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ON THE STRUCTURE OF THE SECOND-ORDER PERIODIC LINEAR DIFFERENTIAL EQUATIONS WITH THE SAME CHARACTERISTIC MULTIPLIERS

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§ 1. INTRODUCTION

The present paper investigates equations of the type

(q) $\quad y'' = q(t)\,y, \qquad q(t + \pi) = q(t) \qquad \text{for } t \in \boldsymbol{R}(=(-\infty, \infty)),\ q \in C^0_{\boldsymbol{R}},$

being oscillatory on $\boldsymbol{R}$ (i.e. every non-trivial solution of (q) has an infinite number of zeros right-and-left of the point $t_0 \in \boldsymbol{R}$). According to the Floquet theory every equation (q) can be associated with a characteristic equation

$$\varrho^2 - A\varrho + 1 = 0, \qquad A \text{ being a constant,}$$

whose roots are called the characteristic multipliers of (q). If $x \in \boldsymbol{R}$ and u, v are solutions of (q) satisfying the initial conditions: $u(x) = 0$, $u'(x) = 1$, $v(x) = 1$, $v'(x) = 0$, then $A = v(x + \pi) + u'(x + \pi)$ (see [2], [6]). Let σ, $\frac{1}{\sigma}$ be the characteristic multipliers of (q). Then there exist independent solutions u, v of (q) satisfying either

$$u(t + \pi) = \sigma u(t), \qquad v(t + \pi) = \frac{1}{\sigma} v(t), \qquad \sigma \neq 0, \tag{1}$$

or

$$u(t + \pi) = \sigma u(t), \qquad v(t + \pi) = \sigma v(t) + u(t), \qquad \sigma^2 = 1, \tag{2}$$

(see [6]).

Let $c \in \boldsymbol{R}$, $d \in \boldsymbol{R}$, $c^2 + d^2 \neq 0$, $q(t) < 0$, $q(t + \pi) = q(t)$ for $t \in \boldsymbol{R}$, $q \in C^2_{\boldsymbol{R}}$. Then $(q_{c,d})$ stands for the equation $y'' = q_{c,d}(t)\,y$, where

$$q_{c,d}(t) := q(t) + \frac{cdq'(t)}{\sqrt{c^2 - d^2 q(t)}} + \sqrt{c^2 - d^2 q(t)}\left(\frac{1}{\sqrt{c^2 - d^2 q(t)}}\right)'', \qquad t \in \boldsymbol{R}.$$

The equation $(q_{c,d})$ was first introduced and investigated by M. Laitoch ([4], [5]).

In [7] there are investigated equations (q) having the same "behaviour". Below we show that (q) and $(q_{c,d})$ have the same behaviour for every $c \in \boldsymbol{R}$, $d \in \boldsymbol{R}$, $c^2 + d^2 \neq 0$.

§ 2. BASIC NOTIONS AND RELATIONS

In accordance with [1] and [2] we say that a function $\alpha : \boldsymbol{R} \to \boldsymbol{R}$, $\alpha \in C_{\boldsymbol{R}}^0$ is a phase of (q) if there exist independent solutions u, v of (q):

$$\operatorname{tg} \alpha(t) = \frac{u(t)}{v(t)} \qquad \text{on } R - \{t \in R, v(t) = 0\}.$$

Every phase α of (q) has the following properties:

$$\alpha \in C_{\boldsymbol{R}}^3, \qquad \alpha'(t) \neq 0 \qquad \text{on } \boldsymbol{R} \qquad \text{and} \qquad \alpha(\boldsymbol{R}) = \boldsymbol{R}. \tag{3}$$

The set of all functions α with the properties of (3) together with the composition rule form the group $\mathfrak{G}$. The set of phases of equation $y'' = -y$ is the subgroup $\mathfrak{E}$ of the group $\mathfrak{G}$. If α is a phase of (q), then $\mathfrak{E}\alpha := \{\varepsilon\alpha, \varepsilon \in \mathfrak{E}\}$ is the set of phases of (q) and $q(t) = -\dfrac{\alpha'''(t)}{2\alpha'(t)} + \dfrac{3}{4}\left(\dfrac{\alpha''(t)}{\alpha'(t)}\right)^2 - \alpha'^2(t)$. The set $\mathfrak{H} := \{\alpha \in \mathfrak{G};\ \alpha(t + \pi) = \alpha(t) + \pi \,.\, \operatorname{sign} \alpha',\ t \in \boldsymbol{R}\}$ is a subgroup of $\mathfrak{G}$ and is called the group of elementary phases.

Let $t_0 \in \boldsymbol{R}$, n be a positive integer and y be a solution of (q), $y(t_0) = 0$ (from now on the trivial solution $y(t) = 0$ on $\boldsymbol{R}$ will be excluded). Denote $\varphi_n(t_0)$ $(\varphi_{-n}(t_0))$ the n^{th} hero of y lying to the right (to the left) of the point t_0. Then the function $\varphi_n(\varphi_{-n})$ is defined on $\boldsymbol{R}$ and called the (central) dispersion with index $n(-n)$ of (q). In place of φ_1 we write φ and speak of the dispersion of (q). The dispersion φ of (q) has the following properties:

$$\varphi \in C_{\boldsymbol{R}}^3, \qquad \varphi(t) > t, \qquad \varphi'(t) > 0, \qquad \varphi(t + \pi) = \varphi(t) + \pi \qquad \text{for } t \in \boldsymbol{R}$$

(see [1], [2]).

Let α be a phase and φ_n be the dispersion with index n of (q). Then the equality $\alpha\varphi_n(t) = \alpha(t) + n\pi \,.\, \operatorname{sign} \alpha'$ holds on $\boldsymbol{R}$.

Following O. Borůvka [2] we say that (q) is of category $(1, k)$, k being a positive integer, if (q) has real characteristic multipliers and there exists $t_0 \in \boldsymbol{R}$: $\varphi_k(t_0) = t_0 + \pi$, with φ_k being the dispersion with index k of (q). Say that (q) is of category $(2, k)$, k being an integer, if (q) has complex characteristic multipliers $(= e^{\pm a\pi i}$, $a \in (0, 1))$ and if there exists a phase α: $\alpha(t + \pi) = \alpha(t) + (a + 2k)\pi$ for $t \in \boldsymbol{R}$.

In accordance with [7] we say that (q_1) and (q_2) have the same behaviour if 1° they have the same characteristic multipliers and 2° they are of the same category and 3° if (2) holds for a suitable pair of solutions of one of the equations (q_1) or (q_2), then it holds also for a suitable pair of solutions of the other equation and the Wronskians of both pairs have the same sign.

Let α be a phase of (q). Then $\mathfrak{E}\alpha\mathfrak{H} := \{\varepsilon\alpha\gamma; \varepsilon \in \mathfrak{E}, \gamma \in \mathfrak{H}\}$ are the phases of equations having the same behaviour as the equation (q) (see [7]).

Let $q \in C_{\boldsymbol{R}}^2$, $q(t) < 0$ on $\boldsymbol{R}$, $c \in \boldsymbol{R}$, $d \in \boldsymbol{R}$, $c^2 + d^2 \neq 0$. If u is a solution of (q), then $U(t) := \dfrac{cu(t) + du'(t)}{\sqrt{c^2 - d^2q(t)}}$ is the solution of $(\mathrm{q}_{c,d})$ and also reversely if $U(t)$ is a solution of $(\mathrm{q}_{c,d})$, then there exists a solution u of (q) with $U(t) = \dfrac{cu(t) + du'(t)}{\sqrt{c^2 - d^2q(t)}}$ for $t \in \boldsymbol{R}$. The equation $(\mathrm{q}_{0,d})$ is investigated at length in [1].

§ 3. THE BEHAVIOUR OF EQUATIONS (q), $(\mathrm{q}_{c,d})$

Let $g \in C_{\boldsymbol{R}}^2$, $q(t) < 0$ on $\boldsymbol{R}$, $c \in \boldsymbol{R}$, $d \in \boldsymbol{R}$, $c^2 + d^2 \neq 0$.

Lemma 1. *Let u be a solution of* (q), $d \neq 0$. *Then the zeros of the functions u, $cu + du'$ are separated on $\boldsymbol{R}$.*

Proof: a) Let $u(t_0) = u(t_1) = 0$, $u(t) \neq 0$ for $t \in (t_0, t_1)$. Then $\lim\limits_{t \to t_0^+} \dfrac{cu(t) + du'(t)}{u(t)} = \operatorname{sign} d \cdot \infty$, $\lim\limits_{t \to t_1^-} \dfrac{cu(t) + du'(t)}{u(t)} = -\operatorname{sign} d \cdot \infty$. Further for $t \in (t_0, t_1)$ is $\left(\dfrac{cu(t) + du'(t)}{u(t)}\right)' = \dfrac{d}{u^2(t)}(q(t)\,u^2(t) - u'^2(t)) \neq 0$ and therefore $\dfrac{cu(t) + du'(t)}{u(t)}$ is a strictly monotone function on (t_0, t_1) and the equation $cu(t) + du'(t) = 0$ has exactly one solution on (t_0, t_1).

b) Let $cu(t_0) + du'(t_0) = 0$, $cu(t_1) + du'(t_1) = 0$ and $cu(t) + du'(t) \neq 0$ for $t \in (t_0, t_1)$. For definiteness let us suppose $cu(t) + du'(t) > 0$ on (t_0, t_1). Then $u(t_0)\,u(t_1) \neq 0$, $\lim\limits_{t \to t_0^+} \dfrac{u(t)}{cu(t) + du'(t)} = \operatorname{sign} u(t_0) \cdot \infty$, $\lim\limits_{t \to t_1^-} \dfrac{u(t)}{cu(t) + du'(t)} = \operatorname{sign} u(t_1) \cdot \infty$. Further $\left(\dfrac{u(t)}{cu(t) + du'(t)}\right)' = \dfrac{d(u'^2(t) - q(t)\,u^2(t))}{(cu(t) + du'(t))^2} \neq 0$ on (t_0, t_1); herefrom $\dfrac{u(t)}{cu(t) + du'(t)}$ is a strictly monotone function on (t_0, t_1) and the equation $u(t) = 0$ has exactly one solution on (t_0, t_1).

Corollary 1. *Let u be a solution of* (q). *Then there exists such a solution U of* $(\mathrm{q}_{c,d})$ *that the zeros of solutions u and U are separated on $\boldsymbol{R}$.*

Proof: Let $U(t) := \dfrac{cu(t) + du'(t)}{\sqrt{c^2 - d^2q(t)}}$, $t \in \boldsymbol{R}$. Then U is the solution of $(\mathrm{q}_{c,d})$ and the statement of Corollary 1 follows from Lemma 1.

Corollary 2. *The equation* $(\mathrm{q}_{c,d})$ *is oscillatory on $\boldsymbol{R}$.*

Proof: By the assumption (q) is oscillatory on $\boldsymbol{R}$ and therefore it follows from Corollary 1 that $(\mathrm{q}_{c,d})$ is also oscillatory on $\boldsymbol{R}$.

Theorem. (q) *and* $(q_{c,d})$ *have the same behaviour.*

Proof: If $d = 0$, then for $c \neq 0$ we have $q_{c,0}(t) = q(t)$, $t \in \mathbf{R}$ and the equations (q) and $(q_{c,0})$ have the same behaviour. Let therefore $d \neq 0$.

a) We show that the equations (q) and $(q_{c,d})$ have the same characteristic multipliers. To this it is sufficient to prove the existence of a number x that for the solutions u, v of (q) and for the solutions U, V of $(q_{c,d})$ satisfying the initial conditions: $u(x) = U(x) = 0$, $u'(x) = U'(x) = 1$, $v(x) = V(x) = 1$, $v'(x) = V'(x) = 0$ it holds

$$v(x + \pi) + u'(x + \pi) = V(x + \pi) + U'(x + \pi).$$

Let $x \in \mathbf{R}$ be a number for which $q'(x) = 0$ (the existence of such an x follows from the assumption of the π-periodicity of q) and let u, v be the solutions of (q): $u(x) = v'(x) = 0$, $u'(x) = v(x) = 1$. Let

$$U(t) := \frac{1}{\sqrt{(c^2 - d^2q(x))(c^2 - d^2q(t))}}(c^2u(t) + cdu'(t) - cdv(t) - d^2v'(t)),$$

$$V(t) := \frac{1}{\sqrt{(c^2 - d^2q(x))(c^2 - d^2q(t))}}(c^2v(t) - cdq(x)\,u(t) + cdv'(t) - d^2q(x)\,u'(t)), \quad t \in \mathbf{R}.$$

Then U, V are the solutions of $(q_{c,d})$; $U(x) = 0$, $V(x) = 1$. Further

$$U'(t) = \frac{1}{\sqrt{(c^2 - d^2q(x))(c^2 - d^2q(t))}}\Bigg[(c^2u'(t) + cdq(t)\,u(t) - cdv'(t) - d^2q(t)\,v(t)) + \frac{d^2q'(t)}{2(c^2 - d^2q(t))}(c^2u(t) + cdu'(t) - cdv(t) - d^2v'(t))\Bigg],$$

$$V'(t) = \frac{1}{\sqrt{(c^2 - d^2q(x))(c^2 - d^2q(t))}}\Bigg[(c^2v'(t) - cdq(x)\,u'(t) + cdq(t)\,v(t) - d^2q(x)\,q(t)\,u(t)) + \frac{d^2q'(t)}{2(c^2 - d^2q(t))} \times (c^2v(t) - cdq(x)\,u(t) + cdv'(t) - d^2q(x)\,u'(t))\Bigg].$$

Consequently $U'(x) = 1$, $V'(x) = 0$ and we have

$$V(x + \pi) + U'(x + \pi) = \frac{1}{c^2 - d^2q(x)}(-cdq(x)\,u(x + \pi) + c^2v(x + \pi) - d^2q(x)\,u'(x + \pi) + cdv'(x + \pi) + c^2u'(x + \pi) + cdq(x)\,u(x + \pi) - cdv'(x + \pi) - d^2q(x)\,v(x + \pi)) = v(x + \pi) + u'(x + \pi)$$

since $q(x + \pi) = q(x)$, $q'(x + \pi) = q'(x) = 0$.

b) We show now: if all solutions of one equation from (q) or $(q_{c,d})$ are π-periodic (π-halfperiodic), then the same is true for the other equation.

If all solutions of (q) are π-periodic (π-halfperiodic) then, evidently, all solutions of $(q_{c,d})$ have this property, too.

Let all solutions of $(q_{c,d})$ be π-halfperiodic. Let us assume that there exists a solution u of (q) with $u(t + \pi) \neq -u(t)$ for $t \in (t_0, t_1)$. Let $z(t) := u(t + \pi) + u(t)$ for $t \in (t_0, t_1)$. Then $z(t) \neq 0$, $z''(t) = q(t) z(t)$. We have next:

$$\frac{cu(t + \pi) + du'(t + \pi)}{\sqrt{c^2 - d^2 q(t + \pi)}} = -\frac{cu(t) + du'(t)}{\sqrt{c^2 - d^2 q(t)}}, \qquad t \in \boldsymbol{R},$$

hence $cu(t + \pi) + du'(t + \pi) = -(cu(t) + du'(t))$, $c(u(t + \pi) + u(t)) = -d(u'(t + \pi) + u'(t))$. Herefrom: $cz(t) = -dz'(t)$ for $t \in (t_0, t_1)$. By differentiating the last equality and after some manipulations we get $cz'(t) = -dz''(t) = -dq(t) z(t) = \frac{d^2}{c} q(t) z'(t), z'(t)(c^2 - d^2 q(t)) = 0$. $c^2 - d^2 q(t) > 0$ for $t \in \boldsymbol{R}$; therefore $z'(t) = 0$ for $t \in (t_0, t_1)$. Thus for this t we obtain $q(t) = 0$ contrary to the assumption $q(t) < 0$. We proceed analogously in case of all solutions of $(q_{c,d})$ being π-periodic.

c) Let σ, $\frac{1}{\sigma}$ be the characteristic multipliers of (q), $\sigma^2 = 1$ and let for the solutions u, v of (q) hold: $u(t+\pi) = \sigma u(t)$, $v(t+\pi) = \sigma v(t) + u(t)$. Let $U(t) := \frac{cu(t) + du'(t)}{\sqrt{c^2 - d^2 q(t)}}$, $V(t) := \frac{cv(t) + dv'(t)}{\sqrt{c^2 - d^2 q(t)}}$, $t \in \boldsymbol{R}$. Then $U(t + \pi) = \sigma U(t)$, $V(t + \pi) = \sigma V(t) + U(t)$, $U'(t) V(t) - U(t) V'(t) = u'(t) v(t) - u(t) v'(t)$.

d) Let (q) and $(q_{c,d})$ be of categories $(1, n)$ and $(1, m)$. Let φ be the dispersion of (q) and let $\bar{\varphi}$ be the dispersion of $(q_{c,d})$. Then there exist $t_0 \in \boldsymbol{R}$, $t_1 \in \boldsymbol{R}$: $\varphi_n(t_0) = t_0 + \pi$, $\bar{\varphi}_m(t_1) = t_1 + \pi$. Because of $(-\infty, \infty) = \bigcup_{\nu=-\infty}^{\infty} \langle \varphi_\nu(t_0), \varphi_{\nu+1}(t_0))$ we can assume without any loss of generality that $t_1 \in \langle t_0, \varphi(t_0))$. From Corollary 1 we get: $\varphi_{j-1}(t_0) < \bar{\varphi}_j(t_1) < \varphi_{j+2}(t_0)$ for $j = 1, 2, 3, \ldots$ Let k be a positive integer. Then

$$\varphi_{(k-1)mn}(t_0) \leqq \varphi_{kmn-1}(t_0) < \bar{\varphi}_{kmn}(t_1) < \varphi_{kmn+2}(t_0) \leqq \varphi_{(k+2)mn}(t_0).$$

Next

$$\varphi_{(k-1)mn}(t_0) = t_0 + (k - 1) m\pi, \qquad \varphi_{(k+2)mn}(t_0) = t_0 + (k + 2) m\pi,$$
$$\bar{\varphi}_{kmn}(t_1) = t_1 + kn\pi$$

hence for every positive integer k:

$$t_0 + (k - 1) m\pi < t_1 + kn\pi < t_0 + (k + 2) m\pi,$$
$$(k(m - n) - m) \pi < t_1 - t_0 < (k(m - n) + 2m) \pi,$$

which is true in case of $m = n$ only.

e) Let the characteristic multipliers of (q) and $(q_{c,d})$ be complex and equal $e^{\pm a\pi i}$, $0 < a < 1$; $(2, n)$ and $(2, m)$ be the categories of (q) and $(q_{c,d})$, respectively. Then there exist a phase α of (q) and a phase α_1 of $(q_{c,d})$: $\alpha(t + \pi) = \alpha(t) + (a + 2n) \pi$,

$\alpha_1(t + \pi) = \alpha_1(t) + (a + 2m)\pi$, $t \in \boldsymbol{R}$. Let φ and $\bar{\varphi}$ be the dispersions of (q) and $(\mathrm{q}_{c,d})$, respectively. From Corollary 1 now follows

$$\varphi_{j-1}(t) < \bar{\varphi}_j(t) < \varphi_{j+1}(t), \qquad t \in \boldsymbol{R}, \quad j = 1, 2, 3, \ldots \tag{4}$$

Let us assume first that $\operatorname{sign} \alpha' = \operatorname{sign} \alpha_1' = 1$ (for $\operatorname{sign} \alpha' = \operatorname{sign} \alpha_1' = -1$ the proof is analogous). Then $n \geqq 0$, $m \geqq 0$. For every positive integer j we have $\alpha(t + j\pi) = \alpha(t) + (aj + 2jn)\pi$, $\alpha_1(t + j\pi) = \alpha_1(t) + (aj + 2jm)\pi$ and therefore

$$\varphi_{[aj]+2jn}(t) \leqq t + j\pi < \varphi_{[aj]+2jn+1}(t),$$
$$\bar{\varphi}_{[aj]+2jm}(t) \leqq t + j\pi < \bar{\varphi}_{[aj]+2jm+1}(t),$$

where $[x]$ stands for the whole part of x. From this and from (4) we come to

$$\varphi_{[aj]+2jm-1}(t) < \bar{\varphi}_{[aj]+2jm}(t) < \varphi_{[aj]+2jn+1}(t),$$
$$\bar{\varphi}_{[aj]+2jn-1}(t) < \varphi_{[aj]+2jn}(t) < \bar{\varphi}_{[aj]+2jm+1}(t)$$

and thus to

$$[aj] + 2jm - 1 < [aj] + 2jn + 1, \qquad [aj] + 2jn - 1 < [aj] + 2jm + 1.$$

Consequently for every positive integer j: $j(m - n) < 1$, $j(n - m) < 1$, which can occur in case of $m = n$ only.

Let $\operatorname{sign} \alpha' = -\operatorname{sign} \alpha_1'$ and without any loss of generality let $\operatorname{sign} \alpha' = 1$. Then $n \geqq 0$, $m < 0$ and from the equalities $\alpha(t + j\pi) = \alpha(t) + (aj + 2jn)\pi$, $-\alpha_1(t + j\pi) = -\alpha_1(t) + (-aj - 2jm)\pi$ we get

$$\varphi_{[aj]+2jn}(t) \leqq t + j\pi < \varphi_{[aj]+2jn+1}(t),$$
$$\bar{\varphi}_{-[aj]-2jm-1}(t) \leqq t + j\pi < \varphi_{-[aj]-2jm}(t).$$

Herefrom and from (4) we have

$$\varphi_{-[aj]-2jm-2}(t) < \bar{\varphi}_{-[aj]-2jm-1}(t) < \varphi_{[aj]+2jn+1}(t),$$
$$\bar{\varphi}_{[aj]+2jn-1}(t) < \varphi_{[aj]+2jn}(t) < \bar{\varphi}_{-[aj]-2jm}(t).$$

Hence for every positive integer j: $-[aj] - 2jm - 2 < [aj] + 2jn + 1$, $[aj] + 2jn - 1 < -[aj] - 2jm$ and consequently also $-2[aj] - 3 < 2j(m + n)$, $-2[aj] + 1 > 2j(m + n)$. From this $m + n \leqq -1$, $2[aj] + 3 > 2j$. Now j_0 be such a positive integer that $[aj_0] < j_0 - 2$. Then $2j_0 < 2[aj_0] + 3 < 2j_0 - 1$, which is a contradiction. Therefore $\operatorname{sign} \alpha' = \operatorname{sign} \alpha_1'$ and thus $m = n$.

The equations (q) and $(\mathrm{q}_{c,d})$ have the same behaviour and this completes the proof of the Theorem.

Remark 1. From our Theorem it follows in particular that (q) and $(\mathrm{q}_{0,d})$ have the same characteristic multipliers as proved in [3].

Remark 2. Following [1] the phase of $(\mathrm{q}_{0,d})$ is called the second phase of (q). Therefore if α is a phase of (q) and β is the second phase of (q), then from the foregoing Theorem and from [7] follows the existence of $\varepsilon \in \mathfrak{E}$ and $\gamma \in \mathfrak{H}$: $\beta = \varepsilon\alpha\gamma$.

Corollary 3. *Let U be a solution of* $(q_{c,d})$. *Then there exists a solution v of* (q) *and* $\gamma \in \mathfrak{H}$:

$$\frac{v(\gamma(t))}{\sqrt{|\gamma'(t)|}} = U(t) \qquad \text{for } t \in \boldsymbol{R}.$$

Remark 3. Corollary 3 may be also expressed as follows: To every solution u of (q) and to $c \in \boldsymbol{R}$, $d \in \boldsymbol{R}$, $c^2 + d^2 \neq 0$ there exists a solution v of (q) and $\gamma \in \mathfrak{H}$:

$$\frac{v(\gamma(t))}{\sqrt{|\gamma'(t)|}} = \frac{cu(t) + du'(t)}{\sqrt{c^2 - d^2 q(t)}} \qquad \text{for } t \in \boldsymbol{R}.$$

Proof: Let U be a solution of $(q_{c,d})$. Then there exists a phase β of $(q_{c,d})$: $U(t) = \frac{\sin \beta(t)}{\sqrt{|\beta'(t)|}}$, $t \in \boldsymbol{R}$ (see [1]). Let α be a phase of (q). Then according to our Theorem and to the Theorem in [7]: $\beta = \varepsilon\alpha\gamma$ for some $\varepsilon \in \mathfrak{E}$ and $\gamma \in \mathfrak{H}$. Let $v(t) := \frac{\sin(\varepsilon\alpha(t))}{\sqrt{|\varepsilon'(\alpha(t)) \,.\, \alpha'(t)|}}$, $t \in \boldsymbol{R}$. Then v is the solution of (q) and

$$\frac{v(\gamma(t))}{\sqrt{|\gamma'(t)|}} = \frac{\sin[\varepsilon\alpha\gamma(t)]}{\sqrt{|(\varepsilon\alpha\gamma(t))'|}} = \frac{\sin \beta(t)}{\sqrt{|\beta'(t)|}} = U(t).$$

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SOUHRN

POZNÁMKA KE STRUKTUŘE PERIODICKÝCH LINEÁRNÍCH DIFERENCIÁLNÍCH ROVNIC 2. ŘÁDU S DANÝMI CHARAKTERISTICKÝMI KOŘENY

SVATOSLAV STANĚK

Rovnice (q_1): $y'' = q_1(t)$, (q_2): $y'' = q_2(t)$, $q_1 \in C_R^0$, $q_2 \in C_R^0$, $q_1(t + \pi) = q_1(t)$, $q_2(t + \pi) = q_2(t)$, mají stejné chování ([7]) jestliže 1° mají stejné charakteristické kořeny, 2° mají stejnou kategorii ([2]) a 3° jestliže (2) platí pro vhodnou dvojici nezávislých řešení jedné z rovnic (q_1) a (q_2), pak to také platí pro vhodnou dvojici nezávislých řešení druhé z nich a wronskiány obou dvojic řešení mají stejné znaménko.

Nechť $q(t) < 0$, $q \in C_R^2$, $q(t + \pi) = q(t)$. V práci je dokázáno, že pro každé $c \in \boldsymbol{R}$, $d \in \boldsymbol{R}$, $c^2 + d^2 \neq 0$ mají rovnice (q) a ($q_{c,d}$), kde

$$q_{c,d}(t) := q(t) + \frac{cdq(t)}{\sqrt{c^2 - d^2 q(t)}} + \sqrt{c^2 - d^2 q(t)} \left(\frac{1}{\sqrt{c^2 - d^2 q(t)}} \right)'', \qquad t \in \boldsymbol{R},$$

stejné chování.

РЕЗЮМЕ

ЗАМЕТКА К СТРУКТУРЕ ПЕРИОДИЧЕСКИХ ЛИНЕЙНЫХ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ ВТОРОГО ПОРЯДКА С ДАННЫМИ ХАРАКТЕРИСТИЧЕСКИМИ КОРНЯМИ

СВАТОСЛАВ СТАНЕК

Уравнения (q_1) : $y'' = q_1(t)\, y$, (q_2): $y'' = q_2(t)\, y$, где $q_1 \in C_R^0$, $q_2 \in C_R^0$, $q_1(t + \pi) = q_1(t)$, $q_2(t + \pi) = q_2(t)$, имеют одинаковое поведение ([7]) если 1° имеют одинаковые характеристические корни, 2° имеют одинаковую категорию ([2]) и 3° если верно соотношение (2) для некоторой пары независимых решений одного из уравнений (q_1) и (q_2), то тот же самое верно для некоторой пары независимых решений остального уравнения и вронскианы этих пар решений имеют согласный знак.

Пусть $q(t) < 0$, $q \in C_R^2$, $q(t + \pi) = q(t)$. В работе доказывается что для всех $c \in \boldsymbol{R}$, $d \in \boldsymbol{R}$, $c^2 + d^2 \neq 0$ имеют уравнения (q) и ($q_{c,d}$), где

$$q_{c.d}(t) := q(t) + \frac{cdq(t}{\sqrt{c^2 - d^2 q(t)}} + \sqrt{c^2 - d^2 q(t)} \left(\frac{1}{\sqrt{c^2 - d^2 q(t)}} \right)'', \qquad t \in \boldsymbol{R},$$

одинаковое поведение.