЕНИЕ ТОЧНОСТИ ПРИ МОДЕЛИРОВАНИЮ ОТНОШЕНИЯ МАЛЫХ ЗНАЧЕНИЙ У ВЫЧИСИЛТЕЛЬНЫХ МАШИН МЕДА 40ТА, МЕДА 41ТЦ И АДТ 3000

Резюме

В статье описано повышение точности моделирования отношения $з = \frac{\text{м}m^{v}}{m^{v}}$ при малых значениях т. Есть описано офо́рмление отношения так, чтобы точность моделирования отношения была хорошая и для малых *m*.

ZVÝŠENÍ PŘESNOSTI PŘI MODELOVÁNÍ PODÍLU MALÝCH HODNOT U POČÍTAČŮ MEDA 40TA, MEDA 41TC a ADT 3000

Souhrn

V článku je popsáno zvýšení přesnosti modelování podílu $Z = \frac{mt^\nu}{t^\nu}$ při malých hodnotách t. Je navržena úprava podílu (vynásobením vhodným koeficientem) tak, aby přesnost modelování podílu byla zachována i pro malá t. Je určeno, od kterých hodnot t nastávají výraznější chyby.

References

[1] Beneš, K.: *Zvýšení přesnosti při modelování podílu dvou funkcí.* Elektrotechnický časopis 28, 1977, č. 7, str. 481—493.

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