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ON THE POLYNOMIALS ORTHOGONAL IN THE INTERVAL $(-\infty, +\infty)$ WITH THE WEIGHT FUNCTION $\exp(-x^8)$

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Summary. In this paper polynomials $H_n(x)$ $n = 0, 1, \ldots$, which are orthonormal with respect to the function $H(x) = \exp(-x^8)$ on the interval $E = (-\infty, +\infty)$ are investigated. Relations coefficients of polynomials and relations (29), (30), (31) for $s_2^{(n)}, s_4^{(n)}, s_6^{(n)}$, where $s_k^{(n)}$ is the sum of k-th powers of the roots or the polynomial $H_n(x)$, are derived. Finaly differential equation (33) for the polynomials $H_n(x)$ is obtained.

Keywords: Polynomials orthogonal, differential equation, Hermitian polynomial.

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1. INTRODUCTION

Definition 1. For $n = 0, 1, \ldots$ let

$$(1) \qquad H_n(x) = \sum_{k=0}^{n} a_k^{(n)} x^{n-k}, \quad a_0^{(n)} > 0,$$

be the *polynomial orthonormal* in the interval $E = (-\infty, \infty)$ with respect to the function

$$(2) \qquad H(x) = \exp(-x^8),$$

i.e. for $m, n = 0, 1, \ldots$

$$(3) \qquad \int_E H_m(x) H_n(x) H(x) \, dx = \delta_{m,n},$$

where $\delta_{m,n} = 0$ for $m \neq n$ and $\delta_{n,n} = 1$.

Remark 1. By a well known theorem (see [1]) for every n there exists one and only one polynomial $H_n(x)$.

Since $H(x)$ is an even function, $H_n(x)$ is an even or an odd function if n is even or odd, respectively, i.e.

$$H_n(-x) = (-1)^n H_n(x).$$

Further, we employ the notation

$$P(x) = \pi_n,$$

which is equivalent to the statement: $P(x)$ is the polynomial whose degree is not higher than n.

In this paper some properties of the integrals connected with the polynomials $H_n(x)$ and with the coefficients of $H_n(x)$ will be derived. Further, we prove the relations (29), (30), (31), where $s_2^{(n)}, s_4^{(n)}, s_6^{(n)}$ is the sum of the second, the fourth, and the sixth power of the roots of the polynomial $H_n(x)$, respectively and $q_n = a_0^{(n-1)}/a_0^{(n)}$. The *differential equation* (33) for these polynomials will be derived, too.

2. THE BASIS PROPERTIES OF THE POLYNOMIALS $H_n(x)$

Lemma 1. *Let* $P(x) = \sum_{k=0}^{n} a_k x^k$. *Then*

$$P(x) = \sum_{k=0}^{n} \alpha_k H_k(x) \tag{4}$$

where

$$\alpha_k = \int_E P(x)\, H_k(x)\, H(x)\, \mathrm{d}x\,. \tag{5}$$

Further

$$\int_E P(x)\, H_n(x)\, H(x)\, \mathrm{d}x = \frac{a_n}{a_0^{(n)}}\,. \tag{6}$$

If $P(x) = \pi_{n-1}$, *then*

$$\int_E P(x)\, H_n(x)\, H(x)\, \mathrm{d}x = 0\,. \tag{7}$$

Proof. The possibility of expressing the polynomial $P(x)$ in the form (4) can be proved by the mathematical induction with respect to n. Using orthonormality of the polynomial $H_n(x)$ the relation (5) is obtained as a consequence of the relation (4). Comparing the coefficients at x^n in relation (4), we get $a_n = a_0^{(n)}\alpha_n$, which is equivalent to (6).

If $P(x) = \pi_{n-1}$, then $a_n = 0$ and (7) is a consequence of (6).

Lemma 2. *Let*

$$q_0 = 0\,, \quad q_n = \frac{a_0^{(n-1)}}{a_0^{(n)}} \quad \textit{for} \quad n > 0\,. \tag{8}$$

Then for $n > 0$ *we have*

$$q_n = \int_E x\, H_n(x)\, H_{n-1}(x)\, H(x)\, \mathrm{d}x\,. \tag{9}$$

Proof. Let us put $P(x) = x\, H_{n-1}(x)$. Then (6) yields

$$\int_E x\, H_{n-1}(x)\, H_n(x)\, H(x)\, \mathrm{d}x = \frac{a_0^{(n-1)}}{a_0^{(n)}} = q_n\,.$$

Lemma 3. (*Recurrence formula for the polynomials* $H_n(x)$.)

(10) $$x\, H_n(x) = q_{n+1}\, H_{n+1}(x) + q_n\, H_{n-1}(x)\,.$$

Proof. Let us put $P(x) = x\, H_n(x)$ in (4) and utilize (5) and (9).

Lemma 4. *Using the notation* (8) *we have*

(11) $$\int_E x^2\, H_n^2(x)\, H(x)\, dx = q_n^2 + q_{n+1}^2\,,$$
$$\int_E x^4\, H_n(x)\, H_{n-2}(x)\, H(x)\, dx = q_{n-1}\, q_n(q_{n+1}^2 + q_n^2)\,.$$

Proof. Let us express $x\, H_n(x)$ using (10), then

$$\int_E [q_{n+1}\, H_{n+1}(x) + q_n\, H_{n-1}(x)]^2\, H(x)\, dx =$$
$$= q_{n+1}^2 \int_E H_{n+1}^2(x)\, H(x)\, dx +$$
$$+ 2q_n\, q_{n+1} \int_E H_{n+1}(x)\, H(x)\, H_{n-1}(x)\, dx +$$
$$+ q_n^2 \int_E H_{n-1}^2(x)\, H(x)\, dx = q_{n+1}^2 + q_n^2\,.$$

The second integral in (11) is obtained from the first since $x^2\, H_{n-2}(x) = q_{n-1} q_n\, H_n(x) + \pi_{n-1}$.

Lemma 5. *For every natural number* k *we have*

(12) $$\int_E x^k\, H_n(x)\, H_{n-k}(x)\, H(x)\, dx = \prod_{i=0}^{k-1} q_{n-i}\,.$$

Proof. a) For $k = 1$ (9) holds.

b) Assuming that (12) is valid for the natural number k we prove that it holds also for $k + 1$:

$$\int_E x^{k+1}\, H_n(x)\, H_{n-k-1}(x)\, H(x)\, dx = \int_E x^k\, H_n(x)\, [x\, H_{n-k-1}(x)]\, H(x)\, dx =$$
$$= \int_E x^k\, H_n(x)\, [q_{n-k}\, H_{n-k}(x) + q_{n-k-1}\, H_{n-k-2}(x)]\, H(x)\, dx =$$
$$= q_{n-k} \int_E x^k\, H_n(x)\, H_{n-k}(x)\, H(x)\, dx = q_{n-k} \prod_{i=0}^{k-1} q_{n-i} = \prod_{i=0}^{k} q_{n-i}\,.$$

Lemma 6. *Using the notation* (9) *we have*

(13) $$\int_E x^3\, H_n(x)\, H_{n-1}(x)\, H(x)\, dx = q_n(q_{n+1}^2 + q_n^2 + q_{n-1}^2)\,.$$

Proof. Using the integral (13) and the relations (10), (11) and (12) we get

$$\int_E x^3\, H_n(x)\, H_{n-1}(x)\, H(x)\, dx = \int_E x^2\, H_n(x)\, [x\, H_{n-1}(x)]\, H(x)\, dx =$$
$$= \int_E x^2\, H_n(x)\, [q_n\, H_n(x) + q_{n-1}\, H_{n-2}(x)]\, H(x)\, dx =$$
$$= q_n \int_E x^2\, H_n^2(x)\, H(x)\, dx + q_{n-1} \int_E x^2\, H_n(x)\, H_{n-2}(x)\, H(x)\, dx =$$
$$= q_n(q_n^2 + q_{n+1}^2) + q_{n-1} q_n q_{n-1}\,.$$

Lemma 7. *For* $k = 0, 1, \ldots$ *we have*

(14) $$x^k H_n(x) = \sum_{i=0}^{k} h^{(k)}_{n+k-2i} H_{n+k-2i}(x),$$

where

(15) $$i \in [1, k] \Rightarrow h^{(k+1)}_{n+k+1-2i} = q_{n+k+1-2i} h^{(k)}_{n+k-2i} + q_{n+k-2i+2} h^{(k)}_{n+k-2i+2},$$

(16) $$i = 0 \Rightarrow h^{(k+1)}_{n+k+1} = q_{n+k+1} h^{(k)}_{n+k} = \prod_{j=1}^{k+1} q_{n+j},$$

(17) $$i = k + 1 \Rightarrow h^{(k+1)}_{n-k-1} = h^{(k)}_{n-k} q_{n-k} = \prod_{j=0}^{k} q_{n-j},$$

and

(18) $$h^{(0)}_n = 1.$$

Proof. The relations (14) and (18) are evident for $k = 0$. If we apply (10) to (14) we get

$$\begin{aligned} x^{k+1} H_n(x) &= x \sum_{i=0}^{k} h^{(k)}_{n+k-2i} H_{n+k-2i}(x) = \\ &= \sum_{i=0}^{k} h^{(k)}_{n+k-2i} [q_{n+k+1-2i} H_{n+k+1-2i}(x) + \\ &+ q_{n+k-2i} H_{n+k-1-2i}(x)] = \\ &= \sum_{i=0}^{k} h^{(k)}_{n+k-2i} q_{n+k+1-2i} H_{n+k+1-2i}(x) + \\ &+ \sum_{i=0}^{k} h^{(k)}_{n+k-2i} q_{n+k-2i} H_{n+k-1-2i}(x) = \\ &= \sum_{i=0}^{k} h^{(k)}_{n+k-2i} q_{n+k+1-2i} H_{n+k+1-2i}(x) + \\ &+ \sum_{i=1}^{k+1} h^{(k)}_{n+k-2i+2} q_{n+k-2i+2} H_{n+k-2i+1}(x). \end{aligned}$$

From the above relation the relations (15), (16) and (17) evidently follow.

Lemma 8. *Using the notation as in Lemma 7, for* $k = 0, 1, \ldots$ *we have*

(19) $$J_{2k} = \int_E x^{2k} H_n^2(x) H(x) \, dx = h^{(2k)}_n.$$

Consequently

(20) $$J_2 = h^{(2)}_n = q_n^2 + q_{n+1}^2,$$

(21) $$J_4 = h^{(4)}_n = \sum_{i=0}^{2} [h^{(2)}_{n+2-2i}]^2 = q_{n+1}^2 q_{n+2}^2 + (q_n^2 + q_{n+1}^2)^2 + q_n^2 q_{n-1}^2,$$

(22) $$J_6 = h_n^{(6)} = \sum_{i=0}^{3} [h_{n+3-2i}^{(3)}]^2 = (q_{n+1}q_{n+2}q_{n+3})^2 + (q_{n-2}q_{n-1}q_n)^2 + $$
$$+ q_{n+1}^2(q_n^2 + q_{n+1}^2 + q_{n+2}^2) + q_n^2(q_{n-1}^2 + q_n^2 + q_{n+1}^2)^2 ,$$

(23) $$J_8 = h_n^{(8)} = \sum_{i=0}^{4} [h_{n+4-2i}^{(4)}]^2 = q_{n+1}^2 q_{n+2}^2 q_{n+3}^2 q_{n+4}^2 +$$
$$+ q_{n-3}^2 q_{n-2}^2 q_{n-1} q_n^2 + q_{n+1}^2 q_{n+2}^2 [q_n^2 + q_{n+1}^2 + q_{n+2}^2 + q_{n+3}^2]^2 +$$
$$+ [q_{n+1}^2 q_{n+2}^2 + (q_n^2 + q_{n+1}^2)^2 + q_n^2 q_{n-1}^2]^2 +$$
$$+ q_n^2 q_{n-1}^2 [q_{n-2}^2 + q_{n-1}^2 + q_n^2 + q_{n+1}^2]^2 .$$

Proof. Using (14).

Lemma 9. *Using the notation* (19) *we have*

(24) $$J_8 = \frac{2n + 1}{8} .$$

Proof. Integrating by parts and using (6), we get

$$J_8 = -\tfrac{1}{8} \int_E x\, H_n^2(x)\, d[\exp(-x^8)] =$$
$$= \tfrac{1}{8} \int_E [H_n^2(x) + 2x\, H_n'(x)\, H_n(x)]\, H(x)\, dx = \tfrac{1}{8}(1 + 2n) .$$

Lemma 10. *Using the notation* (19) *we have*

$$J_{2k} = O(n^{k/r})$$

where $k < r$ *are natural numbers,* $k = 1, 2, 3, 4$.

Proof. Hölder's inequality yields

$$\left|\int_E f(x)\, q(x)\, dx\right| \leqq \left[\int_E |f(x)|^p\, dx\right]^{1/p} \left[\int_E |q(x)|^q\, dx\right]^{1/q} ,$$

where $1/p + 1/q = 1$.

Let us put

$$\frac{1}{p} = \frac{k}{r} \Rightarrow p = \frac{r}{k} > 1 ,$$

$$\frac{1}{q} = 1 - \frac{k}{r} = \frac{r - k}{r} \Rightarrow q = \frac{r}{r - k} ,$$

$$f(x) = x^8 [H_n^2(x)\, H_n(x)]^{k/r} ,$$

$$g(x) = [H_n^2(x)\, H(x)]^{1/q} .$$

We get

$$J_{2k} < J_{2r}^{k/r} . J_0^{k/r} = J_{2r}^{k/r} .$$

Then (24) yields (25).

Lemma 11. *Using the notation* (8) *we have*

$$(26) \qquad q_n = O(n^{1/2r}) .$$

Proof. From (20) and (25) we get

$$q_n^2 < q_n^2 + q_{n+1}^2 = J_2 < c_1 n^{1/r} ,$$

where c_1 is a positive constant which depends neither on n nor on x.

Lemma 12. *Let* $P(x) = \sum_{k=0}^{n} a_k x^{n-k}$, *where* $a_0 = 1$. *We denote the sum of the k-th powers of the zero points of the polynomial* $P(x)$ *as* s_k *for* $k = 0, 1, 2, \ldots$.
Then

$$(27) \qquad x^k P'(x) = P(x) \sum_{i=0}^{k-1} s_i x^{k-1-i} + \pi_{n-1}$$

and

$$(28) \qquad \sum_{i=0}^{k-1} a_i s_{k-i} + k a_k = 0 .$$

The proof is known (see e.g. [2], p. 55).

Lemma 13. *We denote the sum of the zero points of the polynomial* $H_n(x)$ *as* $s_k^{(n)}$. *Then*

$$(29) \qquad s_2^{(n)} = 2 \sum_{k=1}^{n-1} q_k^2 ,$$

$$(30) \qquad s_4^{(n)} = 2 \sum_{k=1}^{n-1} (q_k^4 + 2 q_{k-1}^2 q_k^2) ,$$

$$(31) \qquad s_6^{(n)} = 2 \sum_{k=1}^{n-1} [q_k^6 + 3 q_{k-1}^2 q_k^2 (q_{k-2}^2 + q_{k-1}^2 + q_k^2)] .$$

Proof. 1. Let us denote

$$H_n(x) = \sum_{k=0}^{n} a_k^{(n)} x^{n-k} = a_0^{(n)} \sum_{k=0}^{n} r_k^{(n)} x^{n-k}$$

where

$$r_k^{(n)} = \frac{a_k^{(n)}}{a_0^{(n)}} .$$

Comparing the coefficients at x^{n-2i+1} in equation (10), that is

$$x H_n(x) = q_{n+1} H_{n+1}(x) + q_n H_{n-1}(x) ,$$

we get

(a) $$a_0^{(n)} r_{2i}^{(n)} = q_{n+1} a_0^{(n+1)} r_{2i}^{(n+1)} + q_n a_0^{(n-1)} r_{2i-2}^{(n-1)}.$$

Since $q_{n+1} a_0^{(n+1)} = a_0^{(n)}$ according to (8), relation (a) yields

(b) $$r_{2i}^{(n+1)} - r_{2i}^{(n)} = q_n^2 r_{2i-2}^{(n-1)},$$

and consequently

(c) $$r_{2i}^{(n)} = -\sum_{k=1}^{n-1} q_k^2 r_{2i-2}^{(k-1)}.$$

2. According to Remark 1 we have $s_k^{(n)} = 0$ and $r_0^{(n-1)} = 1$, when k is an odd number. Then from (28) and (c) we get

(d) $$s_2 + 2r_2^{(n)} = 0 \Rightarrow s_2 = 2\sum_{k=1}^{n-1} q_k^2.$$

Further, for $k = 4$ the relation (28) yields

$$s_4^{(n)} + r_2^{(n)} s_2^{(n)} + 4r_4^{(n)} = 0.$$

Using (d) we get

$$s_4^{(n)} - \tfrac{1}{2}(s_2^{(n)})^2 + 4r_4^{(n)} = 0,$$

and finally

$$s_4^{(n+1)} - s_4^{(n)} = \tfrac{1}{2}(s_2^{(n+1)} - s_2^{(n)})(s_2^{(n+1)} + s_2^{(n)}) - 4(r_4^{(n+1)} - r_4^{(n)}).$$

Using (b) and (d) in the above relation we get

$$s_4^{(n+1)} - s_4^{(n)} = q_n^2(s_2^{(n+1)} + s_2^{(n)}) + 4q_n^2 r_2^{(n-1)} = 2q_n^2 s_2^{(n)} +$$

$$+ 2q_n^4 - 2q_n^2 s_2^{(n-1)} = 2q_n^4 + 4q_n^2 q_{n-1}^2.$$

The relation (30) evidently follows from the last relation.

The relation (31) is obtained using (28), (29), and (30).

3. DIFFERENTIAL EQUATION

Lemma 14. *Using the notation as in Lemma 13 we have*

(32) $$x^7 H_n'(x) = (s_6^{(n)} + s_4^{(n)} x^2 + s_2^{(n)} x^4 + nx^6) H_n(x) + \pi_{n-1}.$$

Proof. With regard to Remark 1 the proof follows from (27) for $k = 7$ while $s_1^{(n)} = s_3^{(n)} = s_5^{(n)} = 0$.

Theorem 1. *Using the notation as in* (14) *and* (27) *we have*

(33) $$H_n''(x) - 8x^7 H_n'(x) + 8(s_6^{(n)} + s_4^{(n)} x^2 + s_2^{(n)} x^4 + nx^6) H_n(x) = u_n(x),$$

where

(34) $$u_n(x) = \alpha_{n-2} H_{n-2}(x) + \alpha_{n-4} H_{n-4}(x) + \alpha_{n-6} H_{n-6}(x)$$

and

(35) $$\alpha_{n-2} = 16q_{n-1} q_n[q_{n-2}^4 + q_{n-1}^4 + 2q_{n-2}^2(q_{n-3}^2 + q_{n-1}^2) + + (q_n^2 + q_{n+1}^2)(q_{n-2}^2 + q_{n-1}^2) + q_{n+1}^2 q_{n+2}^2 + + (q_n^2 + q_{n+1}^2)^2 + q_n^2 q_{n-1}^2],$$

(36) $$\alpha_{n-4} = 16q_{n-3}q_{n-2}q_{n-1}q_n(q_{n-4}^2 + q_{n-3}^2 + q_{n-2}^2 + q_{n-1}^2) + + 32q_{n-3}q_{n-2}q_{n-1}q_n(q_n^2 + q_{n+1}^2),$$

(37) $$\alpha_{n-6} = 48q_{n-5}q_{n-4}q_{n-3}q_{n-2}q_{n-1}q_n.$$

Proof. We start from the relation $H^{-1}(x)(d/dx)[H_n'(x) H(x)]$ and utilizing (32) we put

$$A(x) = H^{-1}(x)\frac{d}{dx}[H_n'(x) H(x)] + 8(s_6^{(n)} + s_4^{(n)}x^2 + s_2^{(n)}x^4 + nx^6) H_n(x) =$$
$$= H_n''(x) - 8x^7 H_n'(x) + + 8(s_6^{(n)} + s_4^{(n)}x^2 + s_2^{(n)}x^4 + nx^6) H_n(x) = \pi_{n-1}.$$

According to Lemma 1

$$A(x) = \sum_{k=0}^{n-1} \alpha_k H_k(x),$$

where

(a) $$\alpha_k = \int_E A(x) H_k(x) H(x)\,dx = 8\int_E (s_4^{(n)}x^2 + s_2^{(n)}x^4 + nx^6) . . H_n(x) H_k(x) H(x)\,dx + \int_E H_k(x)\,d[H_n'(x) H(x)].$$

Integrating the last integral by parts we get

(b) $$-\int_E H_k'(x) H_n'(x) H(x)\,dx = \int_E H_n(x)\,d[H_k'(x) H(x)] = = -8\int_E H_n(x)(s_4^{(k)}x^2 + s_2^{(k)}x^4 + kx^6 + \pi_{k-1}) H_k(x) H(x)\,dx.$$

From (a) and (b), as a consequence of orthogonality of the polynomials $H_n(x)$, we obtain the implication

$$k < n - 6 \Rightarrow \alpha_k = 0.$$

Further, $\alpha_{n-1} = \alpha_{n-3} = \alpha_{n-5} = 0$, since the integrand is an odd function. From (a) and (b) for $k = n - 2$ we get

$$\alpha_{n-2} = 8\int_E (s_4^{(n)}x^2 + s_2^{(n)}x^4 + nx^6) H_n(x) H_{n-2}(x) H(x)\,dx -$$
$$- 8\int_E [s_4^{(n)}x^2 + s_2^{(n-2)}x^4 + (n-2)x^6 +$$

$$+ \pi_{n-3}] H_n(x) H_{n-2}(x) H(x) \, dx = 8 \int_E [(s_4^{(n)} - s_4^{(n-2)}) x^2 +$$
$$+ (s_2^{(n)} - s_2^{(n-2)}) x^4 + 2x^6] H_n(x) H_{n-2}(x) H(x) \, dx =$$
$$= 8 \int_E \{2[(q_{n-2}^4 + 2q_{n-3}^2 q_{n-2}^2) + (q_{n-1}^4 + 2q_{n-2}^2 q_{n-1}^2)] x^2 +$$
$$+ 2(q_{n-2}^2 + q_{n-1}^2) x^4 + 2x^6\} H_n(x) H_{n-2}(x) H(x) \, dx =$$
$$= 16[q_{n-2}^4 + 2q_{n-3}^2 q_{n-2}^2 + q_{n-1}^4 + 2q_{n-2}^2 q_{n-1}^2] q_n q_{n-1} +$$
$$+ 16 q_{n-1} q_n (q_n^2 + q_{n+1}^2)(q_{n-2}^2 + q_{n-1}^2) +$$
$$+ 16 q_{n-1} q_n \{q_{n+1}^2 q_{n+2}^2 + (q_n^2 + q_{n+1}^2)^2 + q_n^2 q_{n-1}^2\} .$$

From (a) and (b) for $k = n - 4$ we get

$$\alpha_{n-4} = 8 \int_E (s_4^{(n)} x^2 + s_2^{(n)} x^4 + n x^6) H_n(x) H_{n-4}(x) H(x) \, dx -$$
$$- 8 \int_E [s_4^{(n-4)} x^2 + s_2^{(n-4)} x^4 + (n - 4) x^6 +$$
$$+ \pi_{n-5}] H_n(x) H_{n-4}(x) H(x) \, dx = 8 \int_E [(s_4^{(n)} - s_4^{(n-4)}) x^2 +$$
$$+ (s_2^{(n)} - s_2^{(n-4)}) x^4 + 4x^6] H_n(x) H_{n-4}(x) H(x) \, dx =$$
$$= 16 q_{n-3} q_{n-2} q_{n-1} q_n (q_{n-4}^2 + q_{n-3}^2 + q_{n-2}^2 + q_{n-1}^2) +$$
$$+ 32 q_{n-3} q_{n-2} q_{n-1} q_n (q_n^2 + q_{n+1}^2) .$$

From (a) and (b) for $k = n - 6$ we get

$$\alpha_{n-6} = 8 \int_E (n - n + 6) x^6 H_n(x) H_{n-6}(x) H(x) \, dx =$$
$$= 48 q_n q_{n-1} q_{n-2} q_{n-3} q_{n-4} q_{n-5} .$$

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Súhrn

O POLYNOMOCH ORTOGONÁLNYCH V INTERVALE $(-\infty, +\infty)$ S VÁHOVOU FUNKCIOU $\exp(-x^8)$

VÁCLAV STANČEK

Vyšetrujú sa polynomy $H_n(x)$ $n = 0, 1, \ldots$, ktoré sú ortormálne na intervale $E = (-\infty, \infty)$ vzhľadom na funkciu $H(x) = \exp(-x^8)$. Odvozujú sa vzťahy pre koeficienty týchto polynómov a vzťahy (29), (30), (31) pre $s_2^{(n)}, s_4^{(n)}, s_6^{(n)}$, kde $s_k^{(n)}$ je súčet k-tých mocnín koreňov polyomu $H_n(x)$. Napokon sa odvodzuje diferenciálna rovnica (33) pre polynómy $H_n(x)$.

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