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INCREASE OF ACCURACY IN MACHINE COMPUTATION OF SOME DIFFERENTIAL EQUATIONS HAVING VARIABLE COEFFICIENTS AND OF SOME NONLINEAR DIFFERENTIAL EQUATIONS

KAREL BENEŠ

(Received March 25, 1981)

Dedicated to Prof. Miroslav Laitoch on his 60th birthday

Using a diode multiplier in the network in solving differential equations with variable coefficients or in solving nonlinear differential equations, gives rise to a certain inaccuracy mainly caused by the diode multiplier. (In our case by approximating square dependences by linear sections in the technical realization of the diode multiplier.) This inaccuracy even increases in modelling the quotient of two functions in case when both the numerator and the denominator assume small values.

The present article shows a simple modification by which the accuracy of the modelled quotient is substantially increased even in modelling the quotient when the numerator and the denominator assume small values. There are given some applications in solving some differential equations. See [2].

Introduction and theoretical part

The quotient of two functions $z(t) = \frac{f(t)}{g(t)}$ is modelled by solving the implicit functions

$$F(z(t), f(t), g(t)) \equiv f(t) - z(t)g(t) = 0$$

as shown by the programme diagram in figure 1. For the computing network to be stable, it is necessary that $g(t) < 0$. Using a diode multiplier in the computing circuit gives rise to a certain inaccuracy in the quotient modelled, caused by the

inaccuracy of the diode multiplier. This inaccuracy becomes especially expressive in small values of the numerator and the denominator. It is related with the approximation of the square dependences by n -linear sections in the technical realization

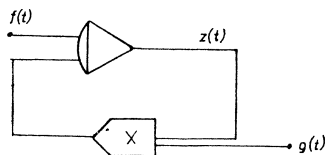


Fig. 1

of the diode multiplier which works on the basis of the relation

$$uv = \left(\frac{u+v}{2}\right)^2 - \left(\frac{u-v}{2}\right)^2.$$

See figure 2 for $z = \frac{mt}{t} = m$, $t \in \langle -1; 0 \rangle$, $n = 5$ for some values m and figure 2a for $z = \frac{-mt^2}{-t^2} = m$, $t \in \langle -1; 0 \rangle$. From the above figures it can be seen

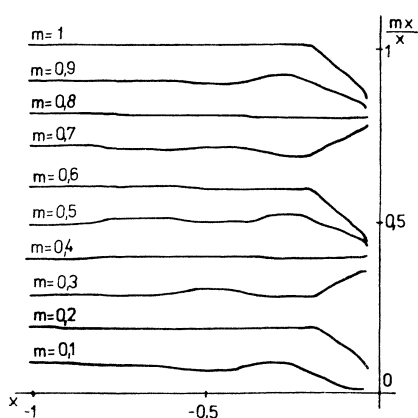


Fig. 2

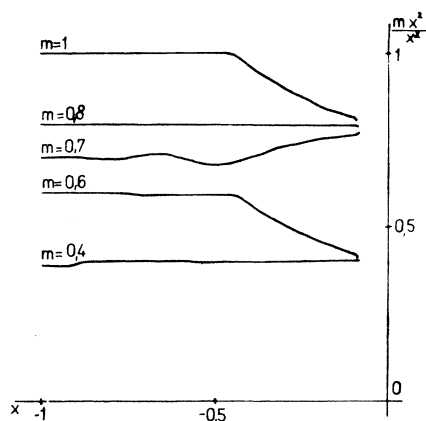


Fig. 2a

that the circuit works with a relatively large error if $m = 1$ and $m = 0.6$, where both the numerator and the denominator assume small values. The above inaccuracy doesnot occur in modelling the quotients $\frac{0,8x}{x}$ and $\frac{0,4x}{x}$. In [1] are defined values

$$m = 0,4; 0,8 \quad \text{for } n = 5 \quad (1)$$

$$m = 0,2; 0,4; 0,6; 0,8; 1 \quad \text{for } n = 10, \quad (2)$$

where the dividing circuit works relatively even at small values t , and

$$m = 0,2; 0,6; 1,0; \quad \text{for } n = 5 \quad (1a)$$

$$m = 0,1; 0,3; 0,5; 0,7; 0,9 \quad \text{for } n = 10 \quad (2a)$$

where the dividing circuit works at small values t with the largest inaccuracy.

Let in the computing network be

$$z = \frac{f(t)}{g(t)}$$

for a given differential equation, where for $t \rightarrow 0$ $f(t) = at^\alpha$, $g(t) = bt^\alpha$, so that

$$\frac{f(t)}{g(t)} = \frac{at^\alpha}{bt^\alpha} = \frac{cf(t)}{g(t)}. \quad (3)$$

Modelling instead of the function $z(t) = \frac{f(t)}{g(t)}$ a suitable multiple of this function, i.e.

$$\frac{qf(t)}{g(t)} = \frac{qct^\alpha}{t^\alpha}, \quad (4)$$

and choosing an appropriate coefficient q , then the quotient $\frac{qf(t)}{g(t)}$ can be modelled with a good accuracy even for small values t . It is necessary to choose the coefficient q so that the relation

$$qc = m, \quad (5)$$

is satisfied, i.e. the expression $qc = m$ must be equal to the given value m according to (1) or (2) in dependence on the given n . In order not to deviate widely from the normalization principle in the input values of the computing units, we evidently adopt the topmost possible value of m .

Remark: We do not consider such cases where the quotient is of the form $z = \frac{mt^\alpha}{t^\beta}$, where $\alpha \neq \beta$, because with respect to the particular values of α and β , the limit of the quotient is either not finite – hence this quotient cannot be modelled for $t \rightarrow 0$, or its limit is equal to zero. Evidently, in such case the accuracy (in modelling the quotient) can be increased by consistent normalization of the numerator and the determinant, only. See equation (20).

Experimental part

The above modification of the modelled quotient finds its application in simulating dynamical systems described by some differential equations having variable coefficients or by nonlinear differential equations.

Let us solve (as an example) a differential equation

$$y'' + 2 \frac{y'}{t} + y = 0 \quad (6)$$

having its initial values $y(0) = 1$, $y'(0) = 0$ in the interval $0 \leq t \leq 1$. It necessary first to determine the $\lim_{t \rightarrow 0} \frac{y'}{t}$; this expression becomes for $t \rightarrow 0$ to be an indefinite expression of the type $\frac{0}{0}$.

The limit of $\frac{y'}{t}$ for $t \rightarrow 0$ will be defined by means of $y(0)$ and $y''(0)$, where the latter may be defined as follows: Putting equation (6) to the form

$$ty'' + 2y' + ty = 0, \quad (7)$$

let us try to find the solution of the above equation in the form

$$y = a_0 + a_1t + a_2t^2 + \dots + a_nt^n. \quad (8)$$

From the last expression we then obtain

$$y' = a_1 + 2a_2t + 3a_3t^2 + \dots + na_nt^{n-1} \quad (8a)$$

$$y'' = 2a_2 + 6a_3t + \dots + n(n-1)a_nt^{n-2}. \quad (8b)$$

Inserting the relations (8), (8a) and (8b) into (7) gives

$$2a_2t + 6a_3t^2 + \dots + n(n-1)a_nt^{n-1} + 2a_1 + 4a_2t + 6a_3t^2 + \dots + 2na_nt^{n-1} + a_0t + a_1t^2 + a_2t^3 + \dots + a_nt^{n+1} = 0. \quad (9)$$

The coefficients a_0 and a_n are furnished in comparing the coefficients with the particular powers of the independent variable t . Seeing that it suffices to specify $y''(0)$ for the calculation of $\lim_{t \rightarrow 0} \frac{y'}{t}$, it is sufficient now to specify at most the coefficient a_2 — as it follows from (8b).

Comparing the coefficients in (9) gives

$$2a_1 = 0,$$

i.e.

$$a_1 = 0, \quad (10)$$

$$2a_2 + 4a_2 + a_0 = 0,$$

i.e.

$$a_2 = -\frac{a_0}{6}. \quad (11)$$

From (8) and from the given initial condition $y(0) = 1$ we obtain $y(0) = a_0 = 1$. Inserting these values into (11) gives

$$a_2 = -\frac{1}{6}, \quad (12)$$

so that $y''(0) = 2a_2 = -\frac{1}{3}$. From (6) we get then

$$\lim_{t \rightarrow 0} \frac{y'}{t} = \frac{-y''(0) - y(0)}{2} = -\frac{1}{3}.$$

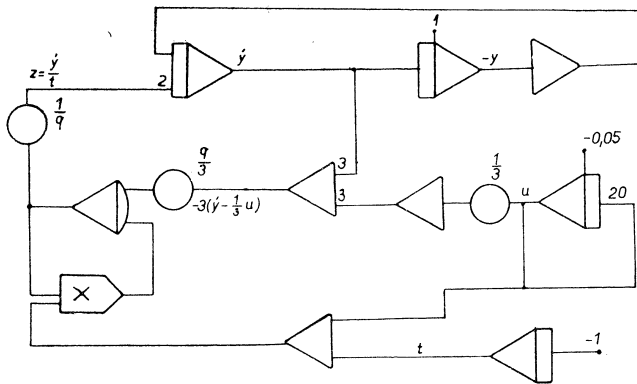


Fig. 3

The programme diagram for solving equation (6) is given in figure 3. The expression $z = \frac{y'}{t}$ is modelled in the form of [2]:

$$qz = \frac{-q(y' + 0,05z(0)e^{-20t})}{-(t + 0,05e^{-20t})}, \quad (13)$$

where $z(0) = \lim_{t \rightarrow 0} \frac{y'}{t} = -\frac{1}{3}$. Equation (6) is programmed in the form

$$-y''' = 2 \frac{y'}{t} + y.$$

With respect to (8a), (10) and (12) it holds for small t

$$z = \frac{y'}{t} \doteq \frac{-\frac{1}{3}t}{t} = \frac{ct}{t}. \quad (14)$$

(The function $z = \frac{\sin t}{t}$ is the solution of (6) with the given initial conditions,

the absolute values of the coefficients a_j with increasing j are decreasing.) Let us choose in (13) $q = 1$, and q such that the expression $q |c| = m$ satisfies the relation (1) (in using the diode multiplier of AP-S, where $n = 5$), $|c| = |z(0)| = \frac{1}{3}$, i.e. $q = 1,2$ and $2,4$. We choose next $q = 1,8$ and $q = 3$. These two values satisfy (1a) when in modelling the quotient of small values the least favorable cases occur. The influence of the quotient q on the accuracy in modelling the

quotient $\frac{y' - \frac{1}{3}u}{t + u}$ can be seen in tables 1, 1a and 1b, where $u = 0,05e^{-20t}$. The accuracy in modelling this quotient is checked by means of the values given in the

tables obtained by a numerical calculation of the quotient $\left(y' - \frac{1}{3}u \text{ and } t + u\right)$

by computer) and of the quotient $\frac{y' - \frac{1}{3}u}{t + u}$ obtained by computer. The fact that

the accuracy in modelling the given quotient depends on an appropriate choice of q before all, can be seen in comparing the results obtained at values $q = 1; 1,2$ and $1,8$ where at the couple $q = 1,2$ and $1,8$ we should (contrarywise) expect greater accuracy of the value $q = 1,8$. Similar results can be obtained in choosing $q = 2,4$ and $q = 3$. The choice of the quotient q is done on the basis of equation (5). Evaluating the tables we devote the main attention to the results with small values of t , i.e. where $t < 0,2$ (cf. [1]). The results with greater values of the independent variable t under different values of q do not differ substantially.

Tab. 1

t	y_{tab}	y	$\delta(y)$	$t + u$	$y' - \frac{1}{2}$	z_{tab}	z	$\delta(z)$
0,000	1,000	1,002	0,002	0,050	-0,016	-0,320	-0,382	-0,062
0,030	1,000	1,002	0,002	0,057	-0,016	-0,281	-0,352	-0,071
0,050	1,000	1,002	0,002	0,068	-0,018	-0,265	-0,327	-0,062
0,100	0,998	1,000	0,002	0,106	-0,028	-0,264	-0,316	-0,052
0,150	0,996	0,999	0,003	0,152	-0,044	-0,289	-0,301	-0,012
0,200	0,993	0,996	0,003	0,201	-0,060	-0,298	-0,291	0,007
0,250	0,989	0,993	0,004	0,250	-0,078	-0,312	-0,294	0,018
0,300	0,985	0,988	0,003	0,300	-0,096	-0,320	-0,300	0,020
0,400	0,973	0,975	0,002	0,400	-0,129	-0,322	-0,309	0,023
0,600	0,941	0,939	-0,002	0,600	-0,188	-0,313	-0,302	0,011
0,800	0,897	0,889	-0,008	0,800	-0,246	-0,307	-0,296	0,011
1,000	0,841	0,824	-0,017	1,000	-0,292	-0,292	-0,288	0,004

$q = 1$

Tab. 1-a

t	y_{tab}	y	$\delta(y)$	$t + u$	$y' - \frac{1}{3}u$	z_{tab}	z	$\delta(z)$
0,000	1,000	1,002	0,002	0,050	-0,016	-0,320	-0,332	-0,012
0,030	1,000	1,002	0,002	0,057	-0,017	-0,298	-0,327	-0,029
0,050	1,000	1,001	0,001	0,068	-0,021	-0,309	-0,326	-0,017
0,100	0,998	1,000	0,002	0,106	-0,032	-0,302	-0,320	-0,018
0,150	0,996	0,999	0,003	0,152	-0,047	-0,309	-0,314	-0,005
0,200	0,993	0,996	0,003	0,201	-0,063	-0,313	-0,308	0,005
0,250	0,989	0,992	0,003	0,250	-0,079	-0,316	-0,304	0,012
0,300	0,985	0,986	0,001	0,300	-0,095	-0,316	-0,306	0,010
0,400	0,973	0,974	0,001	0,400	-0,127	-0,317	-0,318	-0,001
0,600	0,941	0,939	-0,002	0,600	-0,186	-0,310	-0,303	0,007
0,800	0,897	0,890	-0,007	0,800	-0,242	-0,303	-0,302	0,001
1,000	0,841	0,829	-0,012	1,000	-0,291	-0,291	-0,290	0,001

$q = 1,2$

Tab. 1-b

t	y_{tab}	y	$\delta(y)$	$t + u$	$y' - \frac{1}{3}u$	z_{tab}	z	$\delta(z)$
0,000	1,000	1,002	0,002	0,050	-0,016	-0,320	-0,248	0,072
0,030	1,000	1,002	0,002	0,057	-0,021	-0,368	-0,280	0,088
0,050	1,000	1,002	0,002	0,068	-0,026	-0,382	-0,308	0,074
0,100	0,998	1,000	0,002	0,106	-0,039	-0,368	-0,340	0,028
0,150	0,996	0,997	0,001	0,152	-0,052	-0,342	-0,310	0,032
0,200	0,993	0,994	0,001	0,201	-0,066	-0,323	-0,327	-0,004
0,250	0,989	0,990	0,001	0,250	-0,081	-0,324	-0,326	-0,002
0,300	0,985	0,986	0,001	0,300	-0,096	-0,320	-0,321	-0,001
0,400	0,973	0,973	0,000	0,400	-0,127	-0,317	-0,310	0,007
0,600	0,941	0,938	-0,003	0,600	-0,186	-0,310	-0,306	0,004
0,800	0,897	0,890	-0,007	0,800	-0,239	-0,298	-0,296	0,002
1,000	0,841	0,831	-0,010	1,000	-0,289	-0,289	-0,290	-0,001

$q = 1,8$

The function $y = \frac{\sin t}{t}$ is the solution of equation (6) with the given initial conditions; the accuracy in modelling the quotient is in this case without great influence on the solution y , since the solution of equation (6) is limited and according to [3] stable.

The influence of this modification on the accuracy of solutions becomes especially expressive in solving nonconvergent problems where the accuracy is increased

at the very beginning of the operation, as in the case of solving equation

$$ty'' - ty' - y = -2t \quad (15)$$

with the initial conditions $y(0) = 0$, $y'(0) = 1$ in the interval $0 \leq t \leq 1$. The equation will be programmed in the form

$$y'' - y' - \frac{y}{t} = -2. \quad (16)$$

The expression $\frac{y}{t}$ becomes for $t \rightarrow 0$ an indefinite one of the type $\frac{0}{0}$. The limit of this expression for $t \rightarrow 0$ will be defined analogous to (6). The solution of (15) will be understood to be in the form $y = a_0 + a_1t + \dots + a_nt^n$. From the given

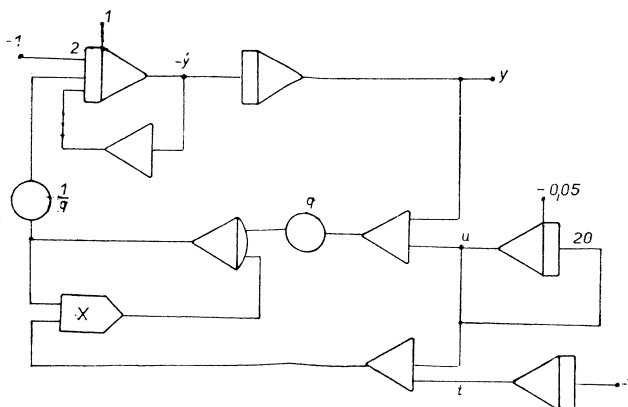


Fig. 4

initial conditions we deduce $a_0 = 0$, $a_1 = 1$, so that the expression $\frac{y}{t}$ may be written in the form

$$\frac{y}{t} = \frac{t + a_2t^2 + \dots + a_nt^n}{t}, \quad \lim_{t \rightarrow 0} \frac{y}{t} = 1.$$

The programme diagram for solving this equation is given in figure 4. The expression $z = \frac{y}{t}$ is modelled in the form

$$qz \doteq \frac{-q(y + 0,05e^{-20t})}{-(t + 0,05e^{-20t})}. \quad (17)$$

On this basis of (1), (1a) and (5) let $q = 1$ and $q = 0,8$. It becomes apparent from tables 2 and 2a that the accuracy in solving equation (15) and in constructing

the quotient $z = \frac{y}{t}$ is better with $q = 0,8$ than with $q = 1$ contrary to what we should expect. The solution of (15) is of the form $y = t$. The influence of the inaccuracy in modelling the quotient $\frac{y}{t}$ becomes especially significant at y' , where $y' = 1$ holds for the right values y' .

Tab. 2

t	y_{tab}	y	$\delta(y)$	y'	$\delta(y')$	$y + u$	$t + u$	z_{tab}	z	$\delta(z)$
0,000	0,000	0,000	0,000	1,002	0,002	0,050	0,050	1,000	0,846	-0,154
0,030	0,030	0,029	-0,001	0,996	-0,004	0,058	0,057	1,017	0,856	-0,161
0,050	0,050	0,049	-0,001	0,994	-0,006	0,069	0,068	1,014	0,868	-0,146
0,100	0,100	0,099	-0,001	0,988	-0,012	0,106	0,106	1,000	0,901	-0,099
0,150	0,150	0,149	-0,001	0,982	-0,018	0,151	0,152	0,993	0,940	-0,053
0,200	0,200	0,198	-0,002	0,978	-0,022	0,199	0,201	0,990	0,977	-0,013
0,250	0,250	0,248	-0,002	0,976	-0,024	0,248	0,250	0,992	0,989	-0,003
0,300	0,300	0,296	-0,004	0,974	-0,026	0,296	0,300	0,986	0,990	0,004
0,400	0,400	0,393	-0,007	0,969	-0,031	0,393	0,400	0,982	0,983	0,001
0,600	0,600	0,583	-0,017	0,958	-0,042	0,583	0,600	0,971	0,984	0,013
0,800	0,800	0,772	-0,028	0,946	-0,054	0,772	0,800	0,965	0,985	0,020
1,000	1,000	0,959	-0,041	0,935	-0,065	0,959	1,000	0,959	0,983	0,024

$q = 1$

Tab. 2-a

t	y_{tab}	y	$\delta(y)$	y'	$\delta(y')$	$y + u$	$t + u$	z_{tab}	z	$\delta(z)$
0,000	0,000	0,000	0,000	1,002	0,002	0,050	0,050	1,000	0,976	-0,024
0,030	0,030	0,031	0,001	1,002	0,002	0,058	0,057	1,017	0,986	-0,031
0,050	0,050	0,052	0,002	1,002	0,002	0,070	0,068	1,029	0,989	-0,040
0,100	0,100	0,102	0,002	1,001	0,001	0,108	0,106	1,018	0,992	-0,026
0,150	0,150	0,152	0,002	1,001	0,001	0,154	0,152	1,013	0,997	-0,016
0,200	0,200	0,202	0,002	1,000	0,000	0,202	0,201	1,005	1,000	-0,005
0,250	0,250	0,252	0,002	1,000	0,000	0,252	0,250	1,008	1,002	-0,006
0,300	0,300	0,301	0,001	1,001	0,001	0,301	0,300	1,003	1,003	0,000
0,400	0,400	0,399	-0,001	1,001	0,001	0,399	0,400	0,998	1,017	0,019
0,600	0,600	0,600	0,000	1,005	0,005	0,600	0,600	1,000	1,023	0,023
0,800	0,800	0,802	0,002	1,006	0,006	0,802	0,800	1,002	1,024	0,022
1,000	1,000	1,004	0,004	1,010	0,010	1,004	1,000	1,004	1,025	0,021

$q = 0,8$

In modelling the quotient $z = \frac{mt^\alpha}{t^\alpha}$ there occur especially unfavourable cases (see [2]) with $\alpha > 1$ (figures 2a and 2b). Such a case occurs for instance in solving equation

$$t^2 y' - y = 2t^3 - t^2 \quad (18)$$

with initial conditions $y(0) = 0$. Its solution is the function $y = t^2$. Equation (18) is programmed in the form $y' - \frac{y}{t^2} = 2t - 1$, $\lim_{t \rightarrow 0} \frac{y}{t^2} = 1$ and the expression $z = \frac{y}{t^2}$ is modelled in the form

$$qz = \frac{-q(y + u)}{-(t^2 + u)}, \quad (19)$$

where $q = 1; 0,8; u = 0,1e^{-10t}$. The programme diagram for solving equation (18) is given in figure 5.

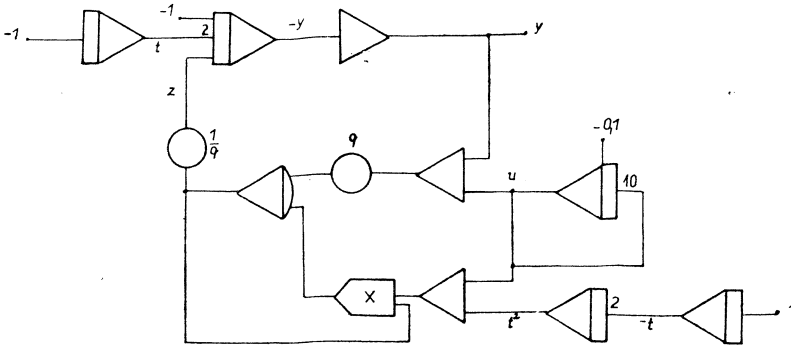


Fig. 5

It becomes apparent from table 3 for $q = 1$ and from table 3a for $q = 0,8$ that in case of $q = 1$ the solution is completely inapplicable due to great inaccuracy in modelling the quotient of small values.

Let us observe another case where for the limit of the modelled quotient $\lim_{t \rightarrow 0} \frac{f(t)}{g(t)} = 0$, ($f(0) = g(0) = 0$) holds. In this case the accuracy can be increased only by normalizing the relative variables, especially such variables, whose quotient is formed, because according to figure 2, there occur great relative errors in modelling the quotient $\frac{mt^\alpha}{t^\alpha}$ with $m \rightarrow 0$. We meet with such a case in solving equation

$$ty'' + ty' - 2y = t \quad (20)$$

with initial conditions $y(0) = 0$, $y'(0) = 0$, whose solutions is the function $y =$

$= 0,5t^2$ and thus $\lim_{t \rightarrow 0} \frac{y}{t} = 0$. The programme diagram for solving equation (20) in the form $y'' = 1 - y' + \frac{2y}{t}$ is given in figure 6; the expression $z = \frac{y}{t}$ is modelled in the form

$$qz = \frac{-qy}{-(t+u)},$$

where q is chosen as $q = 1$, $q = 1,5$ and $q = 2$.

Tab. 3

t	y_{tab}	y	$\delta(y)$	$y + u$	$t^2 + u$	z_{tab}	z	$\delta(z)$
0,000	0,000	0,000	0,000	0,100	0,100	1,000	0,910	-0,090
0,030	0,001	-0,002	-0,003	0,072	0,076	0,947	0,868	-0,079
0,050	0,003	-0,004	-0,007	0,058	0,064	0,906	0,833	-0,073
0,100	0,010	-0,006	-0,016	0,032	0,048	0,666	0,565	-0,101
0,150	0,023	-0,008	-0,031	0,015	0,046	0,326	0,353	0,027
0,200	0,040	-0,020	-0,060	-0,006	0,056	-0,107	-0,085	0,022
0,250	0,063	-0,047	-0,110	-0,039	0,072	-0,541	-0,647	-0,106
0,300	0,090	-0,082	-0,172	-0,077	0,097	-0,793	-0,810	-0,017
0,400	0,160	-0,190	-0,350	-0,188	0,165	-1,139	-1,180	-0,041
0,600	0,360	-0,378	-0,738	-0,376	0,366	-1,027	-1,070	-0,043
0,800	0,640	-0,470	-1,110	-0,470	0,640	-0,734	-0,752	-0,018
1,000	1,000	-0,525	-1,525	-0,525	0,991	-0,529	-0,537	-0,008

$q = 1$

Tab. 3-a.

t	y_{tab}	y	$\delta(y)$	$y + u$	$t + u$	z_{tab}	z	$\delta(z)$
0,000	0,000	0,000	0,000	0,100	0,100	1,000	1,007	0,007
0,030	0,001	0,000	-0,001	0,076	0,077	0,987	1,005	0,018
0,050	0,003	0,001	-0,002	0,065	0,065	1,000	1,004	0,004
0,100	0,010	0,012	0,002	0,048	0,048	1,000	0,996	0,004
0,150	0,023	0,024	0,001	0,045	0,047	0,957	0,987	0,030
0,200	0,040	0,040	0,000	0,053	0,056	0,944	0,977	0,033
0,250	0,063	0,061	-0,002	0,069	0,073	0,945	0,970	0,025
0,300	0,090	0,086	-0,004	0,090	0,096	0,937	0,966	0,029
0,400	0,160	0,152	-0,008	0,154	0,165	0,933	0,950	0,017
0,600	0,360	0,352	-0,008	0,353	0,366	0,937	0,971	0,034
0,800	0,640	0,644	0,004	0,645	0,643	1,003	1,030	0,027
1,000	1,000	1,032	0,032	1,033	0,995	1,038	1,072	0,034

$q = 0,8$

In case of $q = 2$ the solution of equation (20) was obtained with the maximum error $|\delta(y)|_{\max} = 0,045$, in case of $q = 1,5$ with error $|\delta(y)|_{\max} = 0,031$ and in case of $q = 2$ with the error $|\delta(y)|_{\max} = 0,019$.

The above method for increasing the accuracy of solutions finds its application

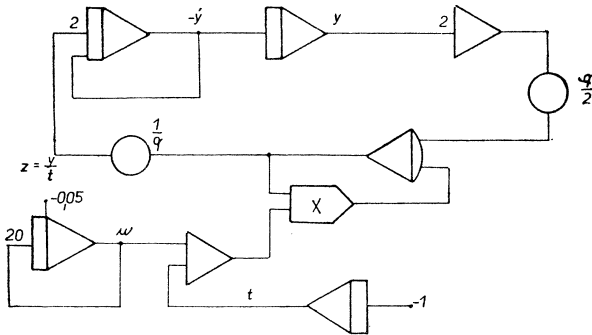


Fig. 6

even in solving some nonlinear differential equations, where the expression $\frac{0}{0}$ appears. We show such a case in solving equation

$$t^2 y'' - t^2 y' - y^2 = -2t \quad (21)$$

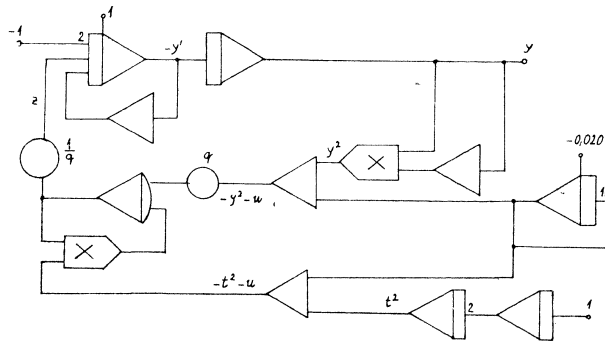


Fig. 7

with the initial conditions $y(0) = 0$, $y'(0) = 1$, programmed in the form

$$y'' = -2 + y' + \frac{y^2}{t^2} \quad (22)$$

Let us suppose it to be of the form

$$y = a_0 + a_1 t + a_2 t^2 + \dots + a_n t^n,$$

i.e.

$$y' = a_1 + 2a_2 t + \dots + na_n t^{n-1}.$$

From the initial conditions $y(0) = 0, y'(0) = 1$ follows $a_0 = 0, a_1 = 1$. Inserting the above relations into relation $\frac{y^2}{t^2}$ and for $t = 0$ we obtain

$$\lim_{t \rightarrow 0} \frac{y^2}{t^2} = \frac{t^2 + a_2^2 t^4 + \dots + 2a_1 a_2 t^3 + \dots + a_n^2 t^{2n}}{t^2} = 1.$$

The programme diagram for solving equation (22) is given in fig. 7. The function $y = t$ is the solution of (21). The expression $z = \frac{y^2}{t^2}$ is modelled in the form $qz = \frac{-q(y^2 + u)}{-(t^2 + u)}$, where $q = 0,850; 0,956, u = 0,020e^{-10t}$. It can be seen from table 4 that the solution with $q = 0,9$, due to great inaccuracy of the dividing circuit for the small values of numerator and the denominator becomes less accurate.

The last equation was solved on the computer MEDA TC, see [4].

Tab. 4.

t	y	$\delta(y)$	y'	$\delta(y')$	$y^2 + u$	$t^2 + u$	z_{tab}	z	$\delta(z)$
0,000	0,001	0,001	1,001	0,001	0,020	0,020	1,000	0,915	-0,085
0,030	0,034	0,004	0,999	-0,001	0,015	0,016	0,937	0,900	-0,037
0,050	0,053	0,003	0,998	-0,002	0,016	0,016	1,000	0,962	-0,038
0,100	0,102	0,002	0,996	-0,004	0,020	0,019	1,052	0,991	-0,060
0,150	0,148	-0,002	0,993	-0,007	0,028	0,028	1,000	0,948	-0,052
0,200	0,196	-0,004	0,992	-0,008	0,044	0,042	1,047	1,003	-0,044
0,250	0,245	-0,005	0,990	-0,010	0,062	0,062	1,000	0,971	-0,029
0,300	0,292	-0,008	0,988	-0,012	0,089	0,088	1,011	0,988	-0,023
0,400	0,389	-0,011	0,984	-0,016	0,154	0,155	0,993	0,975	-0,018
0,600	0,585	-0,015	0,972	-0,028	0,346	0,354	0,977	0,970	-0,007
0,800	0,778	-0,022	0,956	-0,044	0,606	0,628	0,964	0,958	-0,006
1,000	0,973	-0,027	0,933	-0,067	0,948	0,996	0,951	0,945	-0,006

$q = 0,956, y_{\text{tab}} = t.$

Tab. 5.

t	y_t	$\delta(y)$	y'	$\delta(y')$	$y^2 + u$	$t^2 + u$	z_{tab}	z	$\delta(z)$
0,000	0,002	0,002	0,999	-0,001	0,020	0,020	1,000	0,984	-0,016
0,030	0,033	0,003	0,993	-0,007	0,015	0,016	0,937	0,960	0,023
0,050	0,053	0,003	0,996	-0,004	0,016	0,016	1,000	0,986	-0,014
0,100	0,102	0,002	0,992	-0,008	0,019	0,018	1,055	1,038	-0,017
0,150	0,150	0,000	0,994	-0,006	0,029	0,028	1,035	1,036	-0,001
0,200	0,198	-0,002	1,000	0,000	0,044	0,043	1,023	1,011	-0,012
0,250	0,246	-0,004	0,997	-0,003	0,065	0,064	1,015	1,008	-0,007
0,300	0,294	-0,006	0,996	-0,004	0,090	0,088	1,022	1,005	-0,017
0,400	0,392	-0,008	0,996	-0,004	0,156	0,155	1,006	0,995	-0,011
0,600	0,590	-0,010	0,993	-0,007	0,352	0,352	1,000	0,996	-0,004
0,800	0,792	-0,008	0,992	-0,008	0,623	0,626	0,995	1,002	0,007
1,000	0,996	-0,004	0,993	-0,007	0,996	0,998	0,997	1,002	0,005

$$q = 0,850, y_{\text{tab}} = t.$$

Conclusion

In solving differential equations with variable coefficients or some nonlinear differential equations by means of an analog computer when a diode multiplier is used in the network, there arises a certain inaccuracy in the diode multiplier. This inaccuracy even increases in modelling the quotient of two functions in cases when the numerator and the denominator assume small values. This article describes a method in which a simple modification of the modelled quotient enables a substantially increase of accuracy in solutions for such cases.

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Souhrn

**ZVÝŠENÍ PŘESNOSTI STROJOVÉHO ŘEŠENÍ
NĚKTERÝCH DIFERENCIÁLNÍCH ROVNIC
S PROMĚNNÝMI KOEFICIENTY A NĚKTERÝCH
NELINEÁRNÍCH DIFERENCIÁLNÍCH ROVNIC**

KAREL BENEŠ

Práce se zabývá zvýšením přesnosti modelování podílu s použitím diodové násobičky při řešení některých diferenciálních rovnic s proměnnými koeficienty a některých nelineárních diferenciálních rovnic, kdy je třeba vytvářet podíly malých hodnot. Zvýšení přesnosti modelování podílu malých hodnot je dosaženo vynásobením daného podílu vhodným koeficientem.

Резюме

**ПОВЫШЕНИЕ ТОЧНОСТИ МАШИННОГО
РЕШЕНИЯ НЕКОТОРЫХ ДИФФЕРЕНЦИАЛЬНЫХ
УРАВНЕНИЙ С ПЕРЕМЕННЫМИ
КОЕФФИЦИЕНТАМИ И НЕКОТОРЫХ
НЕЛИНЕЙНЫХ ДИФФЕРЕНЦИАЛЬНЫХ
УРАВНЕНИЙ**

КАРЕЛ БЕНЕШ

В статье описан способ улучшения точности моделирования отношения с использованием диодного множительного устройства при решении дифференциальных уравнений с переменными коэффициентами и некоторых нелинейных дифференциальных уравнений когда надо порождать отношения малых значений. Повышение точность моделирования отношения малых значений достигается умножением данного отношения удобным коэффициентом.