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Serpil Pehlivan; Brian Fisher

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# Lacunary strong convergence with respect to a sequence of modulus functions

Serpil Pehlivan1, Brian Fisher

*Abstract.* The definition of lacunary strong convergence is extended to a definition of lacunary strong convergence with respect to a sequence of modulus functions in a Banach space. We study some connections between lacunary statistical convergence and lacunary strong convergence with respect to a sequence of modulus functions in a Banach space.

*Keywords:* lacunary sequence, modulus function, statistical convergence, Banach space

*Classification:* 40A05, 40F05

## 1. Introduction

By a lacunary sequence $\theta = (k_r)$ where $k_0 = 0$, we mean an increasing sequence of positive integers with $h_r = k_r - k_{r-1} \to \infty$ as $r \to \infty$. The intervals determined by $\theta$ will be denoted by $I_r = (k_{r-1}, k_r]$ and the ratio $k_r/k_{r-1}$ will be denoted by $q_r$. The sequence space of lacunary strongly convergent sequences $N_\theta$ was defined by Freedman et al. [4], as follows:

$$N_\theta = \{x = (x_i) \,:\, \lim_{r\to\infty} h_r^{-1} \sum_{i\in I_r} |x_i - l| = 0 \ \text{ for some } \ l\}.$$

Let $\|x\|_\theta = \sup_r (h_r^{-1} \sum_{i\in I_r} |x_i|)$, whenever $x \in N_\theta$. Then $(N_\theta, \|.\|_\theta)$ is a BK-space. $N_\theta^0$ denotes the subset of all sequences which are lacunary strongly convergent to zero. $(N_\theta^0, \|.\|_\theta)$ is also a BK-space.

There is a strong connection between $N_\theta$ and the sequence space $|\sigma_1|$, which is defined by

$$|\sigma_1| = \{x = (x_i) \,:\, \lim_{n\to\infty} n^{-1} \sum_{i=1}^{n} |x_i - l| = 0 \ \text{ for some } \ l\}.$$

In the special case $\theta = (2^r)$, we have $N_\theta = |\sigma_1|$.

The well known space $\hat{c}$, the space of all almost convergent sequences was defined by Lorentz [9]. Later $[\hat{c}]$ the space of strong almost convergence was

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introduced by Maddox [10] and also independently by Freedman et al. [4]. This sequence space was defined as follows:

$$[\hat{c}] = \{x = (x_i) : \lim_{n\to\infty} n^{-1} \sum_{i=p+1}^{p+n} |x_i - l| = 0 \text{ uniformly in } p, \text{ for some } l\}.$$

We denote the space of all sequences which are strongly almost convergent to zero by $[\hat{c}_0]$. In [15], the spaces $[\hat{c}_0]$ and $[\hat{c}]$ were extended to $[\hat{c}_0(f)]$ and $[\hat{c}(f)]$.

Let $X$ be a Banach space. We define $s(X)$ to be the vector space of all $X$-valued sequences, $l_\infty(X)$ the vector space of all bounded X-valued sequences and $c(X)$ the vector space of all convergent X-valued sequences. Thus $x = (x_i) \in l_\infty(X)$, if $\sup \|x_i\| < \infty$, where $x_i \in X$ for $i \in N$. Consequently $l_\infty(X)$ becomes a Banach space with the natural coordinatewise operations and $\|x\| = \sup_i \|x_i\|$ for $x \in l_\infty(X)$.

The notion of a modulus function was introduced by Nakano [13]. We recall that a modulus $f$ is a function from $[0, \infty)$ to $[0, \infty)$ such that (i) $f(x) = 0$ if and only if $x = 0$, (ii) $f(x + y) \le f(x) + f(y)$ for $x, y \ge 0$, (iii) $f$ is increasing and (iv) $f$ is continuous from the right at 0. It follows that $f$ must be continuous on $[0, \infty)$. Connor [2], Maddox [11], [12], Kolk [8], Pehlivan and Fisher[16] and Ruckle [19] used a modulus function to construct sequence spaces.

Now let $S$ be the space of sequences of modulus functions $F = (f_i)$ such that $\lim_{u\to 0^+} \sup_i f_i(u) = 0$. Throughout this paper the sequence of modulus functions determined by F will be denoted by $F = (f_i) \in S$ for every $i \in N$.

The purpose of this paper is to introduce and study a concept of lacunary strong convergence with respect to a sequence of modulus functions in a Banach space.

## 2. Inclusion theorems

We now introduce the generalizations of the lacunary strongly convergent sequences and investigate some inclusion relations.

**Definition 2.1.** Let $F = (f_i)$ be a sequence of modulus functions in $S$. Let $X$ be a Banach space. We define the spaces

$$N_\theta(X) = \{x = (x_i) \in s(X) : \lim_{r\to\infty} h_r^{-1} \sum_{i\in I_r} \|x_i - l\| = 0 \text{ for some } l \in X\},$$

$$N_\theta(X, F) = \{x = (x_i) \in s(X) : \lim_{r\to\infty} h_r^{-1} \sum_{i\in I_r} f_i(\|x_i - l\|) = 0 \text{ for some } l \in X\},$$

$$N_\theta^0(X, F) = \{x = (x_i) \in s(X) : \lim_{r\to\infty} h_r^{-1} \sum_{i\in I_r} f_i(\|x_i\|) = 0\}.$$

$N_\theta(X)$, $N_\theta(X, F)$ and $N_\theta^0(X, F)$ are linear spaces. We consider only $N_\theta(X, F)$. Suppose that $x_i \to l$ in $N_\theta(X, F)$, $y_i \to l'$ in $N_\theta(X, F)$ and $\alpha, \gamma$ are in $C$. Then there exist integers $K_\alpha$ and $M_\gamma$ such that $|\alpha| \leq K_\alpha$ and $|\gamma| \leq M_\gamma$. We have

$$h_r^{-1} \sum_{i \in I_r} f_i(\|\alpha x_i + \gamma y_i - (\alpha l + \gamma l')\|) \\ \leq K_\alpha h_r^{-1} \sum_{i \in I_r} f_i(\|x_i - l\|) + M_\gamma h_r^{-1} \sum_{i \in I_r} f_i(\|x_i - l'\|).$$

This implies that $\alpha x + \gamma y \to \alpha l + \gamma l'$ in $N_\theta(X, F)$. Note that if we put $f_i = f$ for $i \in N$ then $N_\theta(X, F) = N_\theta(X, f)$. We write $N_\theta(X, f) = N_\theta(X)$ for $f(x) = x$.

**Proposition 2.2** ([16]). *Let $f$ be a modulus and let $0 < \delta < 1$. Then for each $\|u\| \geq \delta$, we have $f(\|u\|) \leq 2f(1)\delta^{-1}\|u\|$.*

PROOF:

$$f(\|u\|) \leq f(1 + [\|u\|/\delta]) \leq f(1) + f([\|u\|/\delta]) \leq f(1)(1 + \|u\|/\delta) \leq 2f(1)\|u\|/\delta.$$

where $[\|u\|/\delta]$ denotes the integer part of $\|u\|/\delta$. □

**Theorem 2.3.** *Let $X$ be a Banach space and let $F = (f_i)$ be a sequence of modulus functions in $S$. If $x = (x_i)$ is lacunary strongly convergent to $l$ in $X$, then $x = (x_i)$ is lacunary strongly convergent to $l$ in $X$ with respect to $F$, i.e. $N_\theta(X) \subset N_\theta(X, F)$.*

PROOF: Let $F = (f_i)$ be a sequence modulus functions in $S$ and put $\sup_i f_i(1) = M$. Let $x \in N_\theta(X)$. Then we have

$$A_r(X) = h_r^{-1} \sum_{i \in I_r} \|x_i - l\| \to 0 \quad \text{as} \quad r \to \infty, \quad \text{for some} \quad l \in X.$$

Let $\varepsilon > 0$ and choose $\delta$ with $0 < \delta < 1$ such that $f_i(u) < \varepsilon$ $(i \in N)$ for every $u$ with $0 \leq u \leq \delta$. We can write

$$h_r^{-1} \sum_{i \in I_r} f_i(\|x_i - l\|) = h_r^{-1} \sum_{\substack{i \in I_r \\ \|x_i - l\| \leq \delta}} f_i(\|x_i - l\|) + h_r^{-1} \sum_{\substack{i \in I_r \\ \|x_i - l\| > \delta}} f_i(\|x_i - l\|) \\ \leq h_r^{-1}(h_r \varepsilon) + h_r^{-1} 2M\delta^{-1} h_r A_r(X),$$

by Proposition 2.2. Letting $r \to \infty$, it follows that $x \in N_\theta(X, F)$. □

**Theorem 2.4.** *Let $X$ be a Banach space and $F = (f_i)$ be a sequence of modulus functions. If $\lim_{u \to \infty} \inf_i f_i(u)/u > 0$, then $N_\theta(X, F) = N_\theta(X)$.*

PROOF: If $\lim_{u \to \infty} \inf_i f_i(u)/u > 0$ then there exists a number $c > 0$ such that $f_i(u) > cu$ for $u > 0$ and $i \in N$. We have $x \in N_\theta(X, F)$. Clearly

$$h_r^{-1} \sum_{i \in I_r} f_i(\|x_i - l\|) \geq h_r^{-1} \sum_{i \in I_r} c\|x_i - l\| = ch_r^{-1} \sum_{i \in I_r} \|x_i - l\|,$$

therefore $x \in N_\theta(X)$. By using Theorem 2.3 the proof is complete. □

We now give an example to show that $N_\theta(X,F) \neq N_\theta(X)$ in the case when $\lim_{u\to\infty} \inf_i f_i(u)/u = 0$. Consider the sequence $f_i(x) = x^{1/(i+1)}$ $(i \geq 1,\ x > 0)$ of modulus functions. Now define $x_i = h_r v$ if $i = k_r$ for some $r \geq 1$ and $x_i = \theta$ otherwise, where $v \in X$ and $\|v\| = 1$. This yields

$$h_r^{-1} \sum_{i\in I_r} f_i(\|x_i\|) = h_r^{-1}(f_{k_r}(h_r\|v\|)) = h_r^{-1} h_r^{1/(1+k_r)} \to 0 \quad \text{as} \quad r \to \infty$$

and so $x \in N_\theta(X,F)$. But

$$h_r^{-1} \sum_{i\in I_r} \|x_i\| = h_r^{-1} h_r \|v\| \to 1 \quad \text{as} \quad r \to \infty$$

and so $x \notin N_\theta(X)$.

**Proposition 2.5.** *If $f_i = f$ for $i \in N$, then $[\hat{c}(X,f)] \subset N_\theta(X,f)$ for every lacunary sequence $\theta$, where*
*$[\hat{c}(X,f)] = \{x = (x_i) \in s(X) : \lim_{n\to\infty} n^{-1} \sum_{i=1}^n f(\|x_{i+p} - l\|) = 0$, for some $l \in X$ uniformly in $p\}$.*

To show that $N_\theta^0(X,f)$ strictly contains

$$[\hat{c}_0(X,f)] = \{x = (x_i) \in s(X) : \lim_{n\to\infty} n^{-1} \sum_{i=1}^n f(\|x_{i+p}\|) = 0 \quad \text{uniformly in} \quad p\},$$

we proceed as in [4; p. 513]. We define $x = (x_i)$ by $x_i = v$ if $k_{r-1} < i \leq k_{r-1} + [\sqrt{h_r}\,]$ for some $r$ and $x_i = \theta$ otherwise, where $v \in X$ and $\|v\| = 1$. It follows that $x \notin [\hat{c}_0(X,f)]$. However $x \in N_\theta^0(X,f)$ since

$$h_r^{-1} \sum_{i\in I_r} f(\|x_i\|) = h_r^{-1}[\sqrt{h_r}\,] f(1) \to 0 \quad \text{as} \quad r \to \infty.$$

If $f_i = f$ for $i \in N$ we can show as in [4] that $|\sigma_1(X,f)| = N_\theta(X,f)$ if and only if $1 < \liminf_r q_r \leq \limsup_r q_r < \infty$, where $|\sigma_1(X,f)| = \{x = (x_i) \in s(X) : \lim_{n\to\infty} n^{-1} \sum_{i=1}^n f(\|x_i - l\|) = 0$ for some $l \in X\}$.

**Proposition 2.6.** *Let $X$ be a Banach space. Let $\theta = (k_r)$ be a lacunary sequence with $\liminf_r q_r > 1$ then for any modulus $f$, $|\sigma_1(X,f)| \subset N_\theta(X,f)$.*

Proof: It is enough to show that $|\sigma_1(X,f)|^0 \subset N_\theta^0(X,f)$. Suppose $\liminf_r q_r > 1$. There exists $\delta > 0$ such that $q_r = (k_r/k_{r-1}) \geq 1 + \delta$ for sufficiently large $r$. We have, for sufficiently large $r$, that $(k_r/h_r) \leq (1+\delta)/\delta$ and $(h_r/k_r) \geq \delta/(1+\delta)$. Now write

$$k_r^{-1}\sum_{i=1}^{k_r} f(\|x_i\|) \geq k_r^{-1}\sum_{i\in I_r} f(\|x_i\|) = (h_r/k_r)h_r^{-1}\sum_{i\in I_r} f(\|x_i\|)$$
$$\geq (\delta/(1+\delta))h_r^{-1}\sum_{i\in I_r} f(\|x_i\|),$$

from which we deduce that $|\sigma_1(X,f)|^0 \subset\ N_\theta^0(X,f)$ for any modulus $f$. □

**Proposition 2.7.** *Let $X$ be a Banach space. Let $\theta=(k_r)$ be a lacunary sequence with $\limsup_r q_r < \infty$ then for any modulus $f$, $N_\theta(X,f) \subset |\sigma_1(X,f)|$.*

Proof: Let $x \in N_\theta^0(X,f)$ and $\varepsilon > 0$. There exists $j_0$ such that for every $j \geq j_0$

$$H_j = h_j^{-1}\sum_{i\in I_j} f(\|x_i\|) < \varepsilon.$$

We can also find $M > 0$ such that $H_j \leq M$ for all $j$. If $\limsup_r q_r < \infty$ then there exists $B > 0$ such that $q_r < B$ for every $r$. Now let $n$ be any integer with $k_{r-1} < n \leq k_r$. Then

$$n^{-1}\sum_{i=1}^{n} f(\|x_i\|) \leq k_{r-1}^{-1}\sum_{i=1}^{k_r} f(\|x_i\|) = k_{r-1}^{-1}\Big\{\sum_{i\in I_1} f(\|x_i\|) + \ldots + \sum_{i\in I_r} f(\|x_i\|)\Big\}$$
$$= k_{r-1}^{-1}\Big\{\sum_{j=1}^{j_0}\sum_{i\in I_j} f(\|x_i\|) + \sum_{j=j_0+1}^{r}\ \sum_{i\in I_j} f(\|x_i\|)\Big\}$$
$$\leq k_{r-1}^{-1}\sum_{j=1}^{j_0}\ \sum_{i\in I_j} f(\|x_i\|) + \varepsilon(k_r - k_{j_0})k_{r-1}^{-1}$$
$$= k_{r-1}^{-1}\{h_1H_1 + h_2H_2 + \ldots + h_{j_0}H_{j_0}\} + \varepsilon(k_r - k_{j_0})k_{r-1}^{-1}$$
$$\leq k_{r-1}^{-1}(\sup_{1\leq i\leq j_0} H_i)k_{j_0} + \varepsilon(k_r - k_{j_0})k_{r-1}^{-1} < Mk_{r-1}^{-1}k_{j_0} + \varepsilon B$$

which yields that $x \in |\sigma_1(X,f)|^0$. □

The next result follows from Proposition 2.6 and 2.7.

**Theorem 2.8.** *Let $\theta=(k_r)$ be a lacunary sequence with $1 < \liminf_r q_r \leq \limsup_r q_r < \infty$. Then $|\sigma_1(X,f)| = N_\theta(X,f)$. In particular we have $N_{2^r}(X,f) = |\sigma_1(X,f)|$.*

## 3. Some results on $X$-lacunary statistical convergence

We now introduce natural relationship between lacunary strong convergence with respect to a sequence of modulus functions in Banach space and lacunary statistical convergence in a Banach space. In [3], Fast introduced the idea of statistical convergence, which is closely related to the concept of natural density or asymptotic density of subsets of the positive integers $N$. These ideas were later studied in [1], [5], [17] and [18]. If $K$ is a subset of the positive integers $N$, then $K_n$ denotes the set $\{k \in K : k \leq n\}$ and $|K_n|$ denotes cardinality of $K_n$. The natural density of $K$ is given by $\delta(K) = \lim_{n\to\infty} n^{-1}|K_n|$, see [14]. A sequence $x = (x_i)$ is statistically convergent to $l$ if for every $\varepsilon > 0$,

$$\lim_{n\to\infty} n^{-1}|K(\varepsilon)| = 0,$$

where $K(\varepsilon) = \{i \in N : |x_i - l| \geq \varepsilon\}$ and $|K(\varepsilon)|$ denotes cardinality of $K(\varepsilon)$. The set of all statistically convergent sequences is denoted by $St$.

Recently Fridy and Orhan [6], [7] introduced the following definition of lacunary statistical convergence.

**Definition 3.1.** Let $\theta$ be a lacunary sequence. Then a sequence $x = (x_i)$ is said to be lacunary statistically convergent to a number $l$ if for every $\varepsilon > 0$,

$$\lim_{r\to\infty} h_r^{-1}|K_\theta(\varepsilon)| = 0,$$

where $K_\theta(\varepsilon) = \{i \in I_r : |x_i - l| \geq \varepsilon\}$. The set of all lacunary statistically convergent sequences is denoted by $St_\theta$.

Some results on $St_\theta$-convergence and $St$-convergence were given in [7]. It was shown there that $St = St_\theta$ if and only if $1 < \lim_r \inf q_r \leq \lim_r \sup q_r < \infty$.

**Definition 3.2.** Let $\theta$ be a lacunary sequence. Then a sequence $x = (x_i) \in s(X)$ is said to be $X$-lacunary statistically convergent to an $l \in X$ if for every $\varepsilon > 0$,

$$\lim_{r\to\infty} h_r^{-1}|\{i \in I_r : \|x_i - l\| \geq \varepsilon\}| = 0.$$

The set of all such sequences $x = (x_i)$ is denoted by $St_\theta(X)$.

In the next section we establish inclusion relations between $St_\theta(X)$ and $N_\theta(X, F)$.

**Theorem 3.3.** *Let $F = (f_i)$ be a sequence of modulus functions in $S$. Let $X$ be a Banach space. Then $N_\theta(X, F) \subset St_\theta(X)$ if and only if* $\inf_i f_i(u) > 0$, $(u > 0)$.

Proof: If $\inf_i f_i(u) > 0$ then there exists a number $\alpha > 0$ such that $f_i(u) \geq \alpha$ for $u > 0$ and $i \in N$. Let $x \in N_\theta(X, F)$, $\varepsilon > 0$ and $K_\theta(X, \varepsilon) = \{i \in I_r : \|x_i - l\| \geq \varepsilon\}$ then

$$h_r^{-1} \sum_{i\in I_r} f_i(\|x_i - l\|) \geq h_r^{-1} \sum_{i\in K_\theta(X,\varepsilon)} f_i(\|x_i - l\|) \geq \alpha h_r^{-1}|K_\theta(X, \varepsilon)|$$

and it follows that $x \in St_\theta(X)$.

Conversely we can select subsequence $k_{r_j}$ of the lacunary sequence and choose a number $z \geq \varepsilon > 0$ such that $f_i(z) = 0$ for $i \in I_{r_j}$. Now define a sequence $x = (x_i)$ by putting $x_i = zv$ if $i \in I_{r_j}$ for some $j = 1, 2, \dots$ and $x_i = \theta$ otherwise, where $v \in X$ and $\|v\| = 1$. Then we have $x \in N_\theta(X, F)$ but $x \notin St_\theta$.

**Theorem 3.4.** *Let $F = (f_i)$ be a sequence of modulus functions in $S$. Let $X$ be a Banach space. Then $St_\theta(X) \subset N_\theta(X, F)$ if and only if $\sup_u \sup_i f_i(u) < \infty$.*

Proof: We suppose $T(u) = \sup_i f_i(u)$ and $T = \sup_u T(u)$. Let $x \in St_\theta(X)$. Since $f_i(u) \leq T$ for $i \in N$ and $u > 0$, we have

$$h_r^{-1} \sum_{i \in I_r} f_i(\|x_i - l\|) = h_r^{-1} \Big\{ \sum_{\substack{i \in I_r \\ \|x_i - l\| \geq \varepsilon}} f_i(\|x_i - l\|) + \sum_{\substack{i \in I_r \\ \|x_i - l\| < \varepsilon}} f_i(\|x_i - l\|) \Big\}$$

$$\leq h_r^{-1} \Big\{ T |\{ i \in I_r \, : \, \|x_i - l\| \geq \varepsilon \}| + h_r T(\varepsilon) \Big\}.$$

Taking the limit as $\varepsilon \to 0$, it follows that $x \in N_\theta(X, F)$, proving the sufficiency.

Conversely, suppose that $\sup_u \sup_i f_i(u) = \infty$. Then we have $0 < u_1 < u_2 < \dots < u_{r-1} < u_r < \dots$ such that $f_{k_r}(u_r) \geq h_r$ for $r \geq 1$. We define the sequence $x = (x_i)$ by $x_i = u_r v$ if $i = k_r$ for some $r = 1, 2, \dots$ and $x_i = \theta$ otherwise, where $v \in X$ and $\|v\| = 1$. We have $x \in St_\theta(X)$ but $x \notin N_\theta(X, F)$. □

**Corollary 3.5.** *Let $F = (f_i)$ be a sequence of modulus functions in $S$ and let $X$ be a Banach space. Then $N_\theta(X, F) = St_\theta(X)$ if and only if $\inf_i f_i > 0$ and $\sup_u \sup_i f_i(u) < \infty$. In particular, if $f_i = f$ is a modulus function, we have $N_\theta(X, f) = St_\theta(X)$ if and only if $f$ is bounded.*

## References


[1] Connor J.S., *The statistical and strong p-Cesàro convergence of sequences*, Analysis **8** (1988), 47–63.

[2] ———, *On strong matrix summability with respect to a modulus and statistical convergence*, Canad. Math. Bull. **32** (1989), 194–198.

[3] Fast H., *Sur la convergence statistique*, Colloq. Math. **2** (1951), 241–244.

[4] Freedman A.R., Sember J.J., Raphael M., *Some Cesàro-type summability spaces*, Proc. London Math. Soc. **37** (3) (1978), 508–520.

[5] Freedman A.R.. Sember J.J., *Densities and summability*, Pacific J. Math. **95** (2) (1981), 293–305.

[6] Fridy J.A., Orhan C., *Lacunary statistical summability*, J. Math. Anal. Appl. **173** (1993), 497–504.

[7] ———, *Lacunary statistical convergence*, Pacific J. Math. **160** (1993), 45–51.

[8] Kolk E., *The statistical convergence in Banach spaces*, Acta Comm. Univ. Tartuensis **928** (1991), 41–52.

[9] Lorentz G.G., *A contribution to the theory of divergent sequences*, Acta Math. **80** (1948), 167–190.

[10] Maddox I.J., *A new type of convergence*, Math. Proc. Camb. Philos. Soc. **83** (1978), 61–64.

[11] ———, *Sequence spaces defined by a modulus*, Math. Proc. Camb. Philos. Soc. **100** (1986), 161–166.

[12] ______, *Inclusion between FK spaces and Kuttner's theorem*, Math. Proc. Camb. Philos. Soc. **101** (1987), 523–527.
[13] Nakano H., *Concave modulars*, J. Math. Soc. Japan **5** (1953), 29–49.
[14] Niven I., Zuckerman H.S., *An introduction to the theorem of numbers*, 4th ed., Wiley, New York, 1980.
[15] Pehlivan S., *Sequence space defined by a modulus function*, Erc. Univ. J. Sci. **5** (1989), 875–880.
[16] Pehlivan S., Fisher B., *Some sequence spaces defined by a modulus function*, Math. Slovaca **44** (1994), to appear.
[17] Šalát T., *On statistically convergent sequences of real numbers*, Math. Slovaca **30** (2) (1980), 139–150.
[18] Schoenberg I.J., *The integrability of certain functions and related summability methods*, Amer. Math. Monthly **66** (5) (1959), 361–375.
[19] Ruckle W.H., *FK spaces in which the sequence of coordinate vectors is bounded*, Can. J. Math. **25** (1973), 973–978.

S.D. University, Faculty of Science, Department of Mathematics, 32001 Isparta, Turkey

Department of Mathematics and Computer Science, University of Leicester, Leicester, LE1 7RH, United Kingdom

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