Sborník prací Přírodovědecké fakulty University Palackého v Olomouci. Matematika

Ján Futák

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Sborník prací Přírodovědecké fakulty University Palackého v Olomouci. Matematika, Vol. 16 (1977), No. 1, 61--68

Persistent URL: `http://dml.cz/dmlcz/120052`

1977 — ACTA UNIVERSITATIS PALACKIANAE OLOMUCENSIS
FACULTAS RERUM NATURALIUM — TOM 53

ON ZERO POINTS OF SOLUTIONS OF THE n-TH ORDER NON-LINEAR DELAY DIFFERENTIAL EQUATION

JÁN FUTÁK

(*Received on January 16th, L976*)

Consider a non-linear n-th order differential equation of the form:

$$[p(t)\,y^{(n-1)}]' + \sum_{k=0}^{n-1} r_k(t)\,y^{(k)} + \sum_{k=0}^{n-1}\sum_{i=1}^{m} y^{(k)}[h_i(t)]\,q_{ki}(t)\,F_{ki}(y^{(k)}[h_i(t)]) = g(t), \quad (1)$$

where $n \geqq 3$ is an integer and m is a positive integer.

Next suppose that throughout the paper the following assumptions are fulfilled:

$$p, r_k, q_{ki}, h_i \in C[J \equiv \langle t_0, b), R], \qquad t_0 < b \leqq \infty, p(t) > 0, t \in J,$$
$$\inf_{t \in J}[t - h_i(t)] \geqq d > 0, \qquad i = 1, 2, \ldots, m, k = 0, 1, \ldots, n-1,$$
$$F_{ki} \in C[R, \langle 0, \infty)], \qquad i = 1, 2, \ldots, m, k = 0, 1, \ldots, n-1.$$

Denote $I \equiv (t_0, b)$.

A fundamental initial problem is understood to be the following one (see [5], p. 14):

Let $\Phi(t) = \{\Phi_0(t), \Phi_1(t), \ldots, \Phi_{n-1}(t)\}$ be a vector-function defined and continuous on the initial set

$$E_{t_0} = \bigcup_{i=1}^{m} E_{t_0}^i, \qquad \text{where } E_{t_0}^i = (\inf_{t \in J} h_i(t), t_0\rangle.$$

($E_{t_0}^i$ is a closed interval when $h_i(t)$ attains its inf.)

The problem is to find a solution $y(t)$ of (1) on the interval J that fulfils initial conditions:

$$y^{(k)}(t_0+) = \Phi_k(t_0) = y_0^{(k)}, \qquad y^{(k)}[h_i(t)] \equiv \Phi_k[h_i(t)], \qquad h_i(t) < t_0, \quad (2)$$
$$i = 1, 2, \ldots, m, k = 0, 1, \ldots, n-1.$$

Under above-mentioned assumptions one can use the method of steps for finding a solution of the initial problem (1), (2). Thus the existence and uniqueness of this solution is guaranteed.

Results obtained in this paper for solutions (1), (2) represent a certain generalization of those from [1], [2] and [3]. If in (1) we put $g(t) \equiv 0$ and $F_{ki}(z) \equiv 1$, $i = 1, 2, \dots, m$, $k = 0, 1, \dots, n - 1$, we obtain several assertions from [4].

Now introduce essential inequalities for next considerations.

If a, b are arbitrary real numbers then, the inequalities:

$$\pm 2ab \leqq a^2 + b^2 \tag{3}$$

and

$$\pm 2ab \leqq |a|(1 + b^2) \tag{4}$$

are true.

Similarly, if $a > 0$ and x, b are arbitrary real numbers then one can prove:

$$ax^2 + bx \geqq -\frac{b^2}{4a}. \tag{5}$$

Lemma. *Let $y(t)$ be a solution of the initial problem* (1), (2) *and let* $l = \{0, 1, \dots, n-1\}$. *Then $y(t)$ fulfils the following integral identity*:

$$\begin{aligned} p(t)\, y^{(n-1)}(t)\, y^{(l)}(t) &= p(t_0)\, y_0^{(n-1)}\, y_0^{(l)} + \int_{t_0}^{t} p(s)\, y^{(n-1)}(s)\, y^{(l+1)}(s)\, \mathrm{d}s + \\ &+ \int_{t_0}^{t} g(s)\, y^{(l)}(s)\, \mathrm{d}s - \sum_{k=0}^{n-1} \int_{t_0}^{t} r_k(s)\, y^{(k)}(s)\, y^{(l)}(s)\, \mathrm{d}s - \\ &- \sum_{k=0}^{n-1} \sum_{i=1}^{m} \int_{t_0}^{t} y^{(k)}[h_i(s)]\, y^{(l)}(s)\, q_{ki}(s)\, F_{ki}(y^{(k)}[h_i(s)])\, \mathrm{d}s. \end{aligned} \tag{6}$$

Proof. Identity (6) can be obtained by multiplying (1) with $y^{(l)}(t)$ and integrating from t_0 to t for $t \in J$.

Theorem 1. *Let for any* $t \in J$

$$r_k(t) \leqq 0, \qquad q_{ki}(t) \leqq 0, \qquad k = 0, 1, \dots, n-1, i = 1, 2, \dots, m$$

and

a) $g(t) \geqq 0$, *b)* $g(t) \leqq 0$ *hold.*

If $y(t)$ is a solution of the initial problem (1), (2) *that fulfils*

a) $y_0^{(k)} \geqq 0$, $y_0^{(n-1)} > 0$, $k = 0, 1, \dots, n-2$, $\Phi_k(t) \geqq 0$ *for* $t \in E_{t_0}$, $k = 0, 1, \dots, n-1$, (7)

b) $y_0^{(k)} \leqq 0$, $y_0^{(n-1)} < 0$, $k = 0, 1, \dots, n-2$, $\Phi_k(t) \leqq 0$ *for* $t \in E_{t_0}$, $k = 0, 1, \dots, n-1$,

then $y^{(k)}(t)$, $k = 0, 1, \dots, n-1$ *have no zero points on* I.

Proof. The proof will be done in the case (7) a). The case (7) b) can be proved in a similar way.

Suppose that $y^{(n-1)}(t)$ has zeros on J. Denote $t_1 \in I$ the first zero point of $y^{(n-1)}(t)$ for t increasing. With regard to conditions (7) a) it means that $y^{(n-1)}(t) > 0$ for $t \in \langle t_0, t_1)$. Thus $y^{(k)}(t) > 0$, $k = 0, 1, \ldots, n-2$, for $t \in \langle t_0, t_1)$.

From the differential equation (1) with regard to the assumptions of the theorem we obtain:

$$[p(t)\, y^{(n-1)}(t)]' \geqq 0, \qquad \text{for } t \in \langle t_0, t_1).$$

By integrating the last inequality from t_0 to t, $t \in \langle t_0, t_1\rangle$ we obtain

$$p(t)\, y^{(n-1)}(t) \geqq p(t_0)\, y_0^{(n-1)} > 0,$$

and hence $p(t_1)\, y^{(n-1)}(t_1) > 0$ which is a contradiction because $y^{(n-1)}(t_1) = 0$. Therefore $y^{(n-1)}(t) > 0$ for $t \in J$ and also $y^{(k)}(t) > 0$, $k = 0, 1, \ldots, n-2$, for any $t \in I$ must be true.

Theorem 2. *Let for an* $l \in \{0, 1, \ldots, n-2\}$ *and for any* $t \in J$ $r_{l+1}(t) \in C^1(J)$, $r'_{l+1}(t) - 2r_l(t) \geqq 0$ *hold and let further* $r_k(t) \leqq 0$, $k = 0, 1, \ldots, l-1, l+1, \ldots, n-1$, $q_{ki}(t) \leqq 0$, $k = 0, 1, \ldots, n-1$, $i = 1, 2, \ldots, m$, *a)* $g(t) \geqq 0$, *b)* $g(t) \leqq 0$.

If $y(t)$ *is a solution of the initial problem* (1), (2), *which satisfies*

a) $y_0^{(l)} = 0$, $y_0^{(k)} \geqq 0$, $k = 0, 1, \ldots, l-1, l+1, \ldots, n-2$, $y_0^{(n-1)} > 0$, $\Phi_k(t) \geqq 0$, $k = 0, 1, \ldots, n-1$, $t \in E_{t_0}$,

b) $y_0^{(l)} = 0$, $y_0^{(k)} \leqq 0$, $k = 0, 1, \ldots, l-1, l+1, \ldots, n-2$, $y_0^{(n-1)} < 0$, $\Phi_k(t) \leqq 0$, $k = 0, 1, \ldots, n-1$, $t \in E_{t_0}$,

then $y^{(k)}(t)$, $k = 0, 1, \ldots, n-1$ *have no zero points on* I.

Proof. Similarly as in Theorem 1 we shall prove that the function $y^{(n-1)}(t)$ has no zero point on J. Let $t_1 \in I$ be the first point with $y^{(n-1)}(t_1) = 0$. Then in the case a) $y^{(n-1)}(t) > 0$ is true for $t \in \langle t_0, t_1)$.

If we arrange in the identity (6) the third expression on the right and carry on the indicated integration, we get:

$$p(t)\, y^{(n-1)}(t)\, y^{(l)}(t) + \frac{1}{2} r_{l+1}(t)\, [y^{(l)}(t)]^2 = p(t_0)\, y_0^{(n-1)} y_0^{(l)} +$$

$$+ \frac{1}{2} r_{l+1}(t_0)\, [y_0^{(l)}]^2 + \int_{t_0}^{t} p(s)\, y^{(n-1)}(s)\, y^{(l+1)}(s)\, \mathrm{d}s + \int_{t_0}^{t} \left[\frac{1}{2} r'_{l+1}(s) - r_l(s)\right] \times$$

$$\times [y^{(l)}(s)]^2\, \mathrm{d}s - \sum_{\substack{k=0\\ k\neq l\\ k\neq l+1}}^{n-1} \int_{t_0}^{t} r_k(s)\, y^{(k)}(s)\, y^{(l)}(s)\, \mathrm{d}s - \sum_{k=0}^{n-1} \sum_{i=1}^{m} \int_{t_0}^{t} y^{(k)}[h_i(s)]\, y^{(l)}(s) \times$$

$$\times q_{ki}(s)\, F_{ki}(y^{(k)}[h_i(s)])\, \mathrm{d}s + \int_{t_0}^{t} g(s)\, y^{(l)}(s)\, \mathrm{d}s. \tag{8}$$

With regard to the assumptions of the theorem from (8) we get for $t = t_1$ a contradiction, because the left-side is non-positive and the right one is positive. Therefore it must hold that $y^{(n-1)}(t) > 0$ for $t \in J$. Then $y^{(k)}(t) > 0$, $k = 0, 1, \ldots, n-2$ for $t \in I$, too.

Similarly one can prove the assertion of the theorem in the case b).

Theorem 3. *Let for any $t \in J$ the following inequalities hold:*

$$q_{ki}(t) \leqq 0, \qquad k = 0, 1, \ldots, n-1, i = 1, 2, \ldots, m,$$

$$a)\ g(t) \geqq 0,\ b)\ g(t) \leqq 0,$$

$$r_{2l+1}(t) \leqq \int_{t_0}^{t} r_{2l}(s)\,\mathrm{d}s \leqq 0,\ l = 0, 1, \ldots, E\left(\frac{n}{2} - 1\right),$$

where $E(k)$ means the entire part of k,

$$r_{n-1}(t) \leqq 0, \qquad \text{if } n \text{ is an odd integer.}$$

Suppose that

$$[\sum_{k=0}^{n-1} r_k(t)]^2 + [\sum_{k=0}^{n-1} \sum_{i=1}^{m} q_{ki}(t)]^2 + g^2(t) \equiv 0$$

cannot be true on any subinterval of J.

If $y(t)$ is a solution of the initial problem (1), (2) *for which* (7) *hold, then the functions $y^{(k)}(t)$ $k = 0, 1, \ldots, n-1$, have no zero points on I.*

Proof. Let $t_1 \in I$ be the first point where $y^{(n-1)}(t_1) = 0$. Then in the case a) $y^{(n-1)}(t) > 0$ for $t \in \langle t_0, t_1)$ is valid. After integrating equation (1) from t_0 to t for $t \in I$ and arranging we obtain:

1. For n – even

$$p(t)\,y^{(n-1)}(t) + \sum_{l=0}^{\frac{1}{2}n-1} y^{(2l)}(t) \int_{t_0}^{t} r_{2l}(s)\,\mathrm{d}s = p(t_0)\,y_0^{(n-1)} -$$
$$- \sum_{l=0}^{\frac{1}{2}n-1} \int_{t_0}^{t} [r_{2l+1}(s) - \int_{t_0}^{s} r_{2l}(u)\,\mathrm{d}u]\,y^{(2l+1)}(s)\,\mathrm{d}s -$$
$$- \sum_{k=0}^{n-1} \sum_{i=1}^{m} \int_{t_0}^{t} y^{(k)}[h_i(s)]\,q_{ki}(s)\,F_{ki}(y^{(k)}[h_i(s)])\,\mathrm{d}s + \int_{t_0}^{t} g(s)\,\mathrm{d}s. \tag{9}$$

2. For n – odd

$$p(t)\,y^{(n-1)}(t) + \sum_{l=0}^{\frac{1}{2}(n-3)} y^{(2l)}(t) \int_{t_0}^{t} r_{2l}(s)\,\mathrm{d}s = p(t_0)\,y_0^{(n-1)} -$$
$$- \sum_{l=0}^{\frac{1}{2}(n-3)} \int_{t_0}^{t} [r_{2l+1}(s) - \int_{t_0}^{s} r_{2l}(u)\,\mathrm{d}u]\,y^{(2l+1)}(s)\,\mathrm{d}s - \int_{t_0}^{t} r_{n-1}(s)\,y^{(n-1)}(s)\,\mathrm{d}s -$$
$$- \sum_{k=0}^{n-1} \sum_{i=1}^{m} \int_{t_0}^{t} y^{(k)}[h_i(s)]\,q_{ki}(s)\,F_{ki}(y^{(k)}[h_i(s)])\,\mathrm{d}s + \int_{t_0}^{t} g(s)\,\mathrm{d}s. \tag{10}$$

With respect to the assumptions of the theorem we obtain in (9) and (10) for $t = t_1$ a contradiction. Therefore $y^{(n-1)}(t) > 0$ for $t \in J$. Then $y^{(k)}(t) > 0$, $k = 0, 1, \ldots, n-2$ for $t \in I$, too.

Analogically we can prove the case b). Thus the theorem is proved.

In the next three theorems we shall assume that $\int_{t_0}^{b} | g(t) | \, dt < \infty$.

Theorem 4. *Let for any $t \in J$ there the inequalities*

$$r_k(t) \leqq 0, \qquad q_{ki}(t) \leqq 0, \qquad k = 0, 1, \ldots, n - 1, i = 1, 2, \ldots, m,$$

hold.

If $y(t)$ is a solution of the initial problem (1), (2) *which fulfils the conditions*

a) $y_0^{(k)} \geqq 0$, $k = 0, 1, \ldots, n - 2$, $p(t_0) y_0^{(n-1)} - \int_{t_0}^{b} | g(t) | \, dt > 0$, $\Phi_k(t) \geqq 0$ for $t \in E_{t_0}$, $k = 0, 1, \ldots, n - 1$,

b) $y_0^{(k)} \leqq 0$, $k = 0, 1, \ldots, n - 2$, $p(t_0) y_0^{(n-1)} + \int_{t_0}^{b} | g(t) | \, dt < 0$, $\Phi_k(t) \leqq 0$ for $t \in E_{t_0}$, $k = 0, 1, \ldots, n - 1$,

then $y^{(k)}(t)$, $k = 0, 1, \ldots, n - 1$, have no zero points on I.

Proof. From equation (1) we get:

$$[p(t) y^{(n-1)}(t)]' \geqq - \sum_{k=0}^{n-1} r_k(t) y^{(k)}(t) - \sum_{k=0}^{n-1} \sum_{i=1}^{m} y^{(k)}[h_i(t)] \, q_{ki}(t) F_{ki}(y^{(k)}[h_i(t)]) - | g(t) | .$$

After integrating the last inequality from t_0 to t for $t \in I$ with regard to the assumptions of the theorem in the case a) we get:

$$p(t) y^{(n-1)}(t) \geqq p(t_0) y_0^{(n-1)} - \int_{t_0}^{t} | g(s) | \, ds > 0.$$

Further we proved as in Theorem 1.

Theorem 5. *Let for an $l \in \{0, 1, \ldots, n - 2\}$ and any $t \in J$ $r_{l+1}(t) \in C^1(J)$, $r'_{l+1}(t) - 2r_l(t) - | g(t) | \geqq 0$ hold and further let $r_k(t) \leqq 0$, $k = 0, 1, \ldots, l - 1, l + 1, \ldots, n - 1$, $q_{ki}(t) \leqq 0$, $k = 0, 1, \ldots, n - 1$, $i = 1, 2, \ldots, m$.*

Furthermore suppose that

$$[r'_{l+1}(t) - 2r_l(t) - | g(t) |]^2 + [\sum_{\substack{k=0 \\ k \neq l \\ k \neq l+1}}^{n-1} r_k(t)]^2 + [\sum_{k=0}^{n-1} \sum_{l=1}^{m} q_{ki}(t)]^2 \equiv 0,$$

cannot hold any subinterval of J.

If $y(t)$ is a solution of the initial problem (1), (2) *for which* (7) *is fulfilled and*

$$p(t_0) y_0^{(n-1)} y_0^{(l)} + \frac{1}{2} r_{l+1}(t_0) [y_0^{(l)}]^2 - \frac{1}{2} \int_{t_0}^{b} | g(t) | \, dt \geqq 0,$$

then $y^{(k)}(t)$, $k = 0, 1, \ldots, n - 1$ have no zero points on I.

Proof. If we apply to (8) the inequality (4), we obtain:

$$p(t)\,y^{(n-1)}(t)\,y^{(l)}(t)+\frac{1}{2}r_{l+1}(t)\,[y^{(l)}(t)]^2 \geqq p(t_0)\,y_0^{(n-1)}y_0^{(l)}+\frac{1}{2}r_{l+1}(t_0)\,[y_0^{(l)}]^2-$$
$$-\frac{1}{2}\int_{t_0}^{t}|\,g(s)\,|\,\mathrm{d}s+\int_{t_0}^{t}p(s)\,y^{(n-1)}(s)\,y^{(l+1)}(s)\,\mathrm{d}s+\int_{t_0}^{t}\left[\frac{1}{2}r'_{l+1}(s)-r_l(s)-|\,g(s)\,|\right]\times$$
$$\times[y^{(l)}(s)]^2\,\mathrm{d}s-\sum_{\substack{k=1\\k\neq l\\l+1}}^{n-1}\int_{t_0}^{t}r_k(s)\,y^{(k)}(s)\,y^{(l)}(s)\,\mathrm{d}s-$$
$$-\sum_{k=0}^{n-1}\sum_{i=1}^{m}\int_{t_0}^{t}y^{(k)}[h_i(s)]\,q_{ki}(s)\,F_{ki}(y^{(k)}[h_i(s)])\,\mathrm{d}s.$$

The next part of the proof is similar to that of Theorem 2. In a similar way as Theorem 3 one can prove the following theorem:

Theorem 6. *Let for $t\in J$ the assumptions of Theorem* 3 *be valid with the exception that $g(t)\geqq 0$ and $g(t)\leqq 0$, respectively, is replaced by*

a) $$p(t_0)\,y_0^{(n-1)}-\int_{t_0}^{b}|\,g(t)\,|\,\mathrm{d}t\geqq 0,$$

respectively

b) $$p(t_0)\,y_0^{(n-1)}+\int_{t_0}^{b}|\,g(t)\,|\,\mathrm{d}t\leqq 0.$$

If $y(t)$ is a solution of the initial problem (1), (2) *for which* (7) *holds, then $y^{(k)}(t)$, $k=0,1,\dots,n-1$ have no zero points on I.*

Remark. Similar assertions as in Theorems 4, 5 and 6 can be obtained by using (3), only the assumption $\int_{t_0}^{b}|\,g(t)\,|\,\mathrm{d}t<\infty$ must be replaced by $\int_{t_0}^{b}g^2(t)\,\mathrm{d}t<\infty$.

Theorem 7. *Let for an $l\in\{0,1,\dots,n-2\}$ and for $t\in J$ $r_{l+1}(t)\in C^1(J)$, $r'_{l+1}(t)-2r_l(t)>0$ and $r_k(t)\leqq 0$, $k=0,1,\dots,l-1,l+1,\dots,n-1$, $q_{ki}(t)\leqq 0$, $k=0,1,\dots,n-1$, $i=1,2,\dots,m$, hold.*

If $y(t)$ is a solution of the initial problem (1), (2) *for which* (7) *is true and*

$$p(t_0)\,y_0^{(n-1)}y_0^{(l)}+\frac{1}{2}r_{l+1}(t_0)\,[y_0^{(l)}]^2-\int_{t_0}^{b}\frac{g^2(t)}{2r'_{l+1}(t)-4r_l(t)}\,\mathrm{d}t\geqq 0,$$

then $y^{(k)}(t)$, $k=0,1,\dots,n-1$ have no zero points on I.

Proof. Applying (5) to (8) and rearranging, we obtain the expression:

$$p(t)\,y^{(n-1)}(t)\,y^{(l)}(t)+\frac{1}{2}r_{l+1}(t)\,[y^{(l)}(t)]^2\geqq p(t_0)\,y_0^{(n-1)}y_0^{(l)}+$$
$$+\frac{1}{2}r_{l+1}(t_0)\,[y_0^{(l)}]^2-\int_{t_0}^{t}\frac{g^2(s)}{2r'_{l+1}(s)-4r_l(s)}\,\mathrm{d}s+\int_{t_0}^{t}p(s)\,y^{(n-1)}(s)\,y^{(l+1)}(s)\,\mathrm{d}s-$$

$$-\sum_{\substack{k=0\\k\neq l\\l+1}}^{n-1}\int_{t_0}^{t} r_k(s)\,y^{(k)}(s)\,y^{(l)}(s)\,\mathrm{d}s-\sum_{k=0}^{n-1}\sum_{l=1}^{m}\int_{t_0}^{t} y^{(k)}[h_i(s)]\,y^{(l)}(s)\,q_{ki}(s)\,F_{ki}(y^{(k)}[h_i(s)])\,\mathrm{d}s.$$

From the last inequality with regard to the assumptions of the theorem the assertions of this theorem follow.

Theorem 8. *Let for an* $l\in\{0, 1, \ldots, n-1\}$ *and any* $t\in J$ *the inequalities*

$$r_1(t)<0,\qquad r_k(t)\leqq 0,\qquad k=0,1,\ldots,l-1,l+1,\ldots,n-1,$$
$$q_{ki}(t)\leqq 0,\qquad k=0,1,\ldots,n-1,\ i=1,2,\ldots,m$$

and

$$\int_{t_0}^{b}\frac{g^2(t)}{4r_l(t)}\,\mathrm{d}t>-\infty,\qquad \textit{hold.}$$

If $y(t)$ *is a solution of the initial problem* (1), (2) *for which determined by* (7) *and*

$$p(t_0)\,y_0^{(n-1)}y_0^{(l)}+\int_{t_0}^{b}\frac{g^2(t)}{4r_l(t)}\,\mathrm{d}t\geqq 0,$$

then $y^{(k)}(t)$, $k=0, 1, \ldots, n-1$, *have no zero points on* I.

The proof will be carried out similarly as in Theorem 7 by using the inequality (5) in the identity (6).

Author's address: Ján Futák, Department of Mathematics, Vysoká škola dopravná, Žilina.

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$$[r(t)\,y^{(n-1)}(t)]' + \sum_{k=0}^{n-2} P_k(t)\,y^{(k)}(t) + \sum_{k=0}^{n-2} Q_k(t)\,y^{(k)}[h(t)] = 0.$$

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Shrnutí

O NULOVÝCH BODOCH RIEŠENÍ NELINEÁRNEJ DIFERENCIÁLNEJ ROVNICE n-TÉHO RÁDU S ONESKORENÝM ARGUMENTOM

Ján Futák

V práci sú uvedené postačujúce podmienky k tomu, aby riešenie $y(t)$ začiatočnej úlohy (1), (2) a funkcie $y^{(k)}(t)$, $k = 1, 2, \ldots, n - 1$ nemali na intervale I nulový bod. Výsledky sú získané pomocou istých integrálnych identít.

Резюме

О НУЛЕВЫХ ТОЧКАХ РЕШЕНИЙ НЕЛИНЕЙНОГО ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ n-ОГО ПОРЯДКА С ЗАПАЗДЫВАЮЩИМ АРГУМЕНТОМ

Ян Футак

В статье приведены достаточные условия для того, чтобы решение $y(t)$ начальной задачи (1), (2) и функции $y^{(k)}(t)$, $k = 1, 2, \ldots\ldots, n - 1$ не принимали нулевой точки в промежутке I. Результаты выведены при помощи интегральных тождеств.