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Centralizers on prime and semiprime rings

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Abstract. The purpose of this paper is to investigate identities satisfied by centralizers on prime and semiprime rings. We prove the following result: Let R be a noncommutative prime ring of characteristic different from two and let S and T be left centralizers on R. Suppose that $[S(x), T(x)]S(x) + S(x)[S(x), T(x)] = 0$ is fulfilled for all $x \in R$. If $S \neq 0$ ($T \neq 0$) then there exists λ from the extended centroid of R such that $T = \lambda S$ ($S = \lambda T$).

Keywords: prime ring, semiprime ring, extended centroid, derivation, Jordan derivation, left (right) centralizer, Jordan left (right) centralizer, commuting mapping, centralizing mapping

Classification: 16A12, 16A68, 16A72

This research has been inspired by the work of B. Zalar [11]. Throughout, R will represent an associative ring with center $Z(R)$. A ring R is 2-torsion free if $2x = 0$, $x \in R$ implies $x = 0$. We write $[x, y]$ for $xy - yx$ and make extensive use of basic commutator identities $[xy, z] = [x, z]y + x[y, z]$, $[x, yz] = [x, y]z + y[x, z]$. An additive mapping $D : R \to R$ is called a derivation if $D(xy) = D(x)y + xD(y)$ holds for all $x, y \in R$ and is called a Jordan derivation in case $D(x^2) = D(x)x + xD(x)$ is fulfilled for all $x \in R$. A derivation D is inner if there exists $a \in R$ such that $D(x) = [a, x]$ holds for all $x \in R$. An additive mapping $T : T \to R$ is left (right) centralizer if $T(xy) = T(x)y$ ($T(xy) = xT(y)$) holds for all $x, y \in R$. A centralizer is an additive mapping which is both left and right centralizer. An additive mapping $T : R \to R$ is Jordan left (right) centralizer in case $T(x^2) = T(x)x$ ($T(x^2) = xT(x)$) holds for all $x \in R$. For any fixed element $a \in R$ the mapping $T(x) = ax$ ($T(x) = xa$) is left (right) centralizer. Recall that a ring R is prime in case $aRb = (0)$ implies that either $a = 0$ or $b = 0$ and is semiprime if $aRa = (0)$ implies $a = 0$. Any derivation is a Jordan derivation. The converse is in general not true. A classical result of Herstein [7] asserts that every Jordan derivation on a prime ring of characteristic different from two is a derivation. A brief proof of Herstein theorem can be found in [1]. Cusak [6] has extended Herstein theorem on 2-torsion free semiprime rings (see also [2]). Any left (right) centralizer is a Jordan left (right) centralizer. Zalar [11] has proved that every left (right) Jordan centralizer on a 2-torsion free semiprime ring is a left (right) centralizer. We shall restrict our attention on left centralizers since all

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results presented in this paper are true also for right centralizers because of left-right symmetry. We shall denote by C the extended centroid of a prime ring R. First we list few lemmas.

Lemma 1. *Suppose that the elements a_i, b_i in the central closure of a prime ring R satisfy $\sum a_i x b_i = 0$ for all $x \in R$. If $b_i \neq 0$ for some i then a_i's are C-dependent.*

The explanation of the notions of the extended centroid and the central closure of a prime ring, as well as the proof of Lemma 1, can be found in [8, pp. 20–23] or [9].

Lemma 2. *Let R be a noncommutative prime ring and let $T : R \to R$ be a left centralizer. If $T(x) \in Z(R)$ holds for all $x \in R$, then $T = 0$.*

PROOF: Since $[T(x), y] = 0$ for all $x, y \in R$ we have, putting xz for x, $0 = [T(x)z, y] = [T(x), y]z + T(x)[z, y] = T(x)[z, y]$. Thus we have $T(x)[z, y] = 0$, which gives $T(x)w[z, y] = 0$ for all $x, y, z, w \in R$ whence it follows $T = 0$, otherwise R would be commutative. □

Lemma 3. *Let R be a noncommutative prime ring and let $S : R \to R$, $T : R \to R$ be left centralizers. Suppose that $[S(x), T(x)] = 0$ holds for all $x \in R$. If $T \neq 0$ then there exists $\lambda \in C$ such that $S = \lambda T$.*

PROOF: The linearization (i.e. putting $x+y$ for x) of the relation $[S(x), T(x)] = 0$ gives

$$[S(x), T(y)] + [S(y), T(x)] = 0. \tag{1}$$

Putting in (1) yz for y we obtain $0 = [S(x), T(y)z] + [S(y)z, T(x)] = [S(x), T(y)]z + T(y)[S(x), z] + [S(y), T(x)]z + S(y)[z, T(x)] = T(y)[S(x), z] + S(y)[z, T(x)]$. Thus we have

$$T(y)[S(x), z] + S(y)[z, T(x)] = 0.$$

Putting in the above relation yw for y we obtain

$$T(y)w[S(x), z] + S(y)w[z, T(x)] = 0. \tag{2}$$

Since we have assumed that $T \neq 0$ it follows from Lemma 2 that there exist $x, z \in R$ such that $[T(x), z] \neq 0$. Now (2) and Lemma 1 imply that $S(y) = \lambda(y)T(y)$ where $\lambda(y)$ is from C. Putting in (2) $\lambda(y)T(y)$ for $S(y)$ and $\lambda(x)T(x)$ for $S(x)$ we obtain $(\lambda(x) - \lambda(y))T(y)w[T(x), z] = 0$ for all pairs $y, w \in R$ whence it follows $(\lambda(x) - \lambda(y))T(y) = 0$ since $[T(x), z] \neq 0$. Thus we have $\lambda(x)T(y) = \lambda(y)T(y)$ which completes the proof of the lemma. □

We are now able to prove the first theorem of this paper.

Theorem 4. *Let R be a noncommutative 2-torsion free semiprime ring and $S : R \to R$, $T : R \to R$ left centralizers. Suppose that $[S(x), T(x)]S(x) + S(x)[S(x), T(x)] = 0$ holds for all $x \in R$. In this case we have $[S(x), T(x)] = 0$ for all $x \in R$. In case R is a prime ring and $S \neq 0$ ($T \neq 0$) then there exists $\lambda \in C$ such that $T = \lambda S$ ($S = \lambda T$).*

Proof: We have the relation

$$[S(x), T(x)]S(x) + S(x)[S(x), T(x)] = 0, \ x \in R. \tag{3}$$

Putting in (3) $x + y$ for y we obtain

$$\begin{gathered}[S(x), T(x)]S(y) + S(y)[S(x), T(x)] + [S(x), T(y)]S(x) + S(x)[S(x), T(y)]+\\ [S(y), T(x)]S(x) + S(x)[S(y), T(x)] + [S(y), T(y)]S(x) + S(x)[S(y), T(y)]+\\ [S(y), T(x)]S(y) + S(y)[S(y), T(x)] + [S(x), T(y)]S(y)+\\ S(y)[S(x), T(y)] = 0.\end{gathered} \tag{4}$$

Putting in the above relation $-x$ for x we obtain

$$\begin{gathered}[S(x), T(x)]S(y) + S(y)[S(x), T(x)] + [S(x), T(y)]S(x) + S(x)[S(x), T(y)]+\\ [S(y), T(x)]S(x) + S(x)[S(y), T(x)] - [S(y), T(y)]S(x) - S(x)[S(y), T(y)]-\\ [S(y), T(x)]S(y) - S(y)[S(y), T(x)] - [S(x), T(y)]S(y)-\\ S(y)[S(x), T(y)] = 0.\end{gathered} \tag{5}$$

Combining (4) with (5) we obtain $2[S(x), T(x)]S(y) + 2S(y)[S(x), T(x)] + 2[S(x), T(y)]S(x) + 2S(x)[S(x), T(y)] + 2[S(y), T(x)]S(x) + 2S(x)[S(y), T(x)] = 0$ whence it follows

$$\begin{gathered}[S(x), T(x)]S(y) + S(y)[S(x), T(x)] + [S(x), T(y)]S(x) + S(x)[S(x), T(y)]+\\ [S(y), T(x)]S(x) + S(x)[S(y), T(x)] = 0\end{gathered} \tag{6}$$

since we have assumed that R is 2-torsion free. Putting in the above relation xy for y we obtain

$$\begin{gathered}0 = [S(x), T(x)]S(x)y + S(x)y[S(x), T(x)] + [S(x), T(x)y]S(x)+\\ S(x)[S(x), T(x)y] + [S(x)y, T(x)]S(x) + S(x)[S(x)y, T(x)] =\\ [S(x), T(x)]S(x)y + S(x)y[S(x), T(x)] + [S(x), T(x)]yS(x) + T(x)[S(x), y]S(x)+\\ S(x)[S(x), T(x)]y + S(x)T(x)[S(x), y] + [S(x), T(x)]yS(x) + S(x)[y, T(x)]S(x)+\\ S(x)[S(x), T(x)]y + S(x)^2[y, T(x)].\end{gathered}$$

According to (6) the above calculation reduces to

$$\begin{gathered}S(x)y[S(x), T(x)] + 2[S(x), T(x)]yS(x) + T(x)[S(x), y]S(x)+\\ S(x)T(x)[S(x), y] + S(x)[y, T(x)]S(x) + S(x)[S(x), T(x)]y+\\ S(x)^2[y, T(x)] = 0.\end{gathered} \tag{7}$$

Putting in the above relation $yS(x)$ for y we obtain
$S(x)yS(x)[S(x),T(x)]+2[S(x),T(x)]yS(x)^2+T(x)[S(x),y]S(x)^2+$
$S(x)T(x)[S(x),y]S(x)+S(x)[y,T(x)]S(x)^2+S(x)y[S(x),T(x)]S(x)+$
$S(x)[S(x),T(x)]yS(x)+S(x)^2[y,T(x)]S(x)+S(x)^2y[S(x),T(x)]=0$ which leads according to (7) to

$$S(x)yS(x)[S(x),T(x)]+S(x)^2y[S(x),T(x)]=0. \tag{8}$$

Putting in (8) $T(x)y$ for y we obtain

$$S(x)T(x)yS(x)[S(x),T(x)]+S(x)^2T(x)y[S(x),T(x)]=0. \tag{9}$$

Left multiplication by $T(x)$ gives

$$T(x)S(x)yS(x)[S(x),T(x)]+T(x)S(x)^2y[S(x),T(x)]=0. \tag{10}$$

From (9) and (10) we obtain $[S(x),T(x)]yS(x)[S(x),T(x)]+$
$[S(x)^2,T(x)]y[S(x),T(x)]=[S(x),T(x)]yS(x)[S(x),T(x)]+([S(x),T(x)]S(x)+$
$S(x)[S(x),T(x)])y[S(x),T(x)]=[S(x),T(x)]yS(x)[S(x),T(x)]=0.$ Thus we have

$$[S(x),T(x)]yS(x)[S(x),T(x)]=0.$$

Left multiplication of the above relation by $S(x)$ gives

$$S(x)[S(x),T(x)]yS(x)[S(x),T(x)]=0 \tag{11}$$

for all pairs $x,y\in R$. From (11) it follows

$$S(x)[S(x),T(x)]=0. \tag{12}$$

From (3) and (10) we obtain also

$$[S(x),T(x)]S(x)=0. \tag{13}$$

From (12) one obtains the relation

$$S(y)[S(x),T(x)]+S(x)[S(y),T(x)]+S(x)[S(x),T(y)]=0 \tag{14}$$

(see the proof of (6)). Putting in (14) xy for y we obtain

$$0=S(x)y[S(x),T(x)]+S(x)[S(x)y,T(x)]+S(x)[S(x),T(x)y]=$$
$$S(x)y[S(x),T(x)]+S(x)[S(x),T(x)]y+S(x)^2[y,T(x)]+S(x)[S(x),T(x)]y+$$
$$S(x)T(x)[S(x),y]=S(x)y[S(x),T(x)]+S(x)^2[y,T(x)]+S(x)T(x)[S(x),y].$$

Thus we have the relation $S(x)y[S(x),T(x)] + S(x)^2[y,T(x)]+$ $S(x)T(x)[S(x),y] = 0$ which can be written in the form $S(x)y[S(x),T(x)] +$ $S(x)^2yT(x) - S(x)T(x)yS(x) + S(x)[T(x),S(x)]y = 0$ whence it follows

$$S(x)y[S(x),T(x)] + S(x)^2yT(x) - S(x)T(x)yS(x) = 0 \tag{15}$$

according to (12). Left multiplication of (15) by $T(x)$ gives

$$T(x)S(x)y[S(x),T(x)] + T(x)S(x)^2yT(x) - T(x)S(x)T(x)yS(x) = 0. \tag{16}$$

The substitution $T(x)y$ for y in (15) gives

$$S(x)T(x)y[S(x),T(x)] + S(x)^2T(x)yT(x) - S(x)T(x)^2yS(x) = 0. \tag{17}$$

From (16) and (17) one obtains

$$0 = [S(x),T(x)]y[S(x),T(x)] + [S(x)^2,T(x)]yT(x) + [T(x),S(x)]T(x)yS(x) =$$
$$[S(x),T(x)]y[S(x),T(x)] + ([S(x),T(x)]S(x) + S(x)[S(x),T(x)])yT(x)+$$
$$[T(x),S(x)]T(x)yS(x)$$

which reduces to

$$[S(x),T(x)]y[S(x),T(x)] + [T(x),S(x)]T(x)yS(x) = 0. \tag{18}$$

The substitution $yS(x)z$ for y in (18) gives

$$[S(x),T(x)]yS(x)z[S(x),T(x)] + [T(x),S(x)]T(x)yS(x)zS(x) = 0. \tag{19}$$

On the other hand, right multiplication of (18) by $zS(x)$ leads to

$$[S(x),T(x)]y[S(x),T(x)]zS(x) + [T(x),S(x)]T(x)yS(x)zS(x) = 0. \tag{20}$$

From (19) and (20) we obtain

$$[S(x),T(x)]yA(x,z) = 0, \tag{21}$$

where $A(x,z)$ stands for $[S(x),T(x)]zS(x) - S(x)z[S(x),T(x)]$. The substitution $zS(x)y$ for y in (21) gives

$$[S(x),T(x)]zS(x)yA(x,z) = 0. \tag{22}$$

Left multiplication of (21) by $S(x)z$ leads to

$$S(x)z[S(x),T(x)]yA(x,z) = 0. \tag{23}$$

Combining (22) with (23) we arrive at

$$A(x,z)yA(x,z)=0$$

for all $x,y,z\in R$ whence it follows $A(x,z)=0$. In other words

$$[S(x),T(x)]zS(x)=S(x)z[S(x),T(x)]. \tag{24}$$

The substitution $z=T(x)y$ in (24) gives

$$[S(x),T(x)]T(x)yS(x)=S(x)T(x)y[S(x),T(x)]. \tag{25}$$

The relation (25) makes it possible to replace in (18) $[S(x),T(x)]T(x)yS(x)$ by $S(x)T(x)y[S(x),T(x)]$. Thus we have $[S(x),T(x)]y[S(x),T(x)]-$ $S(x)T(x)y[S(x),T(x)]=0$, which reduces to

$$T(x)S(x)y[S(x),T(x)]=0. \tag{26}$$

Putting in (26) $T(x)y$ for y we obtain

$$T(x)S(x)T(x)y[S(x),T(x)]=0. \tag{27}$$

Multiplying (26) from the left side by $T(x)$ we obtain

$$T(x)^2S(x)y[S(x),T(x)]=0. \tag{28}$$

Subtracting (28) from (27) we obtain $T(x)[S(x),T(x)]y[S(x),T(x)]=0$ which gives putting $yT(x)$ for y

$$T(x)[S(x),T(x)]yT(x)[S(x),T(x)]=0$$

for all pairs $x,y\in R$ whence it follows

$$T(x)[S(x),T(x)]=0. \tag{29}$$

The substitution $yT(x)$ for y in (25) gives because of (29)

$$[S(x),T(x)]yT(x)S(x)=0. \tag{30}$$

From (13) we obtain the relation

$$[S(x),T(x)]S(y)+[S(x),T(y)]S(x)+[S(y),T(x)]S(x)=0$$

(see the proof of (6)). Putting in the above relation xy for y we obtain $0=$ $[S(x),T(x)]S(x)y+[S(x),T(x)y]S(x)+[S(x)y,T(x)]S(x)=[S(x),T(x)]yS(x)+$ $T(x)[S(x),y]S(x)+[S(x),T(x)]yS(x)+S(x)[y,T(x)]S(x)$. Thus we have

$2[S(x),T(x)]yS(x)+T(x)[S(x),y]S(x)+S(x)[y,T(x)]S(x)=0$ which can be written after some calculation in the form

$$[S(x),T(x)]yS(x)+S(x)yT(x)S(x)-T(x)yS(x)^2=0. \tag{31}$$

The relation (24) makes it possible to replace in (31) $[S(x),T(x)]yS(x)$ by $S(x)y[S(x),T(x)]$. Thus we have $0=S(x)y[S(x),T(x)]+S(x)yT(x)S(x)-T(x)yS(x)^2=S(x)yS(x)T(x)-T(x)yS(x)^2$. We have therefore

$$S(x)yS(x)T(x)=T(x)yS(x)^2. \tag{32}$$

Putting in the above relation $T(x)y$ for y we obtain

$$S(x)T(x)yS(x)T(x)=T(x)^2yS(x)^2. \tag{33}$$

Left multiplication of (32) by $T(x)$ leads to

$$T(x)S(x)yS(x)T(x)=T(x)^2yS(x)^2. \tag{34}$$

Combining (33) with (34) we arrive at

$$[S(x),T(x)]yS(x)T(x)=0$$

which gives together with (30)

$$[S(x),T(x)]y[S(x),T(x)]=0$$

for all pairs $x,y\in R$ whence it follows

$$[S(x),T(x)]=0. \tag{35}$$

In case R is a prime ring the relation (35) and Lemma 3 complete the proof of the theorem. □

Corollary 5. *Let R be a 2-torsion free semiprime ring and $T:R\to R$ a left centralizer. Suppose that $[T(x),x]x+x[T(x),x]=0$ holds for all $x\in R$. In this case T is a centralizer.*

Proof: Since the assumptions of Theorem 4 are fulfilled we have

$$[T(x),x]=0$$

for all $x\in R$. According to the above relation we have $T(x^2)=T(x)x=xT(x)$. Thus we have $T(x^2)=xT(x)$ for all $x\in R$. In other words, T is a Jordan right centralizer. By Proposition 1.4 in [11] T is a right centralizer which completes the proof. □

Similarly, putting in Theorem 4 $T(x)=x$ and applying again Proposition 1.4 from [11], we obtain the following result.

Corollary 6. *Let R be a 2-torsion free semiprime ring and $T : R \to R$ a left centralizer. Suppose that $[T(x), x]T(x) + T(x)[T(x), x] = 0$ holds for all $x \in R$. In this case T is a centralizer.*

The above corollaries characterize centralizers among all left centralizers on 2-torsion free semiprime rings. Both of these results as well as Corollaries 8 and 9 at the end of the paper are contributions to the theory of so-called commuting and centralizing mappings. A mapping $F : R \to R$ is centralizing on a ring R if $[F(x), x] \in Z(R)$ for all $x \in R$. In a special case when $[F(x), x] = 0$ for all $x \in R$, a mapping F is called commuting on R. The study of centralizing and commuting mappings was initiated by the classical result of Posner [10], which states that the existence of a nonzero centralizing derivation on a prime ring forces the ring to be commutative. A lot of work has been done during the last twenty years in the field. The work of Brešar [3], [4], [5], where further references can be found, should be mentioned.

We are ready for our next result.

Theorem 7. *Let R be a 2-torsion free noncommutative semiprime ring and let $S : R \to R$, $T : R \to R$ be left centralizers. Suppose that $[[S(x), T(x)], S(x)] = 0$ is fulfilled for all $x \in R$. In this case we have $[S(x), T(x)] = 0$ for all $x \in R$. In case R is a prime ring and $S \neq 0$ ($T \neq 0$) then there exists $\lambda \in C$ such that $T = \lambda S$ ($S = \lambda T$).*

Proof: The relation

$$[[S(x), T(x)], S(x)] = 0, \tag{36}$$

gives (see the proof of Theorem 4)

$$[[S(x), T(x)], S(y)] + [[S(x), T(y)], S(x)] + [[S(y), T(x)], S(x)] = 0. \tag{37}$$

Putting in (37) xy for y we obtain

$$\begin{gathered} 0 = [[S(x), T(x)], S(x)y] + [[S(x), T(x)y], S(x)] + [[S(x)y, T(x)], S(x)] = \\ [[S(x), T(x)], S(x)]y + S(x)[[S(x), T(x)], y] + \\ [[S(x), T(x)]y + T(x)[S(x), y], S(x)] + [[S(x), T(x)]y + S(x)[y, T(x)], S(x)] = \\ S(x)[[S(x), T(x)], y] + [[S(x), T(x)], S(x)]y + [S(x), T(x)][y, S(x)] + \\ [T(x), S(x)][S(x), y] + T(x)[[S(x), y], S(x)] + [[S(x), T(x)], S(x)]y + \\ [S(x), T(x)][y, S(x)] + S(x)[[y, T(x)], S(x)]. \end{gathered}$$

We have therefore

$$\begin{gathered} S(x)[[S(x), T(x)], y] + 3[S(x), T(x)][y, S(x)] + T(x)[[S(x), y], S(x)] + \\ S(x)[[y, T(x)], S(x)] = 0. \end{gathered} \tag{38}$$

Putting in the above relation $yS(x)$ for y we obtain

$$0 = S(x)[[S(x), T(x)], yS(x)] + 3[S(x), T(x)][yS(x), S(x)] + T(x)[[S(x), yS(x)], S(x)] + S(x)[[yS(x), T(x)], S(x)] = S(x)[[S(x), T(x)], y]S(x) + S(x)y[[S(x), T(x)], S(x)] + 3[S(x), T(x)][y, S(x)]S(x) + T(x)[[S(x), y]S(x), S(x)] + S(x)[[y, T(x)]S(x) + y[S(x), T(x)], S(x)] = S(x)[[S(x), T(x)], y]S(x) + 3[S(x), T(x)][y, S(x)]S(x) + T(x)[[S(x), y], S(x)]S(x) + S(x)[[y, T(x)], S(x)]S(x) + S(x)[y, S(x)][S(x), T(x)] + S(x)y[[S(x), T(x)], S(x)].$$

Thus we have according to (36) and (38) $S(x)[y, S(x)][S(x), T(x)] = 0$ which can be written in the form

$$S(x)yS(x)[S(x), T(x)] = S(x)^2 y[S(x), T(x)]. \tag{39}$$

Putting in the above calculation $T(x)y$ for y we obtain

$$S(x)T(x)yS(x)[S(x), T(x)] = S(x)^2 T(x)y[S(x), T(x)]. \tag{40}$$

On the other hand, left multiplication of (39) by $T(x)$ gives

$$T(x)S(x)yS(x)[S(x), T(x)] = T(x)S(x)^2 y[S(x), T(x)]. \tag{41}$$

Subtracting (41) from (40) we obtain

$$0 = [S(x), T(x)]yS(x)[S(x), T(x)] - [S(x)^2, T(x)]y[S(x), T(x)] = [S(x), T(x)]yS(x)[S(x), T(x)] - ([S(x), T(x)]S(x) + S(x)[S(x), T(x)])y[S(x), T(x)].$$

According to the requirement of the theorem one can replace in the above calculation $[S(x), T(x)]S(x)$ by $S(x)[S(x), T(x)]$ which gives

$$[S(x), T(x)]yS(x)[S(x), T(x)] = 2S(x)[S(x), T(x)]y[S(x), T(x)].$$

Left multiplication of the above relation by $S(x)$ gives

$$S(x)[S(x), T(x)]yS(x)[S(x), T(x)] = 2S(x)^2[S(x), T(x)]y[S(x), T(x)]. \tag{42}$$

On the other hand, putting $[S(x), T(x)]y$ for y in (39) we arrive at

$$S(x)[S(x), T(x)]yS(x)[S(x), T(x)] = S(x)^2[S(x), T(x)]y[S(x), T(x)]. \tag{43}$$

Combining (42) with (43) we obtain $S(x)[S(x),T(x)]yS(x)[S(x),T(x)] = 0$ for all pairs $x, y \in R$, whence it follows

$$S(x)[S(x),T(x)] = 0, \tag{44}$$

by semiprimeness of R. From (44) and the assumption of the theorem we have also

$$[S(x),T(x)]S(x) = 0.$$

The rest of the proof goes through in the same way as in the proof of Theorem 4. □

Theorem 7 gives together with Proposition 1.4 from [11] the following characterizations of centralizers among all left centralizers on 2-torsion free semiprime rings.

Corollary 8. *Let R be a 2-torsion free semiprime ring and $T : R \to R$ a left centralizer. Suppose that $[[T(x),x],x] = 0$ holds for all $x \in R$. In this case T is a centralizer.*

Corollary 9. *Let R be a 2-torsion free semiprime ring and $T : R \to R$ a left centralizer. Suppose that $[[T(x),x],T(x)] = 0$ holds for all $x \in R$. In this case T is a centralizer.*

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