Abstracts of theses in mathematics

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ABSTRACTS OF THESES* IN MATHEMATICS
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STRUCTURE OF STEADY RINGS

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(January 8, 1999; supervisor J. Trlifaj)

Dually slender modules are exactly those modules M for which the covariant functor $Hom(M, -)$ commutes with direct sums, particularly finitely generated modules satisfy this condition. For some types of rings, the dually slender modules form a much larger class than the finitely generated ones. The rings for which dually slender modules coincide with the finitely generated ones were termed right steady.

The main goal of the thesis is to provide a modul-theoretical criterion of steadiness for all rings and ring-theoretical criterion of steadiness for particular classes of semiartinian rings.

It is shown in the thesis that a ring is steady if and only if a certain modul contains no infinitely generated dually slender module. It is also proved that the representative class of dually slender modules over a commutative regular ring is only a set and it is given an estimate of the cardinality of each dually slender module. Further, it is shown that commutative regular rings are steady if and only if R^* contains no infinitely generated dually slender submodule. Our approach to steadiness is also through a study of the ω-complete anti-filter, $\mathcal{S}_R$, of all right steady (two-sided) ideals of the ring R. Assume that all ideals of R are countably generated. Then $\mathcal{S}_R$ has a maximal element, M. Now, simply, R is right steady iff $M = R$. It is shown that an arbitrary valuation ring S is steady iff its prime radical $rad(S)$ is countably generated and $S/rad(S)$ contains only countable chains of ideals. There is a couple of examples in the thesis illustrating the limits of the methods employed.

As regards semiartinian rings it is proved in the thesis that a commutative semiartinian ring is steady if and only if no its factors contains an infinitely generated dually slender member of Loewy chain. Moreover, this characterization of steadiness is extended to a larger class of semiartinian rings. They are presented some examples of both steady and non-steady semiartinian rings. In addition, it is shown a relationship between pure extension and steadiness of rings and proved steadiness of polynomial rings in countable many variables over some skew-fields.

*An equivalent to PhD.

ON SOME ASPECTS OF SUBDIFFERENTIALITY OF FUNCTIONS ON BANACH SPACES

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The thesis consists of two independent parts. The first part is called 'Some applications of a subdifferential calculus for non-convex functions on Asplund spaces'. We use a subdifferential calculus for lower semi-continuous functions on Asplund spaces to show that a mean value theorem studied by P.D. Loewen on Banach spaces with Fréchet differentiable norm can be generalized to Asplund spaces. Next we prove existence and uniqueness of the solutions of the Hamilton-Jacobi equations considered in the book of R. Deville, G. Godefroy and V. Zizler. Again, using the subdifferential calculus we extend their result replacing the assumption of existence of a Fréchet differentiable Lipschitz bump function by the assumption of Asplundness.

The second part is called 'On Asplund functions and projectional resolutions of identity'. If the subspace Y of X^* generated by the set of subdifferentials of a continuous convex function f defined on a Banach space X exhibit properties similar to those of duals of Asplund spaces then we call f an Asplund function according to W.K. Tang. We prove that Y admits a projectional resolution of identity. If f is an Asplund function on a weakly Lindelöf determined Banach space then we show that Y admits a weak* lower semicontinuous norm and f can be uniformly approximated by Fréchet differentiable convex functions. All these results are generalizations of analogous well-known results that concern continuous convex functions on Asplund spaces.

CONSTRUCTION OF MARKOV KERNELS WITH APPLICATION FOR MOMENT PROBLEM SOLUTION

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Having given moment problem (or another condition on probability measures on Polish space $\mathbb{Y}$) we usually find the set of solutions to be non-unique. Consider now that for given Polish space $\mathbb{X}$ and all $x \in \mathbb{X}$ we have given admissible set of probability measures $\mathcal{P}_x$ on $\mathbb{Y}$ which consist of solution of given problem with initial condition x. Then for any deterministic initial condition there exists solution, and the natural question is the existence of solution with non degenerate initial distribution λ on $\mathbb{X}$.

This problem is closely related with the existence of universally measurable map $x \in \mathbb{X} \mapsto P^x \in \mathcal{P}_x$ since then there exists probability measure P^λ on $\mathbb{X} \times \mathbb{Y}$

defined as $P^\lambda(B) = \int_{\mathbb{X}} P^x(B_x)\lambda(dx)$, where $B \subset \mathbb{X}\times\mathbb{Y}$ is a universally measurable set, and B_x its section in x. This map forms Markov kernel, which represents conditional distribution of η given $\xi = x$ for two-dimensional random vector (ξ, η) on $\mathbb{X}\times\mathbb{Y}$. Since the set $\mathcal{P} = \bigcup\{x\} \times \mathcal{P}_x$ forms a graph of some multifunction, the problem can be solved using the theory of set-valued functions, namely we study a measurability of set $\mathcal{P}$.

The first part of the thesis contains basic results from measure theory concerning analytic and universally measurable sets, and set-valued analysis. In the second part we study Markov kernels and its support. Namely we are interested in the relation between supports of P^x, λ and P^λ.

The set $\mathcal{P}$ is studied in the third part. We define $(\mathcal{P}, \lambda)$-vector as a random vector (ξ, η) such that the marginal distribution of ξ is λ, and conditional distributions of $(\eta|\xi = x)$ are on $\mathcal{P}_x$. We show sufficient conditions for the existence of such vector, and for the existence of $(\mathcal{P}, \lambda)$-vector with maximal support. Several examples of $(\mathcal{P}, \lambda)$-vectors solving given problems are given as corollaries to the existence theorems.

In the last part we define (b, f) or more general (b, F) problem as a problem to find universally measurable Markov kernel P such that $b(x, P^x) = f(x)(\in F(x))$ a.s. $[\lambda]$, where $b : \mathbb{X} \times \mathcal{M}_1(\mathbb{Y}) \to \mathbb{Z}, f : \mathbb{X} \to \mathbb{Z}(F : \mathbb{X} \to 2^{\mathbb{Z}})$ are given. We prove that under mild conditions on measurability of b, f, F there exists such a Markov kernel, and under bit stronger conditions it is possible to find a solution with maximal support. At the end we illustrate the problem on generalized moment and barycentrical problems.

PROPERTIES OF THE BGG RESOLUTION ON THE SPHERES

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In the last time, there has been great effort in the study of conformally invariant differential operators on manifolds with a given conformal structure. A special subclass of them — so called standard invariant operators, are coming together in sequences called (generalized) Bernstein-Gelfand-Gelfand sequences (BGG sequences for a short).

In the thesis, BGG sequences on the conformally flat homogeneous model (i.e. on the sphere) are studied using elementary tools of representation theory. It is shown, that any sequence of conformally invariant operators acting among global sections of the same natural bundles on the sphere as the BGG sequence does necessarily form a complex. It is a consequence of representational theoretical properties of the spaces of global sections and no specific information concerning a form of invariant operators is needed. Secondly, it is shown that exactness of the sequence on the sphere (up to the last place) is equivalent to certain spectral properties of corresponding invariant operators. It opens a possibility that there can be a direct verification of the mentioned spectral properties of corresponding invariant operators, which would give a simple and elementary proof of the exactness of the BGG resolution on the sphere. Moreover — in fact this was the original aim of the study — the equivalence can be used in other direction for explicit computation of the form of kernels and images of all standard operators on the sphere.

The work is split into three chapters. The first chapter is introductory and reviews a necessary background. The results of the second chapter form the core of the work. It is discussed, separately for even and odd dimensions, the BGG sequence of standard operators, and the kernels and the images of all of them — including also the twistor operators - are explicitly described. The third chapter includes some applications of the previous results (only in the even case), mainly the computation of exact forms of kernels of higher twistor operators on the sphere, Euclidean space and hyperbolic space with metrics of constant curvatures.

CONSTRUCTION OF BERNSTEIN-GELFAND-GELFAND RESOLUTIONS FOR ALMOST HERMITIAN SYMMETRIC STRUCTURES

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The subject of the thesis belongs to the theory of invariant differential operators on manifolds with Almost Hermitian Symmetric Structures, that generalizes the classical conformal structure on manifolds.

There exists a class of standard operators that can be described in a constructive way. The essential tool is finite dimensional representation theory of semisimple Lie algebras. For a given Lie algebra $\mathfrak{g}$, the operators are constructed as projections of associated bundles to certain representation spaces onto irreducible components.

A natural question is a systematization of standard operators. We consider sequences of operators — so-called BGG (Bernstein-Gelfand-Gelfand) sequences.

The BGG sequences are usually described by means of Hasse diagram. It determines the form (shape) of a BGG sequence. To obtain this diagram and individual data in a BGG-sequence, it is necessary to act by the corresponding elements of the Weyl group on the highest weight of $\mathfrak{g}$ (it means the affine Weyl action).

The thesis shows a method how to construct the Hasse diagram directly from the so-called weight graph of the positive part $\mathfrak{g}_1$ of the $|1|$-grading of $\mathfrak{g}$. This gives in fact the same set of operators, and also the individual data (weights of representations and orders of operators) in the sequence (for given initial data) are computed more directly. This method shows a surprising connection between sequences of operators (which is a geometrical notion) and purely representation theoretical properties of the given Lie algebra.

For technical reasons, the method is developed for the cases of AHS structures for which all the weights of $\mathfrak{g}_1$ are extremal. This excludes the odd conformal and symplectic structures, where certain technical modifications are necessary.

The thesis shows a practical recipe for construction of BGG sequences in particular cases, and also the resulting sequence for every instance of an AHS-structure with all weights of $\mathfrak{g}_1$ extremal. Also a formula for computing the orders of resulting operators is given.

SIMULTANEOUS EXTENSION OPERATORS. POROSITY

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This doctoral thesis consists of four papers. The first two papers deal with *simultaneous extension operators*, i.e. linear operators $L\colon X \to Y$ defined on a function space X such that the value $L(f)$ is an extension of f for every function $f \in X$. Linear operators under consideration are positive or continuous ones.

In the paper *Simultaneous extension operators for the density topology* (Real Anal. Exch. **25**, 1999–2000, No. 1), Y is the space of bounded approximately continuous functions on $\mathbf{R}$ and X is the space of bounded Baire one functions restricted to a set $M \subset \mathbf{R}$ of Lebesgue measure zero. We show that a positive (or continuous or isometric) simultaneous extension operator exists if and only if M is scattered.

The second paper *Simultaneous solutions of the weak Dirichlet problem* (to appear in Potential Analysis) is a joint work with J. Lukeš. First we prove that there exists a positive and isometric simultaneous extension operator $L\colon \mathcal{C}(H) \to Y$ where Y is the space of all continuous affine functions on a Choquet simplex K and H is a compact metrizable subset of $\operatorname{ext} K$. This is then generalized to the situation where Y is replaced by a simplicial function space on a Hausdorff compact topological space K and H is a compact metrizable subset of the Choquet boundary. Thus we have a (partial) generalization of a result of J. Bliedtner and W. Hansen (*The weak Dirichlet problem*, J. Reine Angew. Math, 1984) for the space of harmonic functions on an abstract harmonic space.

Next two papers deal with σ-porosity, a notion of smallness (or, more precisely, a family of notions) which is a strengthening of the notion of the first category sets.

The paper *Porosity and compacta with dense ambiguous loci of metric projections* (Acta Univ. Carolinae – Math. et Phys. **39**, 1998, 119–125) is focused on metric projection onto the typical compact set K in a Banach space X. By $\mathrm{R}(K)$ we denote the set of points $x \in X$ which have more than one nearest point in K. If X is strictly convex then the following is known: $\mathrm{R}(K)$ is dense in X for every K with an exception of compacta K belonging to a set M which is of the first category in the space $\mathcal{K}(X)$ of all compact subsets of X. We show that the exceptional set M is even σ-porous. For separable Banach spaces X we give a necessary and sufficient condition for the smallness of the exceptional set M.

In the last paper *Nowhere approximately differentiable and nowhere Hölder continuous functions — Porous sets and Haar null sets* (to appear in Proc. Amer. Math. Soc.) we define a new notion of *HP-small* sets which is stronger both then σ-porosity and Haar nullness and study its properties. Then we show that every continuous function on the unit interval, with an exception of an HP-small set of functions, is nowhere Hölder and has a finite unilateral approximate derivative at no point.

The result is new in several directions, let us try to emphasize them: The most interesting is that we have nowhere *approximate* unilateral derivatives for every $f \in \mathcal{C}([0,1])$ with an exception of a Haar null set in $\mathcal{C}([0,1])$ (this is a generalization of a result of B. R. Hunt published in Proc. Amer. Math. Soc., 1994). This implies that the Besicovitch functions form a Haar null set. Further we have *one* proof instead of two separate proofs for σ-porosity and Haar nullness. The new notion is *stronger* than the two classical notions together (e.g. a hyperplane is *not* HP-small) and it is *stable* with respect to projections with finitely dimensional kernel and w.r.t. Minkovski addition of σ-compact sets. Also, we achieved the *best possible* porosity constant, that is 1. Furthermore, in the Hunt's paper it is shown that the "typical" function is nowhere Hölder and we generalize to nowhere *"ϕ-Hölder"*, where ϕ is an arbitrary fixed modulus of continuity.