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Jaroslav Pospíšil

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THE GRAPHICAL FOURIER ANALYSIS METHOD FOR DETERMINATION OF OPTICAL TRANSFER FUNCTION OF OBJECTIVES FROM THE EDGE IMAGE

JAROSLAV POSPÍŠIL

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1. INTRODUCTION

The edge is a suitable element for measuring the optical transfer function of objectives. The adventages are as follows: a sinusoidal wave grating is not necessary, the amount of light is adequate, the Fourier spectrum of the edge is unlimited. A disadventage is the fact, that the Fourier spectrum of an edge is not flat, but decreases according to the spatial frequency, so that the ratio signal to noise becomes insufficient.

At present some methods have been realized for measuring the optical transfer function ([1], [2], [3], [4]). The graphical Fourier analysis method for determination of optical transfer function of objectives from the edge image is described in this article. By this method it is suitable to determine the optical transfer function directly from the edge image. Thus the equipment for measuring the optical transfer function becomes simplier, because we need to obtain experimentally the light distribution function of the edge image only. But the velocity of obtaining the optical transfer function is low.

2. THEORETICAL CONSIDERATIONS

The optical transfer function is represented generally in two dimensions. For experimental purposes the one-dimensional expression of this function is more suitable. Following [5], the optical transfer function is equal to the normalized Fourier transform of the intensity distribution of the line image $E_d(x)$:

$$g(\sigma) = \frac{\int_{-\infty}^{\infty} E_d(x) \exp\left[-i\, 2\pi\sigma x\right] \mathrm{d}x}{\int_{-\infty}^{\infty} E_d(x)\, \mathrm{d}x}, \tag{1}$$

where σ is the spatial frequency expressed in lines per millimeter.

We may express the distribution of the line image $E_d(x)$ in the following form

$$E_d(x) = \frac{dE_e(x)}{dx}, \tag{2}$$

where $E_e(x)$ is the light distribution of the image of a sharp edge by the objective under test.

Substituting from (2) into (1), we obtain

$$g(\sigma) = \frac{\int\limits_{-\infty}^{\infty} \frac{dE_e(x)}{dx} \exp\left[-i\, 2\pi\sigma x\right] dx}{\int\limits_{-\infty}^{\infty} \frac{dE_e(x)}{dx}\, dx}. \tag{3}$$

We can write the expression (3) in the following form:

$$g(\sigma) = \frac{\int_{-\infty}^{\infty} \exp\left[-i\, 2\pi\sigma x\right] d[E_e(x)]}{\int_{-\infty}^{\infty} d[E_e(x)]},$$

or

$$g(\sigma) = q \int_{-\infty}^{\infty} \exp\left[-i\, 2\pi\sigma x\right] d\,[E_e(x)], \tag{4}$$

where

$$q = \frac{1}{\int_{-\infty}^{\infty} d[E_e(x)]}. \tag{5}$$

The equivalent expression of equation (4) for discrete values $k\,\Delta x$ of the coordinate x is as follows:

$$g(n\,\Delta\sigma) = q\{\sum_{k=-m}^{k=-1} \Delta E_e(k\,\Delta x) \exp\left[-i\, 2\pi n\,\Delta\sigma k\,\Delta x\right] + \\ + \sum_{k=0}^{k=m} \Delta E_e(k\,\Delta x) \exp\left[-i\, 2\pi n\,\Delta\sigma k\,\Delta x\right]\},$$

where

$$q = \frac{1}{\sum_{k=-m}^{k=m} \Delta E_e(k\,\Delta x)},$$

and $n\,\Delta\sigma$ are the discrete values of spatial frequency σ. The maximum value m is chosen so, that

$$\Delta E_e(m\,\Delta x) = \Delta E_e(-m\,\Delta x) = 0.$$

If we suppose $k \geqq 0$, then

$$g(n\,\Delta\sigma) = q\left\{\left[\frac{\Delta E_e(o)}{2} + \sum_{k=1}^{k=m} \Delta E_e(k\,\Delta x)\exp(-i\,2\pi n\,\Delta\sigma k\,\Delta x)\right] + \right.$$
$$\left. + \left[\frac{\Delta E_e(o)}{2} + \sum_{k=1}^{k=m} \Delta E_e(-k\,\Delta x)\exp(i\,2\pi n\,\Delta\sigma k\,\Delta x)\right]\right\}, \quad (6)$$

or

$$g(n\,\Delta\sigma) = q[e(n\,\Delta\sigma)^+ + e(n\,\Delta\sigma)^-], \quad (7)$$

where

$$e(n\,\Delta\sigma)^+ = \frac{\Delta E_e(o)}{2} + \sum_{k=1}^{k=m} \Delta E_e(k\,\Delta x)\exp(-i\,2\pi n\,\Delta\sigma k\,\Delta x), \quad (8)$$

$$e(n\,\Delta\sigma)^- = \frac{\Delta E_e(o)}{2} + \sum_{k=1}^{k=m} \Delta E_e(-k\,\Delta x)\exp(i\,2\pi n\,\Delta\sigma k\,\Delta x). \quad (9)$$

The equations (7), (8) and (9) are fundamental for the graphical method for obtaining the optical transfer function from the image of the edge by lens under test. The Fourier transformation of the light distribution $E_e(x)$ is obtainable in the complex plane (Fig. 2). Upon using the graphical way we obtain $e(n\,\Delta\sigma)^+$ and $e(n\,\Delta\sigma)^-$ and then

$$e(n\,\Delta\sigma) = e(n\,\Delta\sigma)^+ + e(n\,\Delta\sigma)^-.$$

Combining the end points of vectors $e(n\,\Delta\sigma)$ for various spatial frequencies $n\,\Delta\sigma$, we obtain the function $e(n\,\Delta\sigma)$ in the complex plane. After normalization (6) we receive the function $g(\sigma)$.

To express the optical transfer function graphically in the form of functions $\tau(\sigma)$ and $\Theta(\sigma)$, which are related by following equations

$$g(\sigma) = \tau(\sigma)\exp[-i\Theta(\sigma)],$$
$$\tau(\sigma) = \sqrt{\mathrm{Re}\,g(\sigma)^2 + \mathrm{Im}\,g(\sigma)^2}$$
$$\Theta(\sigma) = \tan^{-1}\frac{\mathrm{Im}\,g(\sigma)}{\mathrm{Re}\,g(\sigma)},$$

we must to measure the values of the normalized vectors $qe(n\,\Delta\sigma)$ and their angle according to the real axis in the complex plane (Fig. 2). The function $\tau(\sigma)$ is a modulation transfer function and the function $\Theta(\sigma)$ is often called the phase transfer function.

3. THE EXAMPLE OF DETERMINATION OF THE OPTICAL TRANSFER FUNCTION

Fig. 1 shows one of results of measuring the light distribution function $E_e(x)$ of the edge image by lens under test, obtained by scanning method. The result is related to the photographic objective Auto-Takumar Asahi Opt. Co., 1 : 3.5, $f = 35$ mm in on – axis position of the edge for monochromatic light $\lambda = 5\,875$ Å and defocusing $\Delta z = 0.12$ mm. The tangential azimuth of the edge and of the scanning slit was set during the measuring. The relative aperture of the lens under test was 1 : 3.5.

Fig. 2 shows the graphical construction of vectors $e(n\,\Delta\sigma)$ and in Fig. 3 is the result in the form of function $\tau(\sigma)$ (function $\Theta(\sigma)$ is zero because of the on-axis image position). $\tau(\sigma)_c$ is the modulation transfer function obtained directly by the scanning method with the opto-electric Fourier analysis by equipment [6] for the same image conditions.

The numerical results and differences

$$\Delta(\sigma) = \tau(\sigma) - \tau(\sigma)_c,$$

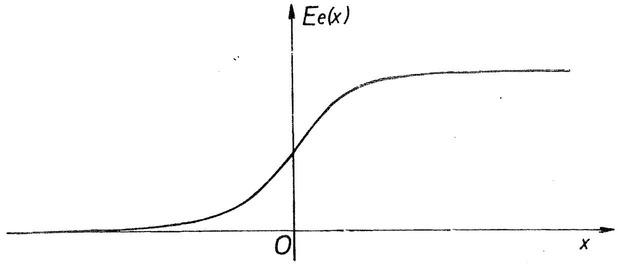

Fig. 1. The graphical representation of the light distribution function $E_e(x)$ of the edge image.

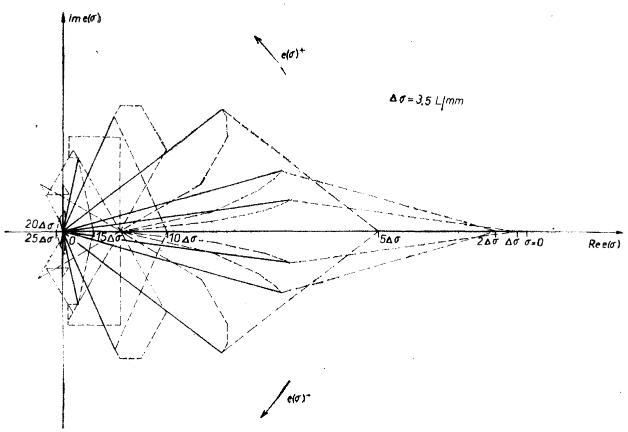

Fig. 2. The graphical construction of vectors $e(n\Delta\sigma)$ for the determination of optical transfer function.

are in table I. We see the sufficient agreement of both results in the frequency range used.

4. CONCLUSIONS

The graphical Fourier analysis method is suitable for determination of the optical transfer function from the edge image by lens under test with sufficient accuracy. The use of this method is limited for frequency range in which the frequency components are sufficiently large.

The advantage of this method is in its simplicity. The method is well suited for laboratory use.

Table I

$\sigma[L/mm]$	$\tau(\sigma)$	$\tau(\sigma)_c$	$\Delta(\sigma)$
0.0	1.000	1.000	0.000
3.5	0.983	0.994	—0.011
6.9	0.948	0.925	0.023
17.3	0.681	0.625	0.056
34.7	0.222	0.200	0.022
52.0	0.065	0.112	—0.047
69.4	0.000	0.012	—0.012
86.7	—0.015	—0.031	0.016

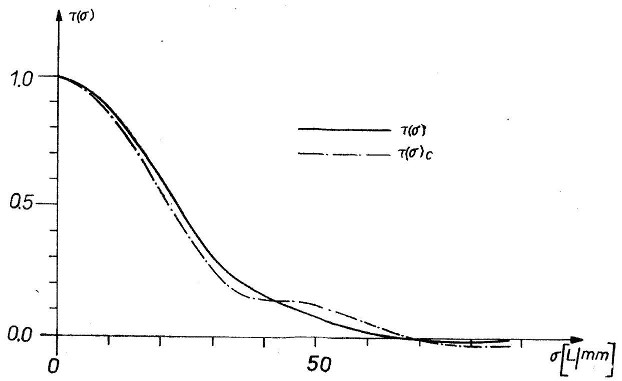

Fig. 3. The modulation transfer functions $\tau(\sigma)$, $\tau(\sigma)_c$, obtained by the described method and by the scanning method with the opto-electric Fourier analysis for the same image conditions.

REFERENCES

[1] *Sayanagi K.:* Oyo-Butsuri, **27,** 10 (1958), 636.
[2] *Rosenhauer K., Rosenbruch K. J.:* Zeitsch. Instr., **67** (1959), 179.
[3] *Rosenbruch K. J., Rosenhauer K.:* Optik, **12,** (1964), 652.
[4] *Sproson W. N.:* BBC Quarterly, **8** (1957), 55.
[5] *Hopkins H. H.:* Proc. Roy. Soc. A **231** (1955), 91.
[6] Instruments for Measuring Optical Transfer Function (JOERA Japan, 1964), 3.

Shrnutí

METODA GRAFICKÉ FOURIEROVY ANALÝZY PRO ZJIŠŤOVÁNÍ OPTICKÉ PŘENOSOVÉ FUNKCE OBJEKTIVŮ Z OBRAZU BŘITU

JAROSLAV POSPÍŠIL

V práci je popsán grafický způsob Fourierovy transformace, vhodný pro zjišťování optické přenosové funkce objektivů z obrazu břitu. Dále je uveden příklad grafického určování optické přenosové funkce pro případ osového zobrazení fotografickým objektivem. Získaný výsledek je srovnán s výsledkem, obdrženým snímací metodou opticko-elektrické Fourierovy analýzy za stejných zobrazovacích podmínek. Ukazuje se dobrý souhlas výsledků a vhodnost popsané metody pro laboratorní zjišťování optických přenosových funkcí.

Резюме

МЕТОД ГРАФИЧЕСКОГО АНАЛИЗА ФОУРЬЕ ДЛЯ УСТАНОВЛЕНИЯ ФУНКЦИИ КОНТРАСТНОСТИ ОБЕКТИВОВ ИЗ ИЗОБРАЖЕНИЯ ОБЕКТА РЕЗКО ПОГРАНИЧНОГО КОНТРАСТА

ЯРОСЛАВ ПОСПИШИЛ

В статье описан способ преобразования Фоурье подходящий для установления функции контрастности объективов из изображения объекта резко пограничного контраста.

Кроме то, в статье показан пример графического установления функции контрастности при изображении фотографическом в оси объективом. Приобретенный результат сравнивает с результатом, полученным прямо методом съемки оптического-электрического анализа Фоурье при одинаковых условиях изображения.

Проявляется согласие результатов с удобностью описанного метода для установления функций контрастности в лаборатории.