Abstracts of theses in mathematics

Commentationes Mathematicae Universitatis Carolinae, Vol. 44 (2003), No. 4, 751--765

Persistent URL: http://dml.cz/dmlcz/119430

ABSTRACTS OF THESES* IN MATHEMATICS

defended recently at Charles University, Prague

CLOSE 2-GROUPS

ZHUKAVETS Natalia, Department of Algebra, Faculty of Mathematics and Physics, Charles University, Sokolovská 83, 186 75 Prague 8, Czech Republic
(March 22, 2002; supervisor A. Drápal)

The thesis is devoted to a detailed study of the one quarter distances of small 2-groups. However, some general results are obtained as well.

For two finite groups $G(\circ)$ and $G(*)$ on the same underlying set, their (Hamming) distance $\mathrm{dist}(\circ, *)$ is defined as the number of pairs $(a, b) \in G \times G$ with $a \circ b \neq a * b$. We say that groups G_1 and G_2 of the same order can be positioned in distance d, if there exist groups $G(\circ) \cong G_1$ and $G(*) \cong G_2$ with $\mathrm{dist}(G(\circ), G(*)) = d$.

In 2000 Aleš Drápal proved that if $G(\circ)$ and $G(*)$ are two 2-groups of order n and $\mathrm{dist}(\circ, *) < n^2/4$, then $G(\circ) \cong G(*)$. This estimate is the best possible one, and since it seems that there exist really a lot of examples of non-isomorphic 2-groups that are in the one quarter distance, the following conjecture has been formulated:

Conjecture. Let H and K be two finite 2-groups of the same order n. Then there exists a chain of groups $G_1, \ldots, G_m$ such that $H \cong G_1$, $K \cong G_m$, $m > 1$, and for each i, $1 \leq i \leq m-1$, G_i and G_{i+1} can be positioned in the distance $n^2/4$.

One of the main result of the thesis is a deeper analysis of the above conjecture. We describe two different methods that construct, given a group $G(\circ)$, groups $G(*)$ with $\mathrm{dist}(\circ, *) = n^2/4$. First we consider groups with isomorphic normal subgroups of index 2 and 4, and then generalize the obtained results. In new constractions it is assumed that the common subgroup is normal both in $G(\circ)$ and $G(*)$, and that the corresponding quotient group is cyclic or dihedral.

All the mentioned methods are applied to groups of order 16 and 32, and the corresponding tables appear in Appendices. We have found several classes of finite 2-groups for which the Conjecture holds. Such classes for example are (1) finite abelian 2-groups, (2) finite 2-groups which possess an elementary abelian subgroup of index 2, (3) 2-groups of order ≤ 32. One section is devoted to non-abelian 2-groups with a cyclic subgroup of index 2.

We used the GAP computer package to obtain certain results and to verify certain conjectures.

*An equivalent to PhD.

SIMPLE COLORINGS AND SIMPLE HOMOMORPHISMS

SMOLÍKOVÁ Petra, Department of Applied Mathematics, Faculty of Mathematics and Physics, Charles University, Malostranské nám. 25, 118 00 Prague 1, Czech Republic
(April 26, 2002; supervisor J. Nešetřil)

Graph coloring problems arise from a variety of applications including scheduling, frequency assignment, and others and constitute one of the most active research domains in graph theory. However, even the simplest graph coloring problem turns out to be very difficult to solve.

We restrict out attention to *digraphs* — sets of vertices joined by a set of arcs which contains no loops (arcs joining a vertex with itself). An *oriented coloring* of a digraph G is a mapping which assigns to each vertex of G an integer (color) in such a way that vertices joined by an arc are assigned distinct colors and if there is an arc aiming from a vertex colored by i to a vertex colored by j, then there is no arc in the opposite direction in G — from a vertex colored by j to a vertex colored by i. This already classical notion introduced by Raspaud and Sopena has attracted a lot of attention in the last decade.

We attempt to answer the question whether the results obtained for oriented colorings of digraphs are still valid if vertices joined by an arc can share the same color (constant mappings are excluded as trivial in this case). We call such a generalized coloring a *simple coloring* of a digraph. We introduce further generalizations of classical "oriented" notions as simple homomorphism, simple chromatic number, simple clique, etc. We deal with simple chromatic numbers of planar graphs, acyclic simple colorings, the number of simple cliques between all tournaments, the possible sets of colors of a simple coloring of a digraph, the complexity of simple colorings, density of the class of all digraphs ordered by the existence of simple homomorphisms between them, category defined by simple homomorphisms and further concepts.

We show that by omitting the condition of any pair of adjacent vertices in a digraph being colored by distinct colors, the notion of an oriented coloring is not deeply simplified and many results known for oriented colorings may be extended to its "simple" variants. We prove, for instance, that if there is a planar digraph with oriented chromatic number k, then there is a planar digraph which needs k colors for any of its simple colorings.

COFINAL CHAINS IN MODULE THEORY AND REPRESENTATIONS OF DISTRIBUTIVE LATTICES

RŮŽIČKA Pavel, Department of Algebra, Faculty of Mathematics and Physics, Charles University, Sokolovská 83, 186 75 Prague 8, Czech Republic
(April 4, 2002; supervisor J. Trlifaj)

P.C. Eklof and J. Trlifaj [1], [5] defined a class of strongly dense lattices, the

lattices possessing a continuous cofinal strictly decreasing chain (shortly c.d.c.) of their nonzero elements. The length of its shortest c.d.c. is defined to be the dimension of a strongly dense lattice. For a strongly dense modular lattice of uncountable dimension κ, the Γ-invariant is defined, which is an element of the Boolean algebra $\mathcal{B}(\kappa)$ of all subsets of κ modulo the filter generated by all closed unbounded sets.

A right module over an associative ring is strongly uniform provided its submodule lattice is strongly dense. The dimension and the Γ-invariant of a strongly uniform module are defined as the dimension and the Γ-invariant of its submodule lattice. We were originally motivated by the following problem:

The Γ-invariant problem. *For an uncountable cardinal κ, which elements of $\mathcal{B}(\kappa)$ different from the element represented by κ are Γ-invariants of strongly uniform modules over regular rings?* [1, Problem 2.3], *or* [5, Problem 3]

We apply the idea of P.C. Eklof and J. Trlifaj [1], who proved that every algebraic distributive lattice isomorphic to the lattice of two-sided ideals of a locally matricial algebra is at the same time isomorphic to the lattice of submodules of a right module over a locally matricial algebra (which is a unit-regular ring). Thus, we study the question which algebraic lattices are the lattices of two-sided ideals of locally matricial algebras.

It is well known that such a lattice is distributive. We present constructions of G.M. Bergman [1] who proved that every algebraic distributive lattice with at most $\aleph_0$ compact elements and every algebraic strongly distributive lattice (i.e., a lattice isomorphic to the lattice of all hereditary subsets of a partially ordered set) are isomorphic to the lattice of two-sided ideals of a locally matricial algebra. On the other hand, F. Wehrung [6] constructed an algebraic distributive lattice with $\aleph_2$ compact elements which is not isomorphic to the lattice of two-sided ideals of any regular ring.

The main result of the thesis is the realization of every algebraic distributive lattice whose poset of compact elements is closed under finite meets as the lattice of two-sided ideals of a locally matricial algebra [4]. Applying the Eklof's and Trlifaj's idea, this result leads to the solution of the Γ-invariant problem.

The lattice of two-sided ideals of a locally matricial algebra is isomorphic to the ideal lattice of its Grothendieck group. The Grothendieck group of a regular ring is a dimension group, that is a directed, unperforated partially ordered Abelian group with interpolation. We construct an algebraic distributive lattice of size $\aleph_2$ which is not isomorphic to the ideal lattice of any dimension group [3].

References

[1] Bergman G.M., *Von Neumann regular rings with tailor-made ideal lattices*, unpublished notes (1986).

[2] Eklof P.C., Trlifaj J., *Γ-invariants for dense lattices*, Algebra Universalis **40** (1998), 427–445.

[3] Růžička P., *A distributive semilattice not isomorphic to the maximal semilattice quotient of the positive cone of any dimension group*, J. Algebra **228** (2003), 290–300.
[4] Růžička P., *Lattices of two-sided ideals of locally matricial algebras and the* Γ*-invariant problem*, preprint.
[5] Trlifaj J., *Modules over non perfect rings*, Advances in Algebra and Model Theory (1996), Gordon & Breach, Philadelphia, 471–492.
[6] Wehrung F., *A uniform refinement property of certain congruence lattices*, Proc. Amer. Math. Soc. **127** (1999), 363–370.

TIME SERIES WITH CHANGING PARAMETERS

JANEČKOVÁ Hana, Department of Probability and Mathematical Statistics, Faculty of Mathematics and Physics, Charles University, Sokolovská 83, 186 75 Prague 8, Czech Republic
(June 18, 2002; supervisor Z. Prášková)

The thesis deals with several generalizations of a basic random coefficient autoregressive (RCA) model. Research is focused mainly on a heteroscedastic RCA model of order 1 (RCA(1)). The studied model is of the form $X_t = b_t X_{t-1} + Y_t$, where b_t are random autoregressive parameters and $\{Y_t\}$ is an error process with changing variances $\sigma_t^2 = \mathrm{E}\, Y_t^2$.

In this model ordinary least squares (OLS), weighted least squares (WLS), conditionally weighted least squares (CWLS) and maximum likelihood (ML) estimators of a parameter $\beta = \mathrm{E}\, b_t$ are studied. Their strong consistency and asymptotic normality are proved under various sets of assumptions. These results are reached for RCA(1) models with $\{b_t - \beta\}$ and $\{Y_t\}$ being either independent sequences or martingale differences. Asymptotic properties are derived in cases when variances σ_t^2 and $\sigma_B^2 = \mathrm{E}(b_t - \beta)^2$ are both known and unknown. Furthermore, it is shown that under normality assumption CWLS and ML estimators coincide.

In addition to β, unknown parameters σ_t^2 and σ_B^2 are estimated. Two different methods of estimation are presented and in both cases proofs of strong consistency and asymptotic normality of all estimators are given under assumption of seasonal heteroscedasticity.

Besides RCA models, some generalizations of autoregressive conditional heteroscedastic (ARCH) models are defined and studied. Non-stationary ARCH(q) model of the form $X_t = \varepsilon_t \left(a_t + \sum_{i=1}^{q} c_i X_{t-i}^2 \right)^{\frac{1}{2}}$ with varying parameters a_t is introduced. Similarly as in case of RCA models, strong consistency and asymptotic normality of unknown parameters in non-stationary ARCH(1) model are proved. Moreover, it is shown that heteroscedastic RCA(p+q) process and AR(p) process with non-stationary ARCH(q) errors are second order equivalent.

As a theoretical tool, strong laws of large numbers and central limit theorems are used. Concepts of martingale differences, martingales, mixingales and near-epoch dependent (NED) processes are utilized. Theoretical results are ac-

companied by several numerical simulations.

STOCHASTIC CALCULUS IN ECONOMETRICS

VEČEŘ Jan, Department of Probability and Mathematical Statistics, Faculty of Mathematics and Physics, Charles University, Sokolovská 83, 186 75 Prague 8, Czech Republic
(June 18, 2002; supervisor J. Štěpán)

This thesis is a collection of three independent, but closely related articles. Each chapter represents one paper. Therefore references to equations, theorems, etc., are done within the same chapter.

In Chapter 1, we study options on a traded account. In terms of the actions available to the buyer, the options we study are more general than a class of options known as *passport options*; in terms of the model of the underlying asset they are more restrictive. Using probabilistic techniques, we find the value of these options, the optimal strategy of the buyer, and the hedging strategy the seller should use in response to a (not necessarily optimal) strategy by the buyer.

In Chapter 2, arithmetic average Asian options are studied. It is observed that the Asian option is a special case of the option on a traded account. The price of the Asian option is characterized by a simple one-dimensional partial differential equation which could be applied to both continuous and discrete average Asian option. The article also provides numerical implementation of the pricing equation. The implementation is fast and accurate even for low volatility and/or short maturity cases.

In Chapter 3, we study passport options when the underlying stock is driven by a Poisson jump process. We prove that "short when ahead long when behind" strategy remains optimal if the contract is terminated at the time of the k-th jump.

FINITE ELEMENT METHOD FOR A PROBLEM WITH NONLINEAR BOUNDARY CONDITIONS

SVÁČEK Petr, Department of Numerical Mathematics, Faculty of Mathematics and Physics, Charles University, Sokolovská 83, 186 75 Prague 8, Czech Republic
(June 27, 2002; supervisor K. Najzar)

In the present thesis we are concerned with a particular class of numerical techniques for determining the approximate solution of partial differential equations: finite element method. The thesis can be divided mainly into two fields: the first part is devoted to the numerical analysis of a solution of a nonlinear problem, the second part is devoted to several topics connected with the implementation of the higher-order FEM. With the use of the implementation on the problem from the first part provides a number of numerical results.

The numerical analysis part of the study is devoted to the solution of an elliptic problem in bounded two- or three- dimensional domains. The problem is equipped with the nonlinear Newton boundary condition. This problem appears, e.g., in the modelling of electrolysis procedures. The nonlinear term describes the turbulent flow in a boundary layer.

First, we consider the numerical approximation of a problem on a two dimensional polygonal domain. We are concerned with the approximation with the aid of higher order elements, the solvability, and the convergence. Next, with the help of uniform monotonicity, error estimates are shown. The effect of approximation of a nonpolygonal domain with a polygonal one is studied in the case of use of linear finite elements. The error estimates for such a situation are shown.

Further, next part of the thesis is devoted to the numerical solution of the problem in a three-dimensional domain, where we considered the situation with a limited nonlinearity only. For such a case, it is possible to show existence and uniqueness of the solution and convergence to the weak solution. The general situation remains still unsolved.

In the third part, numerical tests are presented. The tests were computed by using the implementation of higher order finite element method.

HARMONIC ANALYSIS IN RIEMANNIAN GEOMETRY

DUŠEK Zdeněk, Mathematical Institute, Faculty of Mathematics and Physics, Charles University, Sokolovská 83, 186 75 Prague 8, Czech Republic
(September 11, 2002; supervisor O. Kowalski)

A *g.o. space* is a homogeneous Riemannian manifold $(G/H, g)$ on which every geodesic is an orbit of a one-parameter subgroup of the group G. Each g.o. space gives rise to certain rational maps called *geodesic graphs*. We are particularly interested in the case when the geodesic graphs are of nonlinear character. Then we study the algebraic structure of geodesic graphs and particularly the degree of nonlinearity. Up to recently only linear geodesic graphs or non-linear geodesic graphs of degree 2 were observed. The existence of a linear geodesic graph in G/H is equivalent with the natural reductivity of the g.o. space G/H.

A *Riemannian g.o. manifold* (the name *Riemannian g.o. space* was used earlier) is a homogeneous Riemannian manifold (M, g) on which every geodesic is an orbit of a one-parameter group of isometries. Every Riemannian g.o. manifold can be identified with a g.o. space $(G/H, g)$, where G is a convenient subgroup of the full connected group of isometries of M. In general, there may exist more than one such group G. A *degree* of a Riemannian g.o. manifold (M, g) is the minimum of degrees of all geodesic graphs constructed for all possible g.o. spaces $(G/H, g)$ where $G \subseteq I_0(M)$ and $M = G/H$. The existence of a linear geodesic graph in some g.o. space $(M = G/H, g)$ is equivalent with the natural reductivity of the manifold M.

H-type groups have been introduced by A. Kaplan. The 6-dimensional H-type group with 2-dimensional center was the first example of a Riemannian g.o. manifold which is not naturally reductive. All H-type groups which are g.o. spaces have been classified by C. Riehm.

In the thesis, the general case of the H-type group with 2-dimensional or 3-dimensional center and the 13-dimensional H-type group with 5-dimensional center are studied. The main result of the thesis are geodesic graphs of degree 3 and 6 on the 13-dimensional H-type group with 5-dimensional center. There are two choices how to express this group as a g.o. space $(G/H, g)$. For the choice of the smaller group $G \subset I_0(M)$ the minimal degree of a geodesic graph in $(G/H, g)$ is equal to 6 and for the choice $G = I_0(M)$ the minimal degree is equal to 3. This group provides the first examples of g.o. spaces for which the minimal degree of nonlinearity of geodesic graphs is higher than two (it is 6, or 3, respectively). The degree of the manifold itself is equal to 3.

MEASURABILITY OF SETS OF POINTS OF DIFFERENTIABILITY OF FUNCTIONS BANACH SPACES

ŠMÍDEK Michal, Department of Mathematical Analysis, Faculty of Mathematics and Physics, Charles University, Sokolovská 83, 186 75 Prague 8, Czech Republic
(September 27, 2002; supervisor P. Holický)

This thesis is motivated by a problem of a measurability of a set of points at which a function on a Banach space is differentiable. It is known that the set of points at which a mapping on a normed linear space to a Banach space is Fréchet differentiable, is Borel measurable of type $\mathcal{F}_{\sigma\delta}$, [Z, Theorem 2]. This set is $\mathcal{G}_\delta$ for a convex continuous function $f : X \longrightarrow \mathbb{R}$ on a Banach space, [P, Proposition 1.25].

The first chapter consists of a paper written together with Petr Holický and Luděk Zajíček [HSZ] which deals with the measurability of the set of points at which a continuous convex function f on a Banach space X is Gâteaux differentiable (hereafter notated $G(f)$). By Mazur's theorem, $G(f)$ is a dense $\mathcal{G}_\delta$ set for a convex continuous function f on a separable Banach space, [P, 1.20]. In [T, Theorem 1] M. Talagrand constructed a continuous convex function f on $l_1(\mathfrak{c})$ such that the set $G(f)$ is not (in particular) nonmeasurable. In [HSZ], such a function is constructed on two types of spaces. The first type are spaces with a fundamental system of uncountable cardinality. The second type of spaces are spaces which contain a subspace isomorphic to $l_1(\Gamma)$ for Γ of uncountable cardinality. It gives a negative answer to a question of Rainwater in [R, p. 320] whether for a convex continuous function $f : X \longrightarrow \mathbb{R}$ the set $G(f)$ is Borel measurable, whenever the space X is GDS.

The differentiability of a convex continuous function on a Banach space is connected with properties of special upper semicontinuous multivalued mappings. The second chapter deals with measurability of a selector for upper semicontinuous

multivalued mappings which are not necessarily mappings with compact convex values. There is used a property of a topological space to have a σ–scattered network for the definition of the range of a multivalued mapping. Some corollaries are given which strengthen and modify the results of Jayne and Rogers [JR, Theorem 4 and 8] and Hansell's and Oncina's Theorem [HO, Theorem 3.1].

The third chapter consist of a paper [Sm] which deals with measurability of two concrete subsets of the space $\mathcal{B}_b([0,1])$ of bounded Borel measurable functions $f : [0,1] \longrightarrow \mathbb{R}$ endowed with the supremum metric.

References

[HO] Hansell R.W., Oncina L., *Generalized First Class Selectors for Upper Semi-Continuous Setvalued maps in Banach Spaces*, preprint.

[HSZ] Holický P., Šmídek M., Zajíček L., *Convex functions with non-Borel set of Gâteaux differentiability points*, Comment. Math. Univ. Carolinae **39.3** (1998), 469–482.

[JR] Jayne J.E., Rogers C.A., *Borel selectors for upper semi-continuous set-valued maps*, Acta Mathematica **155** (1985), 41–79.

[P] Phelps R.R., *Convex Functions, Monotone Operators and Differentiability*, vol. 1364, Springer–Verlag, 1993.

[R] Rainwater J., *A class of null sets associated with convex functions on Banach spaces*, Bull. Austral. Math. Soc. **42** (1990), 315–322.

[Sm] Šmídek M., *Measurability of some sets of Borel measurable functions on* $[0,1]$, Acta Math. Univ. Comeniae **LXIV** (1995), no. 2, 247–254.

[T] Talagrand M., *Deux exemples de fonctions convexes*, C.R. Acad. Sci. Paris, Serie A – 461 **288** (1979), 461–464.

[Z] Zajíček L., *Fréchet differentiability, strict differentiability and subdifferentiability*, Czechoslovak Math. J. **41, 116** (1991), 471–489.

MARKOV MODELS IN SIGNALLING SYSTEMS

KLAPKA Štěpán, Department of Numerical Mathematics, Faculty of Mathematics and Physics, Charles University, Sokolovská 83, 186 75 Prague 8, Czech Republic

(September 27, 2002; supervisor V. Janovský)

This thesis deals with some special problems, which originate in Discrete Time Markov Chains (DTMC) modelling of the part of signalling systems. The goal of the thesis is to apply a new understanding of numerical analysis to the modelling of the safety relevant systems. Safety in context of this thesis means the probability that a system will either perform its functions correctly or will discontinue its functions in a manner that does not disrupt the operation of other systems or compromise the safety of any people associated with the system. Safety is a measure of the fail-safe capability of a system; if the system does not operate correctly, the system is at least expected to fail in a safe manner.

The first chapter introduces to the problem of DTMC modelling by 10 general questions. The first 6 questions are answered in the second chapter. In this chapter the basic notations are defined and the basic results of DTMC modelling theory are presented. The identities for the first (II.21) and second (II.23) statistical moment play an important role in the second chapter. These identities are introduced in a special non-irreducible form.

In the third chapter identities for statistical moments of a general form for irreducible DTMC are considered. At the first step the identity (III.2) is established for a matrix F of the global probability of transition. The existence and uniqueness of the solution (III.5-6), which is delivered from (III.2), is stated in Theorem 3.1. The proof is based on the construction of the blocks of the matrix F. This result is useful for the following Theorems (3.2, 3.3) for the first and second statistical moment in the irreducible case. In example A and B at the end of the third chapter it is demonstrated how the results can be used for calculation.

The fourth chapter deals with the numerical method for computation of a stationary probability vector of DTMC. Interesting results are presented in part 4.6. This part deals with the convergence of the aggregation/disaggregation method (IAD), as Algorithm 4.1, for a special matrix structure. The timeout modelling usually derives this structure of the transition matrix P. The main results are in the scope of Theorems 4.5 and 4.6. For a special rank-one structure of the non-diagonal blocks of the transition matrix P the IAD method has the finite number of iterations.

In the fifth chapter the methods for computation of the first statistical moment are examined. In part 5.2 some numerical experiments are presented. The computation method is based on the identities for statistical moments. The results point to a very slow convergence of the iterations in case of the nearly decomposable matrix.

The sixth chapter is focused on several technologies, which lead to Markov models, as they are Continuous Time Markov Chain or Discrete Time Markov Chain. The main purpose is to sketch the sources of the transition matrix P.

The seventh chapter is dedicated to Petri Nets (PN). The reason for this is that Pleas/Transition PN or Timed PN methodology are usually used for a qualitative safety analysis of signalling systems. The stochastic extension of PN leads to the Continuous Time Markov Chain models, too.

In the last chapter two models that are used in developing the Electronic Line Block System ABE-1 in the company AŽD Praha Ltd. are presented. The first model represents a subsystem of ABE-1, which controls Line Block direction (Line Block Direction Control - LBDC). The LBDC is modelled in an error mode, when the control interface is failed. The statistical characteristics of DTMC model are the quantitative measure of the LBDC algorithm ability of fail-safe behaviour. The next model describes a communication subsystem of ABE-1. The key role of the communication subsystem is played by the measurement of a bit-rate error

of a transmission layer. It is impossible to transmit the safety relevant data, when the bit-rate error is greater than a required limit. The DTMC model of the communication subsystem demonstrates this property.

KAREL RYCHLÍK (1885–1968)

HYKŠOVÁ Magdalena, Mathematical Institute, Faculty of Mathematics and Physics, Charles University, Sokolovská 83, 186 75 Prague 8, Czech Republic
(October 23, 2002; supervisor J. Bečvář)

The thesis is devoted to the life and work of the Czech mathematician Karel Rychlík and it consists of eight parts. The first of them contains the brief sketch of Rychlík's life, the survey of his scientific and pedagogical activities and the detailed biography. The following five chapters discuss particular groups of Rychlík's publications: *Algebra and Number Theory*; *Works on Mathematical Analysis*; *Textbooks, Popularizing Papers, Translations*; *Karel Rychlík and Bernard Bolzano*; *Other Works on History of Mathematics*. These chapters are conceived separately, each of them is provided with the conclusion and the list of references.

The seventh chapter presents the list of Rychlík's publications, reviews and lectures at Charles University, at the Czech Technical University and in the Union of Czech Mathematicians and Physicists. The thesis ends with the pictorial appendix, the survey of abbreviations and the name index.

ON ORIENTED COVERS AND DECOMPOSITIONS OF EULERIAN GRAPHS

MAXOVÁ Jana, Department of Applied Mathematics, Faculty of Mathematics and Physics, Charles University, Malostranské nám. 25, 118 00 Prague 1, Czech Republic
(October 30, 2002; supervisor J. Nešetřil)

In this thesis we concentrate on two famous graph theoretic problems concerning circuits in graphs. The *Faithful circuit cover problem* (given a weight function w on $E(G)$ when there exists a collection $\mathcal{C}$ of circuits in G such that every edge appears in exactly $w(e)$ elements of $\mathcal{C}$) was partly solved by Alspach, Goddyn and Zhang. A special case of this problem ($w(e) = 2$ for every edge) is a remarkably difficult problem known as the *Circuit double cover conjecture* stated independently by Szekeres and Seymour.

These two problems, in particular their oriented versions, motivated this thesis and are central to much of it. We first give an overview of various approaches to the CDC conjecture. In Chapter 3 we introduce a new approach to the conjecture via *generalized chord diagrams* of Eulerian trails. These diagrams are generalizations of *Gauss diagrams* of knots.

In Chapters 4 and 5 we demonstrate some important differences between the

circuit double covers and the oriented circuit double covers. We prove that in many cases (such as faithful circuit covers, fractional faithful circuit covers, small circuit covers or perfect path double covers) there is a large gap between the corresponding notions in the directed and undirected case. In the last chapter we concentrate on another possible approach to the CDC conjecture via *compatible circuit decompositions* of Eulerian graphs. In this thesis we address this problem in full generality by considering the set $\mathcal{S}(G, \mathcal{F})$ of all decompositions of G into closed trails with respect to a given set $\mathcal{F}$ of forbidden transitions at every vertex. We further study transformations between elements of the set $\mathcal{S}(G, \mathcal{F})$ and discuss the algorithmic complexity of problems concerning the size of the set $\mathcal{S}(G, \mathcal{F})$.

SEPARATION AXIOMS IN DENSE SUBSETS

MURTINOVÁ EVA, Department of Mathematical Analysis, Faculty of Mathematics and Physics, Charles University, Malostranské nám. 25, 118 00 Prague 1, Czech Republic
(November 13, 2002; supervisor P. Simon)

The thesis continues research started by O. T. Alas et al. in [ATTWY] and by A.V. Arhangel'skii and L. Ludwig in [AL]. Some problems raised in the quoted papers are solved there. It is based on a triple of articles: [M1], [M2], [M3].

In the first part the author considers the question if (or under which conditions) there are dense subsets of topological spaces with stronger separation axioms than the originating spaces. It is shown that every countable Hausdorff space X with $\pi w(X) < \mathfrak{d}$ has a dense Urysohn subspace and that

$$\mathfrak{p} = \min\{w(X);\ |X| = \aleph_0\ \ \&\ \ X \text{ is a } T_2 \text{ space without dense } T_3 \text{ subspace}\}.$$

This equality completes a result from [ATTWY] by construction of a countable Hausdorff space of weight $\mathfrak{p}$, which has no dense regular subspaces.

A further example of a first countable Hausdorff space (of cardinality $\max(\aleph_2, \mathfrak{c})$) without dense Urysohn subspaces is presented. It demonstrates (together with [AW, Example 2]) that the results obtained for countable spaces cannot be generalized to first countable spaces.

In the chapter on topological operations it is shown that dense subsets of spaces entering finite products need not reflect properties of dense subsets of the products — the following theorem provides many counterexamples.

Theorem. *For every topological space X there is a space Y such that $X \times Y$ contains a dense hereditarily normal zero-dimensional subspace.*

At every step of the proof, the spaces associated to X one starts with have strong separation properties. For lower separation axioms, the author describes pairs of spaces without dense subspaces satisfying a stronger axiom, whose product has such a subspace.

In the second part a weakening of axioms T_3 and T_4 is considered, in the sense that dense subsets of closed sets are separated instead of the whole sets. For normality, the notions (α-normal, β-normal) have been introduced in [AL]. We add an analogous definition of α-regularity.

The thesis contains examples showing that these notions do not coincide with each other neither with classical separation axioms. Namely, there is a β-normal Tychonoff space which is not normal, and, consequently, a method from [Jo] allows us to modify it to obtain a regular non-Tychonoff β-normal space. Also, an α-normal non-regular Hausdorff space can be produced as a quotient. Another example of this sort with simpler structure is constructed: there is an α-normal non-regular Hausdorff space, first countable at each point except one.

On the other hand, every first countable α-regular (in particular, α-normal) Hausdorff space is regular. These classes also coincide for countable spaces of a small character: if X is a countable α-regular T_1 space, $w(X) < \mathfrak{p}$, then X is regular.

At least consistently, the restriction of weight cannot be omitted: under Martin's Axiom for countable partially ordered sets or $\mathfrak{b} = \mathfrak{c}$ or under a somewhat weaker assumption there exists a countable Hausdorff α-normal non-regular space.

Then the problem of preservation of the mentioned properties by topological operations is discussed. The separation axioms we deal with inherit to subspaces in an analogous manner to normality and regularity. In contrast to regularity, α-regularity is not preserved by products:

Theorem. *For every non-regular T_1 space X there is $\kappa \leq \chi(X)$ such that $X \times A(\kappa)$ is not α-regular, where $A(\kappa)$ stands for the one-point compactification of a discrete set of cardinality κ.*

References

[AL] Arhangel'skii A. V., Ludwig L., *On α-normal and β-normal spaces*, Comment. Math. Univ. Carolinae **42.3** (2001), 507–519.

[ATTWY] Alas O. T., Tkachenko M. G., Tkachuk V. V., R. G. Wilson, I. V. Yaschenko, *On dense subspaces satisfying stronger separation axioms*, Czech Jour. Math. **51** (2001), 15–27.

[AW] Alas, O. T., R. G. Wilson, *Dense subspaces of first countable Hausdorff spaces*, Questions Answers Gen. Top. **17** (1999), 199–202.

[Jo] Jones F. B., *Hereditarily separable, non-completely regular spaces,*, Topology Conference (Virginia Polytech. Inst. and State Univ., Blacksburg, VA, 1973), Lecture Notes in Math., vol. 375, Springer, Berlin, 1974, pp. 149–152.

[M1] Murtinová E., *A β-normal Tychonoff space which is not normal*, Comment. Math. Univ. Carolinae **43.1** (2002), 159–164.

[M2] Murtinová E., *On α-regularity*, submitted.

[M3] Murtinová E., *Remarks on dense subspaces*, to appear in Czechoslovak Math. J.

SOME PROBLEMS OF RECURSIVE METHODS IN TIME SERIES ANALYSIS

FRANĚK Petr, Department of Probability and Mathematical Statistics, Faculty of Mathematics and Physics, Charles University, Sokolovská 83, 186 75 Prague 8, Czech Republic
(November 27, 2002; supervisor T. Cipra)

The thesis is focused on state-space models and on modification of the Kalman filter recursions with improved sensitivity to outlying observations.

Chapters 1 and 2 contain brief summary of the state-space models theory and the derivation of the filtering and smoothing problems solutions — the Kalman filter and smoother. State-space models' parameter estimation is addressed in Chapter 3.

In the following chapters the linear state-space model

$$\mathbf{y}_t = \mathbf{H}_t\mathbf{x}_t + \mathbf{v}_t$$
$$\mathbf{x}_t = \mathbf{F}_t\mathbf{x}_{t-1} + \mathbf{G}_t\mathbf{w}_t$$

is assumed in the thesis with m-variate observations $\mathbf{y}_t$ and n-variate unobservable state vectors $\mathbf{x}_t$, system matrices $\mathbf{H}_t$, $\mathbf{F}_t$ and $\mathbf{G}_t$ assumed to be known and satisfying stability conditions sufficient for the Kalman filter in this system to converge to a steady state. The random error terms $\mathbf{w}_t$ and $\mathbf{v}_t$ are assumed to be mutually and serially independent.

Two sources of outlying observations are dealt: the additive outliers model (AO) with $\mathbf{v}_t \sim (1-\varepsilon)\,N(\mathbf{0}, \mathbf{Q}_t) + \varepsilon\,H(\mathbf{v})$ and $\mathbf{w}_t \sim N(\mathbf{0}, \mathbf{R}_t)$ and the innovation outliers model (IO) with $\mathbf{v}_t \sim N(\mathbf{0}, \mathbf{Q}_t)$ and $\mathbf{w}_t \sim (1-\varepsilon)N(\mathbf{0}, \mathbf{R}_t) + \varepsilon\,H(\mathbf{w})$, $H(\cdot)$ being some heavy-tailed distribution, $\mathbf{Q}_t$ and $\mathbf{R}_t$ being assumed to be known covariance matrices and $\varepsilon \in (0; 1)$ being the parameter determining the ratio of the contaminated observations.

The effect of the outlying observations produced by AO or IO model on the estimates produced by the Kalman filter performed with the assumption of non-contaminated error terms $\mathbf{v}_t$ and $\mathbf{w}_t$ is then discussed in Chapter 4. In Chapters 5 and 6 robustified alternatives to the Kalman filter are proposed. The new filtering algorithms are then used in Chapter 7 to obtain smoothing algorithm and EM-estimates of the system matrices with improved sensitivity to outlying observations.

Most of the algorithms presented in the thesis were implemented in the form of XploRe *quantlets* — macros written in the XploRe programming language.

THE MATHEMATICAL ANALYSIS OF COMPONENTS OF THE RISK IN THE INSURANCE OF PERSONS

ČÍŽEK Martin, Department of Probability and Mathematical Statistics, Faculty of Mathematics and Physics, Charles University, Sokolovská 83, 186 75 Prague 8, Czech Republic
(April 4, 2002; supervisor P. Mandl)

The aim of submitted thesis was to create models describing a risk of an insurance of persons.

The first chapter includes an application of a model for single premium made by Parker [2]. The total measure of the risk is divided into two components, investment and insurance risk. A sensitivity of the model to parameters is examined in a numerical illustration. A change of some parameters of model of interest rate and portion of profit were found to have only small influence, while the other changes are more significant. The difference of insurance risk is mostly greater then the difference of investment risk. The influence of technical amendments on the value of the risk and its components is also proved.

The model of a risk reserve for portfolio of policies is described in the second chapter. The aim of work was to determine its minimal initial value, for which all future values are nonnegative with sufficiently high probability. The influence of parameters on this initial value is examined in the numerical illustration. The variation of some parameters (the portion of profit, a technical interest rate) has only small influence, while another (a size of portfolio, a kind of insurance, a way of premium, a measure of credibility, initial costs) have distinctly higher influence. Absolutely the greatest influence has the size of portfolio and it depends neither on the kind of insurance nor on the way of premium.

The model describing a risk and its components in a health insurance is created in the third chapter.

The submitted thesis proved an acquisition of examination of variables describing the risk in the insurance of persons. There exist also other methodologies of studying of the risk measure, for example a loss or a gain is described in [1].

References

[1] Bühlmann H. (1995), *Life Insurance with Stochastic Interest Rates*, Financial Risk in Insurance (Ed. G. Ottaviani), Springer-Verlag, Berlin, 1995, pp. 1–24.

[2] Parker G., *Stochastic Analysis of the Interaction between Investment and Insurance Risk*, North American Actuarial Journal **1** (1997), 55–84.

MODELS OF RANDOM SETS AND THEIR STATISTICAL ANALYSIS

MRKVIČKA Tomáš, Mathematical Institute, Faculty of Mathematics and Physics, Charles University, Sokolovská 83, 186 75 Prague 8, Czech Republic
(December 12, 2002; supervisor J. Rataj)

A general non-stationary point process with known non-stationary structure and unknown intensity parameter λ is considered. The family of linear (first order), unbiased estimators for the intensity λ is considered. There is found a necessary and sufficient condition for a linear, unbiased estimator to have minimum variance from all linear, unbiased estimators of the intensity λ. The solution of the condition leads to the Fredholm integral equation of the second type.

The optimal estimator is numerically calculated for Matérn cluster process and Matérn hard-core process. Afterward, it is shown that there are "nearly" no other processes then Poisson and Cox for which $\widehat{\lambda_C}$ (the natural estimator of λ which counts the points only) is optimal.

At the end there are simulations which compare the optimal estimator with the natural one.