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Josef Srovnal

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THE APPLICATION OF PLUECKER'S LINE-GEOMETRY TO THE STUDY OF POLAR PROPERTIES OF QUADRICS

JOSEF SROVNAL

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The polar properties of quadrics in the three-dimensional space may be studied in the usual way by making use of methods of either analytic or synthetic geometry. The main purpose of the present article is to suggest an approach to this problem from the viewpoint of Pluecker's line-geometry.

The subject of our considerations will merely be the regular quadrics in the real projective space P_3 extended to imaginary elements. Any quadric Q_2 in this space may be viewed as ruled and all lines of the space $P_3(i)$ may be mapped on the points of the fourdimensional quadric Q_4 in the five-dimensional space $P_5(i)$. It is thus possible to use this way in our study of some polar properties of quadrics by means of Pluecker's coordinates of the lines.

If an equation of quadric

$$\sum_{i,j=1}^{4} a'_{ij}x_i x_j = 0$$

is given, where $a'_{ij} = a'_{ji}(i, j = 1, 2, 3, 4)$ are real numbers not simultaneously equal to zero, then it is always possible to carry this equation by a convenient real transformation into the canonical form

$$\sum_{i=1}^{4} a_{ii}x_i^2 = 0. \tag{1}$$

Inasmuch as merely regular quadrics are considered, it holds $a_{11}a_{22}a_{33}a_{44} \neq 0$ and to the arbitrary line r of the space $P_3^{(i)}$ defined by different points $A = (a_1, a_2, a_3, a_4)$, $B = (b_1, b_2, b_3, b_4)$ there exists one and only one conjugate polar r' with respect to the quadric (1), which may be determined as the intersection of polar planes α, β of points A, B. The equations of these polars are

$$\sum_{i=1}^{4} x_i f_i(A) = 0, \qquad \sum_{i=1}^{4} x_i f_i(B) = 0,$$

where $f_i(A) = a_{ii}a_i$, $f_i(B) = a_{ii}b_i$ are adjoint linear forms to the quadratic form (1) in points A, B.

The Pluecker's point-coordinates r_{ij} of the line r are the two-rowed determinants

$$r_{ij} = \begin{vmatrix} a_i & a_j \\ b_i & b_j \end{vmatrix} \qquad i \neq j, \qquad i, j = 1, 2, 3, 4,$$

constructed of elements of the matrix

$$\begin{pmatrix} a_1 a_2 a_3 a_4 \\ b_1 b_2 b_3 b_4 \end{pmatrix}$$

in a given order $r_{12}, r_{13}, r_{14}, r_{34}, r_{42}, r_{23}$, which we will denote by r_i, $(i = 1, 2, \ldots, 6)$ according to the scheme:

$$r_1 = r_{12}, \qquad r_2 = r_{13}, \qquad r_3 = r_{14},$$
$$r_4 = r_{34}, \qquad r_5 = r_{42}, \qquad r_6 = r_{23}.$$

Similarly may be determined the plane coordinates $\varrho_i'(i = 1, 2, \ldots, 6)$ of the conjugate polar r' by means of minors of the matrix

$$\begin{pmatrix} f_1(A) & f_2(A) & f_3(A) & f_4(A) \\ f_1(B) & f_2(B) & f_3(B) & f_4(B) \end{pmatrix},$$

whose elements are coefficients at variables in equations of the polar planes α, β. It holds again

$$\varrho_1' = \varrho_{12}', \qquad \varrho_2' = \varrho_{13}', \qquad \varrho_3' = \varrho_{14}',$$
$$\varrho_4' = \varrho_{34}', \qquad \varrho_5' = \varrho_{42}', \qquad \varrho_6' = \varrho_{23}',$$

where

$$\varrho_{ij}' = \begin{vmatrix} f_i(A) & f_j(A) \\ f_i(B) & f_j(B) \end{vmatrix} \qquad i \neq j, \quad i, j = 1, 2, 3, 4.$$

Hence it follows

$$\varrho_{ij}' = a_{ii} a_{jj} r_{ij}.$$

Introducing instead of the polar coordinates $\varrho_i'(i = 1, 2, \ldots, 6)$ of the line r' its point-coordinates r_i', then we obtain from the known proportion

$$r_1' : r_2' : r_3' : r_4' : r_5' : r_6' = \varrho_4' : \varrho_5' : \varrho_6' : \varrho_1' : \varrho_2' : \varrho_3',$$

the relation

$$r_1' : r_2' : r_3' : r_4' : r_5' : r_6' = a_{33}a_{44}r_4 : a_{44}a_{22}r_5 : a_{22}a_{33}r_6 : a_{11}a_{22}r_1 : a_{11}a_{33}r_2 : a_{11}a_{44}r_3. \tag{2}$$

If r is an arbitrary line whose Plucker's point-coordinates are r_i, then the coordinate of its conjugate polar r' with respect to the quadric $\sum_{i=1}^{4} a_{ii}x_i^2 = 0$ are determined by means of the proportion (2).

Let us now observe more closely the conjugate polars r, r' from the viewpoint of their mutual position. Three cases are to be distinguished here: 1) Lines r, r' coincide with a single generating line of a quadric; 2) lines r, r' are crossed and intersect themselves in the point of a quadric; 3) lines r, r' are skew.
When required the line r to be self-polar then for the coordinates of lines r, r' it must hold

$$r_i = kr_i' \qquad k \neq 0, i = 1, 2, \ldots, 6.$$

By applying the proportion (2), these relations can be written in the form

$$\begin{aligned} r_1 &= ka_{33}a_{44}r_4, & r_2 &= ka_{44}a_{22}r_5, \\ r_3 &= ka_{22}a_{33}r_6, & r_4 &= ka_{11}a_{22}r_1, \\ r_5 &= ka_{11}a_{33}r_2, & r_6 &= ka_{11}a_{44}r_3. \end{aligned} \tag{3}$$

Since the numbers r_i cannot vanish simultaneously, it follows e.g. from the first and fourth relation (provided $r_1 \neq 0$) that

$$k^2 = \frac{1}{a_{11}a_{22}a_{33}\,a_{44}}.$$

With respect to the assumption regarding the rank of the quadric (1) the denominator of the fraction on the right side is non-zero; we see also that $k = 0$ is not possible. By extraction we obtain

$$|k| = \frac{1}{\sqrt{a_{11}a_{22}a_{33}a_{44}}}. \tag{4}$$

Hence it follows:

If the quadric $\sum_{i=1}^{4} a_{ii}x_i^2 = 0$ *includes the real self-polar lines then it holds that* $a_{11}a_{22}a_{33}a_{44} > 0$.

By real transformation of type $x_i = \lambda_i x_i'$, the equation of the quadric (1) may be carried into one and only one of the following forms:

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = 0, \tag{5a}$$

$$x_1^2 + x_2^2 + x_3^2 - x_4^2 = 0, \tag{5b}$$

$$x_1^2 + x_2^2 - x_3^2 - x_4^2 = 0. \tag{5c}$$

The quadric (5a) actually satisfies the condition $a_{11}a_{22}a_{33}a_{44} > 0$, but it contains no real lines as this condition is not sufficient. After a modification of the equation (5a) to the form

$$x_1^2 + x_3^2 = -(x_4^2 + x_2^2),$$

this equation may be satisfied so that we put

$$\begin{aligned} x_1 - ix_3 &= \lambda(x_4 + ix_2), &\quad \text{or} \quad x_1 - ix_3 &= -\lambda(x_4 - ix_2), \\ \lambda(x_1 + ix_3) &= -x_4 = ix_2, & \lambda(x_1 + ix_3) &= x_4 + ix_2, \end{aligned}$$

where λ is an arbitrary parameter. In either case the pairs of equations represent different planes determining one line m, n of a quadric respectively. If the Pluecker's plane coordinates of these lines are expressed and carried over to point coordinates, then we obtain

$$\begin{aligned} m_1 &= \mu_4 = (-1 + \lambda^2)\, i, & n_1 &= \nu_4 = (1 - \lambda^2)\, i, \\ m_2 &= \mu_5 = 2\lambda i, & n_2 &= \nu_5 = -2\lambda i, \\ m_3 &= \mu_6 = 1 + \lambda^2 & n_3 &= \nu_6 = \lambda^2 + 1, \\ m_4 &= \mu_1 = (-1 + \lambda^2)\, i, & n_4 &= \nu_1 = (-1 + \lambda^2)\, i, \\ m_5 &= \mu_2 = 2\lambda i, & n_5 &= \nu_2 = 2\lambda i, \\ m_6 &= \mu_3 = \lambda^2 + 1, & n_6 &= \nu_3 = -1 - \lambda^2. \end{aligned}$$

It becomes apparent now that the lines m, n are not real. Either of the lines m, n belongs to another regulus and it holds

$$m_1 = m_4,\ m_2 = m_5,\ m_3 = m_6; \qquad n_1 = -n_4,\ n_2 = -n_5,\ n_3 = -n_6.$$

The quadric (5b) includes no real lines, because it is $a_{11}a_{22}a_{33}a_{44} < 0$, however it does include the imaginary lines, which can easily be verified in the above mentioned way.

The quadric (5c) satisfying the condition $a_{11}a_{22}a_{33}a_{44} > 0$ contains the real self-polar lines. It is easy to find that for the Pluecker's coordinates of these lines it holds

$$\begin{aligned} m_1 &= \mu_4 = 1 + \lambda^2, & n_1 &= \nu_4 = 1 + \lambda^2, \\ m_2 &= \mu_5 = -2\lambda, & n_2 &= \nu_5 = 2\lambda, \\ m_3 &= \mu_6 = -\lambda^2 + 1, & n_3 &= \nu_6 = \lambda^2 - 1, \\ m_4 &= \mu_1 = 1 + \lambda^2, & n_4 &= \nu_1 = -1 - \lambda^2, \\ m_5 &= \mu_2 = 2\lambda, & n_5 &= \nu_2 = 2\lambda, \\ m_6 &= \mu_3 = -1 + \lambda^2, & n_6 &= \nu_3 = -1 + \lambda^2. \end{aligned}$$

It may also be seen from these relations that it holds

$$m_1 = m_4,\ m_2 = -m_5,\ m_3 = -m_6; \qquad n_1 = -n_4,\ n_2 = n_5,\ n_3 = n_6.$$

These results can be derived from the relation (3) and (4) where for the quadric (5c) is $k = \pm 1$.

All our further reasonings might now concern whichever of quadrics (5a), (5b), (5c). However we shall carry out only one reasoning regarding the quadric (5c) including the real generating lines. Thus, according to the foregoing, the Pluecker's coordinates are arbitrary lines m of the regulus ${}^1R$

$$m = (m_1, m_2, m_3, m_1, -m_2, -m_3),$$

and likewise for the line n of the regulus ${}^2R$ it holds

$$n = (n_1, n_2, n_3, -n_1, n_2, n_3).$$

Should these sextuple of numbers define lines, then the points of space $P_5(i)$ corresponding to them must lie on quadric Q_4. In other words, the condition of incidence*) is to be satisfied for either of lines m, n:

$$\varphi(m, m) = m_1^2 - m_2^2 - m_3^2 = 0$$

$$\varphi(n, n) = n_1^2 - n_2^2 - n_3^2 = 0.$$

These relations can be satisfied for the real m_i, n_i, $(i = 1, 2, 3)$ just so, that $m_1 \neq 0$, $n_1 \neq 0$. Geometrically this requirement implies that the lines m, n do not intersect the edge O_1O_2 of the coordinate tetrahedron, i.e. the edge O_1O_2 does not intersect the quadric Q_2 in the real points.

It is easy to see that two different lines of the same regulus do not intersect. If these lines are denoted by m, n it holds for them

$$\varphi(m, n) = 2(m_1n_1 - m_2n_2 - m_3n_3).$$

With respect to the assumption $m_1 \neq 0$, $n_1 \neq 0$ we can put $m_1 = n_1$. When it is required the lines m, n to be incindent, then it must hold $\varphi(m, n) = 0$ and at the same time $\varphi(m, n) = 0$, $\varphi(n, n) = 0$. Combining these relations yields

$$-(m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2 = 0,$$

and with respect to the equality $m_1 = n_1$ it follows also $m_2 = n_2$, $m_3 = n_3$, i.e. the lines m, n coincide. Thus, two lines of the same regulus are incident if and only if they are identical. Being different, they have no point in common.

On the other hand, however, each two lines of distinct reguli intersect because of $m \in {}^1R$, $n \in {}^2R$ is $\varphi(m, n) = 0$, as immediately follows by direct calculation.

Let us now turn our attention to conjugate polars r, r' being incident, yet not coinciding with the only generating line of a quadric. It is then $\varphi(r, r') = 0$ and from relations (3) and (4) we can again provide the coordinates of line r' in case the coordinates of line r are given:

$$r = (r_1, r_2, r_3, r_4, r_5, r_6),$$

$$r' = (r_4, -r_5, -r_6, r_1, -r_2, -r_3).$$

If ϱ is the tangent plane of a quadric defined by lines r, r' with a contact point R in the intersection of lines r, r', then an arbitrary tangent of the quadric lying in the tangent plane ϱ may be expressed as line of a linear bundle whose base lines are r, r' and the vertex the point R. For any such tangent it holds:

$$t = \lambda_1 r + \lambda_2 r'. \tag{6}$$

The line t' which is polar conjugate to the line t belongs to this bundle as well, as

*) The condition of incidence of two arbitrary lines p, q in Pluecker's coordinates p_i, q_i $(i = 1, 2, \ldots, 6)$ will be denoted by symbol $\varphi(p, q) = 0$, where $\varphi(p, q) = p_1q_4 + p_2q_5 + p_3q_6 + p_4q_1 + p_5q_2 + p_6q_3$.

follows from the rank of matrix constructed of the coordinates of lines t', r, r' which equals to 2. Let us now determine the coordinates of generating lines of a quadric which the tangent plane ϱ is passing through. Should the line t be the generating line of a quadric then for its coordinates it must hold $t_1 = t_4$, $t_2 = -t_5$, $t_3 = -t_6$ or $t_1 = -t_4$, $t_2 = t_5$, $t_3 = t_6$, according to which regulus the line t belongs. Replacing the coordinates of lines t by the coordinates of conjugate polars r, r' from (6), where we besides express the coordinates of lines r' by means of the coordinates of line r, we derive for the ratio of numbers $\lambda_1 : \lambda_2$ the relation

$$\left| \frac{\lambda_1}{\lambda_2} \right| = 1,$$

where the value 1 of this ratio determines the line of the regulus ${}^1R$, and the value -1 determines the line of the regulus ${}^2R$. Thus, for lines $m \in {}^1R$, $n \in {}^2R$ there hold the relations

$$m = r + r' \qquad n = r - r',$$

where r, r' are the arbitrary conjugate quadrics and m, n are the generating lines wherein the tangent plane, determined by the polars r, r', intersects the quadric.

Before treating the skew conjugate polars, let us observe one property of conjugate polars. If r, r', s, s' are two pairs of conjugate polars then it holds

$$\varphi(r, s) = \varphi(r', s'),$$
$$\varphi(r, s') = \varphi(r', s),$$

as results from the direct calculation. If now one pair of conjugate polars, e.g. the pair s, s' coicides with the self-polar line m of the quadric then it holds $\varphi(r, m) = \varphi(r', m)$ for $m \in {}^1R$ and $\varphi(r, m) = -\varphi(r', m)$ for $m \in {}^2R$. Herefrom the following corollary may be derived:

If the polar r intersects an arbitrary generating line of a quadric then the conjugate polar r' intersects this line as well.

If now r, r' are skew conjugate polars and the line r intersects the quadric in points M, N and the line r' in points M', N', then no two of these points coincide (because it would contradict the requirement for skewness of both polars) and there are generating lines of the quadric passing pairwise through each of these points. Each of these lines belongs to another regulus. These generating lines determine on the quadric the distorted quadrangle with vertices M, M', N, N', whose segments are the conjugate polars r, r'. Thus, each pair of conjugate polars determines in a one-to-one way the corresponding distorted quadrangle. The generating lines determining the sides of this quadrangle are established from the given conjugate polars r, r' by solving the system of equations

$$\varphi(r, m) = 0, \qquad \varphi(r, n) = 0,$$
$$\varphi(m, m) = 0, \qquad \varphi(n, n) = 0,$$

where $m \in {}^1R$, $n \in {}^2R$. Any of these systems consists of one linear and of one quadratic equation in coordinates of lines m, n, the solution of which yields four number triples determining the generating lines sought. Conversely too, if an arbitrary distorted quadrangle on a quadric is given, determined by lines m, n, p, q, then the corresponding conjugate polars will be fixed by solving the system

$$\begin{aligned} \varphi(r, m) &= 0, \\ \varphi(r, n) &= 0, \\ \varphi(r, p) &= 0, \\ \varphi(r, q) &= 0, \\ \varphi(r, r) &= 0. \end{aligned}$$

The first four equations are linear in $r_i (i = 1, 2, \ldots, 6)$, the last is a quadratic one. Since r_i are homogeneous coordinates, then by solving this system we arrive to an unique determination of two values regarding the ratio of coordinates r_i. It is just the coordinate of the line r and its conjugate polars r' which are determined by the generating lines m, n, p, q, of the quadric.

Let me conclude this section with some remarks of how the foregoing may be used to prove some theorems of conjugate polars. If r, r', s, s' are two pairs of conjugate polars, and these four lines are hyperboloidic (i.e. in a hyperboloidic position, where infinitely many of their transversals exist), then we may express an arbitrary one of them, e.g. the line r, as a linear combination of those remained, i.e. it holds

$$r = \lambda_1 r' + \lambda_2 s + \lambda_3 s'.$$

It can be shown that choosing $\lambda_1 = 1$ it holds $\lambda_2 = -\lambda_3$.

Suppose now that the conjugate polars r, r' and two lines m, n of the regulus ${}^1R$ intersect the line c which does not belong to the regulus ${}^2R$. Then it holds that

$$\begin{aligned} \varphi(r, c) &= 0, \\ \varphi(r', c) &= 0, \\ \varphi(m, c) &= 0, \\ \varphi(n, c) &= 0. \end{aligned}$$

Summarizing the first two relations and joining this sum to the remaining two relations, then after a modification and introduction of $c_1 + c_4 = \mu_1$, $c_5 - c_2 = \mu_2$, $c_6 - c_3 = \mu_3$ we obtain

$$\begin{aligned} (r_1 + r_4)\,\mu_1 + (r_2 - r_5)\,\mu_2 + (r_3 - r_6)\,\mu_3 &= 0, \\ m_1\mu_1 + \qquad m_2\mu_2 + \qquad m_3\mu_3 &= 0, \\ n_1\mu_1 + \qquad n_2\mu_2 + \qquad n_3\mu_3 &= 0. \end{aligned} \tag{7}$$

Since at least one of the numbers μ_1, μ_2, μ_3 must be different from zero (for if $\mu_1 = \mu_2 = \mu_3 = 0$ the line would belong to c of the regulus ${}^2R$, which contradicts our assumption), the relations (7) may be satisfied only so that it holds

$$\begin{vmatrix} r_1 + r_4 & r_2 - r_5 & r_3 - r_6 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{vmatrix} = 0;$$

consequently the matrix constructed of the coordinates of the lines r, r', m, n has rank 3. Thus the lines r, r', m, n are linearly dependent and hence hyperboloidic. From the properties of conjugate polars then follows that any of their transversals intersects them in the quadruple of points which are separated harmonically. This result may be summed up in the theorem:

If two conjugate polars and two generating lines belonging to the same regulus of a quadric intersect one and the same line not belonging to another regulus, then the four first lines are hyperboloidic and are forming the harmonic division.

This theorem immediately proves the second theorem:

Given an arbitrary polar tetrahedron of a quadric, then the generating lines of the same regulus intersecting one edge of this tetrahedron (and thus the opposite edge as well) are hyperboloidic with the second and the third pair of the opposite edges of the tetrahedron and they are separated with these pairs harmonically.

Let us now consider two pairs of conjugate polars r, r', s, s' being hyperboloidic. The pair r, r' determines the generating lines $m, m' \in {}^1R$ and $p, p' \in {}^2R$ and similary the pair s, s' determines the lines $n, n' \in {}^1R$, $q, q' \in {}^2R$ constructing the skew quadrangles on the quadric. It holds

$$\begin{aligned} \varphi(r, m) &= 0, & \varphi(s, n) &= 0, \\ \varphi(r, m') &= 0, & \varphi(s, n') &= 0, \\ \varphi(r, p) &= 0, & \varphi(s, q) &= 0, \\ \varphi(r, p') &= 0, & \varphi(s, q') &= 0. \end{aligned}$$

Expressing these relations in the coordinates and putting

$$r = \lambda_1 r' + \lambda_2 s + \lambda_3 s',$$

where for simplicity we choose $\lambda_1 = 1$ and take into consideration the relation $\lambda_2 = -\lambda_3$, it may be written

$$r = r' + \lambda_2(s - s').$$

Replacing by means of this equation the coordinates of the lines s, s' by coordinates of the lines r, r' in relations

$$\begin{aligned} \varphi(s, n) &= 0, \\ \varphi(s, n') &= 0, \end{aligned}$$

and multiplying these by the coefficient $\lambda_2 \neq 0$ and joining to them the relations

$$\begin{aligned} \varphi(r, m) &= 0, \\ \varphi(r, m') &= 0, \end{aligned}$$

then on the ground of the hyperboloidic position of lines m, m', n, n', it can be derived that these lines coincide with the only pair of skew lines each of which belongs to the regulus ${}^1R$.

Conversely, if the conjugate polar r, r', s, s' intersect two different generating lines m, m' belonging to the same regulus of a quadric, then it holds that

$$\begin{aligned} \varphi(r, m) &= 0, \\ \varphi(r, m') &= 0, \\ \varphi(s, m) &= 0, \\ \varphi(s, m') &= 0. \end{aligned}$$

From the first and third equation it follows $m_1 : m_2 : m_3 = k_1 : k_2 : k_3$, from the second and fourth equation $m'_1 : m'_2 : m'_3 = k_1 : k_2 : k_3$, where

$$k_1 = \begin{vmatrix} r_5 - r_2 & r_6 - r_3 \\ s_5 - s_2 & s_6 - s_3 \end{vmatrix}, \quad k_2 = \begin{vmatrix} r_6 - r_3 & r_1 + r_4 \\ s_6 - s_3 & s_1 + s_4 \end{vmatrix}, \quad k_3 = \begin{vmatrix} r_1 + r_4 & r_5 - r_2 \\ s_1 + s_4 & s_5 - s_2 \end{vmatrix}.$$

If at least one of the determinants k_1, k_2, k_3 is different from zero then the lines m, m' coincide, which contradicts the assumption. Thus it must be $k_1 = k_2 = k_3 = 0$, whence it follows that the rank of the matrix constructed of the coordinates of lines r, r', s, s' is equal to 3. This shows that these lines are linearly dependent i.e. hyperboloidic. The foregoing results are expressed in the following theorem:

Two pairs of polar lines are hyperboloidic if and only if two different generating lines of the same regulus of a quadric intersect.

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Resumé

UŽITÍ PŘÍMKOVÉ GEOMETRIE KE STUDIU POLÁRNÍCH VLASTNOSTÍ KVADRIK

JOSEF SROVNAL

Tato práce pojednává o polárních vlastnostech kvadrik s použitím přímkové geometrie. Autor vychází ze známých výsledků odvozených v trojrozměrném projektivním prostoru, rozšířeném o imaginární prvky. Úvahy, které jsou v práci obsaženy, se týkají pouze regulárních kvadrik.

Zavedeme-li imaginární body, můžeme považovat každou kvadriku za přímkovou a protože lze každé přímce přiřadit jedno-jednoznačně uspořádanou šestici čísel, možno touto cestou studovat polární vlastnosti kvadrik pomocí Plückerových souřadnic přímek.

Předmětem úvah jsou přímky polárně sdružené vzhledem ke kvadrice, klasifikované z hlediska jejich vzájemné polohy (polární přímky splývající, různoběžné a mimoběžné).

V závěru je uvedeno, jak lze metodou přímkové geometrie dokázat některé věty o sdružených polárách.

Zusammenfassung

DIE ANWENDUNG DER LINIENGEOMETRIE IM STUDIUM DER POLAREIGENSCHAFTEN VON QUADRIKEN

JOSEF SROVNAL

Diese Arbeit betrachtet einige Polareigenschaften von Quadriken bei Anwendung der Plückerschen Liniengeometrie. Dabei geht der Author von den bekannten, im dreidimensionalen um imaginäre Elemente erweiterten Projektivraum abgeleiteten Resultaten aus. Die in der Arbeit enthaltenen Betrachtungen finden ihre Anwendung nur auf reguläre Quadriken.

Durch Einführung der imaginären Punkte kann man jede Quadrik als eine Regelquadrik ansehen und in diesem Sinne mit Hilfe der Plückerschen Linienkoordinaten die Polareingenschaften von Quadriken studieren, da man jeder Linie eine eineindeutig angeordnete Sechserreihe von Zahlen zuordnen kann.

Die besagten Betrachtungen umfassen zur Quadrik polarkonjugierte Linien in bezug auf ihre gegenseitige Lage (zusammenfallende, sich schneidende und windschiefe Polarlinien).

Abschliessend wird es gezeigt, wie einige Sätze von konjugierten Polaren mittels der Liniengeometrie sich beweisen lassen.