Aleksandr I. Veksler

Narrow spaces, products of topological spaces and supertiny sequences of A. Szymański

Časopis pro pěstování matematiky, Vol. 114 (1989), No. 4, 366--373

Persistent URL: http://dml.cz/dmlcz/118391

NARROW SPACES, PRODUCTS OF TOPOLOGICAL SPACES AND SUPERTINY SEQUENCES OF A. SZYMAŃSKI*)

A. I. VEKSLER, Leningrad

(Received April 25, 1987)

Summary. The first part is devoted to narrow spaces in connection with products of topological spaces. Examples and open questions are also considered. The second part is devoted to tiny and supertiny sequences in the sense of A. Szymański. In particular, it is shown that a compact space is narrow iff it contains a supertiny sequence. A counterexample to a conjecture of A. Szymański is also given.

Keywords: Narrow space, product of topological spaces, tiny sequence, supertiny sequence.

AMS Subject Classification: 54B05, 54G30, 54G99, 54B10.

1. INTRODUCTION

In [5] the author introduced the following notions: a dominant sequence (DS), a narrow space, a pivot and, in particular, solid and soft pivots. These notions were further investigated in [2], [6]–[9]. In [8] the author gave a survey of the results obtained on this subject until 1984. The present work can be considered as a continuation of [8]. Section 2 contains (mostly without proofs) some results from [9], which are devoted to narrow spaces in connection with their relations to products of topological spaces. Section 3 contains some examples and open questions. In particular, these examples give answers to three questions posed in [8]. Almost all examples from this section are based on Section 2. Section 4 is devoted to tiny sequences and supertiny sequences of A. Szymański. In particular, it is shown (Theorem 7) that a compact space is narrow if and only if it contains a supertiny sequence. Besides, a counterexample to a hypothesis of A. Szymański [4] is presented.

The letters X and Y always stand for topological spaces, the letter G (or F and Φ) stands for open (closed, respectively) sets. Further F_n and F'_n will always denote closed nowhere dense (c.n.d.) sets. Under Θ-sets we understand n.d. zero-sets. Under a P'-set (a P-set) in X we understand any F such that if $F \subset A$ for some G_δ-set A, then $F \subset \operatorname{cl} \operatorname{int} A$ ($F \subset \operatorname{int} A$, respectively). A symbol of the type $\{E_n\}$ always means $\{E_n : n \in \omega\}$.

*) *Talk presented 6th Prague Topological Symposium 1986.*

To make a work selfcontained we recall the definitions of the main notions used below. A dominant sequence (DS) on X is any sequence $\{F_n\}$ of c.n.d. sets satisfying the following two conditions: (a) $\bigcup\{F_n : n \in \omega\}$ is dense in X, and (b) for each c.n.d. F'_n with $F'_n \cap F_n = \emptyset$ $(n \in \omega)$, the union $\bigcup\{F'_n : n \in \omega\}$ is n.d. in X. Any set M of the form $M = \bigcup\{F_n : n \in \omega\}$, where $\{F_n\}$ is an arbitrary DS, is called a pivot. A space X is said to be narrow if X has a DS.

Let M be a pivot in X. (i) M is called solid if each sequence $\{F_n\}$ of c.n.d. sets such that $F_n \uparrow M$ (i.e. $F_1 \subset F_2 \subset \ldots$ and $\bigcup\{F_n : n \in \omega\} = M$) is necessarily a DS. (ii) M is called soft if there exist two sequences (of c.n.d. sets) $\{F_n\}$ and $\{F'_n\}$ such that $F_n \uparrow M$, $F'_n \cap F_n = \emptyset$ $(n \in \omega)$, and $\bigcup\{F'_n : n \in \omega\}$ is dense in X.

2. NARROW SPACES AND PRODUCTS OF TOPOLOGICAL SPACES

Proposition 1 ([9]). *Let M be a meager F_σ-set in X and let $Z = X \times Y$. Then the following statements are true*: 1) *If $\{F_n \times Y\}$ is a DS on Z, then $\{F_n\}$ is a DS on X. If Y is a compact space, then the converse is also true.* 2) *Let Y be compact and let M be a pivot in X. Then $M \times Y$ is a pivot in Z. If X is a Baire space, then the converse is also true.* 3) *M is a solid pivot in X provided $M \times Y$ is a solid pivot in Z. If X and Y are compact, then the converse is also true.*

Remark. As Theorem 3 below shows, the assumption of compactness of Y is essential (at least in some cases).

Proposition 2 ([9]). *Let M be a solid pivot in a compact space X, $F_n \uparrow M$, and let H be a dense F_σ-set in X. Then $\{F_n \cap H\}$ is a DS on H.*

Theorem 3 ([9]). *Let the following conditions be satisfied: M is a dense meager F_σ-set in a compact space X, $F_n \uparrow M$, Y is a metric compact space and T is a dense set in Y. A necessary and, in case $S = Y \setminus T$ is dense in Y, sufficient condition for M to be a solid pivot is that $\{F_n \times Y\}$ is a DS on $X \times T$.*

Taking the space Q of rational numbers for T, we obtain a criterion of a solid pivot in compact spaces. Now we present a construction of narrow spaces.

Let $X^{(k)}$ be a topological space, $x_0^{(k)} \in X^{(k)}$ $(k \in \omega)$ and $x_0 = (x_0^{(1)}, x_0^{(2)}, \ldots)$. We denote by $X = N(\{X^{(k)}, x_0^{(k)}\})$ the σ-product of spaces $X^{(k)}$ with center x_0 and endow X with the box-product topology. An infinite σ-product of the type $A_1 \times \ldots \times A_k \times \ldots$ will always denote a set in the σ-product X.

Theorem 4 ([9]). *If $X^{(k)}$ is a compact space and $x_0^{(k)}$ is a nonisolated point in $X^{(k)}$ $(k \in \omega)$, then $X = N(\{X^{(k)}, x_0^{(k)}\})$ is a narrow space and even a pivot in itself with a DS of compact sets $F_n = \{x \in X : x^{(k)} = x_0^{(k)}$ for $k > n\}$. If $x_0^{(k)}$ is a P'-point in $X^{(k)}$ for all $k \geqq k_0$, then X is a solid pivot, otherwise X is a soft pivot.*

Proof. Let us take $p \in \omega$ and represent X in the form

$$X = (X^{(1)} \times \ldots \times X^{(p)}) \times (X^{(p+1)} \times \ldots) = Y_p \times \bar{X}.$$

Obviously, $Pr_{\bar{X}}$ is a closed mapping (since Y_p is compact). For any $A \subset X$ we denote $\bar{A} = Pr_{\bar{X}}\ A$ and $\tilde{A} = \{(x_0^{(1)}, \ldots, x_0^{(p)})\} \times \bar{A}$. Let Φ be closed and $\Phi \cap F_p = \emptyset$. Evidently $\tilde{\Phi}$ is a c.n.d. set. Note also that $\tilde{\Phi} \cap F_p = \emptyset$. Otherwise we would have $\tilde{\Phi} \cap F_p = \{x_0\}$, i.e. $\Phi \cap F_p \neq \emptyset$, a contradiction.

Let now Φ_n be a c.n.d. set in X and $\Phi_n \cap F_n = \emptyset$ $(n \in \omega)$. To prove the first part of the theorem we need to verify only that $\bigcup\{\Phi_n: n \in \omega\}$ is a n.d. set. Take an arbitrary nonvoid open basic set G in X. Then G can be written as follows

$$G = G^{(1)} \times \ldots \times G^{(m)} \times U^{(m+1)} \times U^{(m+2)} \times \ldots,$$

where all factors are nonvoid and open, and $x_0^{(k)} \in U^{(k)}$. We have already proved that $\tilde{\Phi}_p \cap F_p = \emptyset$ for every $p \in \omega$. Hence, without loss of generality we may assume that $\tilde{\Phi}_p \subset \Phi_p$, i.e. $x \in \Phi_p$ implies $\{(x_0^{(1)}, \ldots, x_0^{(p)})\} \times \{\bar{x}\} \in \Phi_p$. Since $x_0 \in F_p$, we have $x_0 \bar{\in} \Phi_p$. Consequently, there is an open set $G'_p \ni x_0$ such that $G'_p \cap \Phi_p = \emptyset$. Let $G'_p = U_p^{(1)} \times U_p^{(2)} \times ..$ $(p \in \omega)$, where $x_0^{(k)} \in U_p^{(k)}$ $(k \in \omega)$ and $U^{(k)} \supset U_1^{(k)} \supset \supset U_2^{(k)} \supset \ldots$ $(k \geqq m + 1)$. Put

$$G_p = X^{(1)} \times \ldots \times X^{(p)} \times U_p^{(p+1)} \times U_p^{(p+2)} \times \ldots.$$

Then $G_p \supset G'_p$ and $G_p \cap \Phi_p = \emptyset$. Otherwise we would have $\tilde{G}_p \cap \tilde{\Phi}_p \neq \emptyset$, and since $G'_p \supset \tilde{G}_p$, $\Phi_p \supset \tilde{\Phi}_p$, this would imply $G'_p \cap \Phi_p \neq \emptyset$, a contradiction.

Now we put $G_0 = G^{(1)} \times \ldots \times G^{(m)} \times U_m^{(m+1)} \times U_{m+1}^{(m+2)} \times U_{m+2}^{(m+3)} \times \ldots$. Then $\emptyset \neq G_0 \subset G$ and $G_p \supset G_0$ for all $p \geqq m$. Hence $G_0 \cap (\bigcup\{\Phi_n: n \geqq m\}) = \emptyset$. But this means that $\bigcup\{\Phi_n: n \in \omega\}$ is a n.d. set in X.

Let $x_0^{(k)}$ be a P'-point in $X^{(k)}$ for all $k \geqq k_0$. Then it is easy to see that F_k is a P'-set for the same k. This and the equality $X = \bigcup\{F_n: n \in \omega\}$ imply (in view of Theorem 19 in [8]) that X is a solid pivot in βX. But then Proposition 10.3 in [5] implies that X is a solid pivot in itself. If $x_0^{(k)}$ is not a P'-point in $X^{(k)}$, then in $X^{(k)}$ there exists a Θ-set $\Theta^{(k)} \ni x_0^{(k)}$. Thus x_0 belongs to a Θ-set $\Theta_k = X^{(1)} \times \ldots \times X^{(k-1)} \times \Theta^{(k)} \times \times X^{(k+1)} \times \ldots$ in X. Obviously $\Theta_{k+1} \supset F_k$. Hence, if an infinite number of points $X_0^{(k)}$ are not P'-points, then X can be covered by a sequence of Θ-sets and by Theorem 9 from [8] the pivot X is soft.

3. EXAMPLES AND QUESTIONS

Example 1. Let X and Y be compact spaces with a solid pivot $M_X \subset X$ and a soft pivot $M_Y \subset Y$. By Proposition 1, $Z = X \times Y$ is a narrow space with a solid pivot $M_X \times Y$ and a soft pivot $X \times M_Y$. This solves in the affirmative question Q1 from [8]. If $F_n \uparrow M_X$ and $\Phi_n \uparrow M_Y$ are DS on X and Y, respectively, then $F_n \times \Phi_n = (F_n \times Y) \cap (X \times \Phi_n)$ is a DS on Z in view of Proposition 1.1 above and Proposition 3.3 from [8]. Moreover, $F_n \times \Phi_n$ is a n.d. subset (in the induced topology) both of $F_n \times Y$ and of $X \times \Phi_n$. Earlier (see [8], Example 4) it was shown that the space $u\omega_1$ exhibits analogous properties. However, the existence of a soft pivot in $u\omega_1$ depends on some set-theoretical assumptions.

Example 2. Let $\{X^{(k)}\}$ be a sequence of compact narrow spaces with DS $\{F_n^{(k)}: n \in \omega\}$ and let $X = \Pi\{X^{(k)}: k \in \omega\}$. Put

$$F_{n,m} = F_n^{(1)} \times \ldots \times F_n^{(m)} \times \Pi\{X^{(k)}: k \geqq m + 1\} \subset X\,.$$

Then $\{F_{n,m}: n \in \omega\}$ is a DS on X for each m, and $F_{n,m+1}$ is a n.d. subset of $F_{n,m}$ in the (induced) topology on $F_{n,m}$. If all the initial pivots $M^{(k)} = \bigcup\{F_n^{(k)}: n \in \omega\}$ are solid in $X^{(k)}$, then the pivots $M_m = \bigcup\{F_{n,m}: n \in \omega\}$ are solid in X $(m \in \omega)$. However, if $M^{(1)}, \ldots, M^{(p)}$ are solid pivots and $M^{(p+1)}$ is a soft pivot, then the pivots $M_1 \ldots M_p$ are solid in X, and $M_{p+1} \ldots$ are soft.

Example 3. Let X be an arbitrary compact space having both a solid pivot M_1 and a soft pivot M_2 (see Example 1). In view of Theorems 7 and 9 from [8] we can assume without loss of generality that there exist on X a DS of P'-sets $F_n \uparrow M_1$ and a DS of Θ-sets $\Theta_n \uparrow M_2$. Hence M_2 is a soft pivot in itself. On the other hand, one can easily see that M_2 is the union of the increasing sequence $\{F_n \cap M_2\}$ of n.d. P'-subsets in M_2. This means that Theorem 7 from [8] cannot be generalized to normal topological spaces, which is the negative answer to question Q3 from [8].

Example 4. Under the conditions of Theorem 3, let $\{F_n\}$ be a DS and let the pivot M be not solid. Then $\{F_n \times Y\}$ is a DS on $Z = X \times Y$, but the restrictions $\{F_n \times T\}$ and $\{F_n \times S\}$ of this DS to two sets $X \times T$ and $X \times S$ mutually complemented in Z fail to be DS.

Example 5. Let $X^{(k)}$ be a unit circle with $x_0^{(k)} = (1, 0)$. Then $N(\{X^{(k)}, x_0^{(k)}\})$ is a topological group under the natural group operation. By Theorem 4 the topological space of this group is a narrow space and a soft pivot in itself.

Example 6. Let $X^{(k)}$ be a countable compact space with a single nonisolated point $x_0^{(k)}$, i.e. $X^{(k)}$ is homeomorphic to $\alpha\omega$. Then $N(\{X^{(k)}, x_0^{(k)}\})$ is a countable normal narrow space which is a soft pivot in itself. This example was constructed by P. Simon (see [8], Example 9). The first example of this kind was constructed (under CH) by V. I. Malyhin [2].

Example 7. Let X be the absolute of the Stone-Čech compactification of a countable normal narrow space (see Example 6). By virtue of Corollary 1 to Proposition 17 and Corollary to Proposition 2 from [8], X is also narrow and, obviously, a separable extremally disconnected compact space.

Another example of a similar space was constructed by A. Szymański [4]. His space is homeomorphic to a P-set S in ω^*, and S (under MA) is not a retract of $\beta\omega$.

In [8] it was asked (Q2) if there exists a narrow compact space X with a pivot M such that the space $X \setminus M$ is narrow as well. We will give the affirmative answer to this question.

Example 8. Let X be a basically disconnected compact space with a solid pivot. Then there is a smaller pivot M which is the union of an increasing DS of n.d. P-sets $\{\Phi_n\}$. We embed each Φ_n in a netting F_n (recall [10] that a netting is a set $X \setminus \bigcup\{X_\alpha: \alpha \in A\}$ where $\{X_\alpha\}$ is an arbitrary maximal family of pairwise disjoint open and closed sets in X). Put $Y = X \setminus M$, $\bar{F}_n = F_n \cap Y$ and show that Y is a narrow space with DS $\{F_n\}$.

First of all we note that the intersection of the netting F_n and the n.d. P-set Φ_k is a n.d. subset of F_n ([10], Theorem 4). Hence the set $\bar{F}_n = F_n \setminus \bigcup\{F_n \cap \Phi_k: k \in \omega\}$ is dense in F_n. Consequently, $\bigcup\{\bar{F}_n: n \in \omega\}$ is dense in X and, moreover, in Y.

Further we show that if $F = X \setminus \bigcup\{X_\alpha: \alpha \in A\}$ is a netting in X and F' is a c.n.d. set in Y which is disjoint from $\bar{F} = F \cap Y$, then $\mathrm{cl}_X F' \cap F = \emptyset$. Indeed, putting $F'_\alpha = F' \cap X_\alpha$, we will show that all but a finite number of the sets F'_α are empty. If not, there exist $x_k \in F'_{\alpha_k}$ $(k \in \omega)$. Put $H = \mathrm{cl}_X \{x_k: k \in \omega\}$. Since Φ_n is a P-set and $x_k \bar{\in} \Phi_n$, we have $H \cap \Phi_n = \emptyset$. Hence $H \subset Y$, $H \subset F'$. Since F is a netting, we have $F \cap H \neq \emptyset$ and $F \cap F' \neq \emptyset$. But this contradicts the closedness of F' in Y and the disjointness $\bar{F} \cap F' = \emptyset$. Thus, there is only a finite number of α with $F'_\alpha \neq \emptyset$. This implies that $\mathrm{cl}_X F' \cap F = \emptyset$.

Finally, let F'_n be a c.n.d. subset in Y and $F'_n \cap \bar{F}_n = \emptyset$. Then (as proved above) $(\mathrm{cl}_X F'_n) \cap F_n = \emptyset$. Hence $\bigcup\{\mathrm{cl}_X F'_n: n \in \omega\}$ is a n.d. set in X, since $\{F_n\}$ is a DS on X. Consequently, $\bar{F}_n$ is a DS on Y, i.e. $Y = X \setminus M$ is a narrow space.

To conclude this section we pose several questions.

Q1. Is it true that $X \times Q$ is not a narrow space, provided X is a separable compact space?

Q2. Do narrow spaces X and Y exist for which $X \times Y$ is not narrow?

Q3. Do non-narrow X and Y exist for which $X \times Y$ is narrow?

Q4. (see [8], Q4). Let X be a compact space and M a dense meager F_σ-set in X. To find (simple enough) characterizations for M to be a soft pivot. In case M is already a pivot in X such characterizations are known ([8], Theorem 9).

Q5. To find a simple enough characterization of pivots in compact spaces.

Q6. Does there exist a soft pivot M in an extremally disconnected compact space X satisfying the Souslin property, such that M covers no nonvoid Θ-set in X?

In view of Theorem 9 from [8] a solution of Q5 would imply a solution to Q4.

4. NARROW SPACES AND SUPERTINY SEQUENCES OF A. SZYMAŃSKI

A. Szymański in [4] introduced the following notion.

Definition 1. A sequence $\{\mathscr{P}_n: n \in \omega\}$ of families of open sets in X is called a *tiny sequence* (TS) if the following two conditions are satisfied: a) $\bigcup\mathscr{P}_n$ is dense in X

for each $n \in \omega$, b) for any finite family $\mathscr{P}'_n \subset \mathscr{P}_n$ $(n \in \omega)$ the set $\bigcup\{(\bigcup\mathscr{P}'_n): n \in \omega\}$ is not dense in X.

In any space without the Souslin property a TS always exists. This led Szymański to another definition.

Definition 2. A TS $\{\mathscr{P}_n: n \in \omega\}$ is called supertiny (STS), if for any $G \neq \emptyset$ there exists $m \in \omega$ such that $\{\mathscr{P}_n \mid G: n \geqq m\}$ is a TS on G.

A. Szymański has established the following proposition.

Proposition 5. *Let $\{\mathscr{P}_n: n \in \omega\}$ be a STS in a compact space X and let $F_n = X \setminus \bigcup\mathscr{P}_n$. Then $\{F_n\}$ is a DS on X.*

To prove this proposition we will make use of the following criterion of DS (see [8], Proposition 13 and Theorem 12): The condition $(*)$ is necessary and, in case of a normal X, sufficient for a sequence F_n of c.n.d. sets to be a DS:

$(*)$ if W_n is a regular closed (r.c.) set in X and $W_n \cap F_n = \emptyset$ $(n \in \omega)$, then there exists no $G \neq \emptyset$ such that $\bigcup\{W_n \cap G: n \geqq m\}$ is dense in G for each $m \in \omega$.

Proof of Proposition 5. Let us assume that $\{F_n\}$ is not a DS. Then by $(*)$ there exists $G \neq \emptyset$ and a r.c. W_n such that $W_n \cap F_n = \emptyset$ $(n \in \omega)$ but $\bigcup\{W_n \cap G: n \geqq m\}$ is dense in G for each $m \in \omega$. Obviously, $W_n \subset \bigcup\mathscr{P}_n$. Since W_n is a compact space, there exists a finite $\mathscr{P}'_n \subset \mathscr{P}_n$ such that $\bigcup\mathscr{P}'_n \supset W$. Hence $\bigcup\{(\bigcup\mathscr{P}'_n \mid G): n \geqq m\}$ is dense in G $(m \in \omega)$. But this is impossible, since $\{\mathscr{P}_n: n \in \omega\}$ is a STS in X, a contradiction.

Now we will show that the converse statement is also true.

Proposition 6. *Let X have a π-base of rc. sets, let $\{F_n\}$ be a DS on X and let $\mathscr{P}_n$ be a maximal family of pairwise disjoint open sets such that $(U \subset \mathscr{P}_n \Rightarrow F_n \cap \operatorname{cl} U = \emptyset)$. Then $\{\mathscr{P}_n: n \in \omega\}$ is a STS in X.*

Proof. The existence of a π-base of r.c. sets in X implies that $\bigcup\mathscr{P}_n$ is dense in X and $\bigcup\mathscr{P}_n \mid G$ is dense in G for any open G and each $n \in \omega$. Let us assume that $\{\mathscr{P}_n: n \in \omega\}$ is not a STS. Then there exists $G \neq \emptyset$ such that for any $m \in \omega$, $\{\mathscr{P}_n \mid G: n \geqq m\}$ is not a TS in G. This means that for any $m \in \omega$ and $n \geqq m$ there exists a finite $\mathscr{P}^m_n \subset \mathscr{P}_n$ such that $\bigcup\{(\bigcup\mathscr{P}^m_n \mid G): n \geqq m\}$ is dense in G. Define

$$W_n = \bigcup\{(\bigcup\{\operatorname{cl} U: U \in \mathscr{P}^k_n\}): k \leqq n\}\,.$$

Then W_n is a r.c. set in X and $W_n \cap F_n = \emptyset$ $(n \in \omega)$, but for any $m \in \omega$, $\bigcup\{W_n \cap G: n \geqq m\}$ is dense in G. This contradicts $(*)$ above. Hence, $\{\mathscr{P}_n: n \in \omega\}$ is a STS in X.

As an immediate corollary to Propositions 5 and 6 we obtain the following result.

Theorem 7. *A compact space is narrow if and only if it has a STS.*

As we see, A. Szymański's notion of a space X with a STS is close to that of a narrow space and they coincide for compact spaces. Szymański proved that any Y

dense in X has a STS if and only if X has a STS. Thus, in the class of completely regular spaces the study of spaces with STS is reduced to the study of compact spaces with STS. Hence, for example, the negative answer to question Q2 for spaces with STS is trivial. This sort of reduction to compact spaces is apparently not possible for narrow spaces. A. Szymański [4] showed that in ZFC the following statement cannot be proved: "compact spaces without TS are precisely those which are coabsolute with dyadic compact spaces" and he conjectured that this statement is consistent with ZFC. The following example disproves it.

Example 9. Let X be the extremally disconnected compact Stone space of the complete Boolean algebra of Lebesgue measurable sets (mod 0) in the interval $[0, 1]$. Then X has no TS though X is not coabsolute with any dyadic compact space.

Let us admit that $\{\mathscr{P}_n: n \in \omega\}$ is a TS in X. We may assume that each $\mathscr{P}_n$ consists of pairwise disjoint open and closed sets. Let μ be a measure on X generated by the Lebesgue measure. Since $\mu E = 0$ for any n.d. set $E \subset X$, we obtain $\mu(\bigcup \mathscr{P}_n) = 1$ for each $n \in \omega$. Pick up a finite $\mathscr{P}'_n \subset \mathscr{P}_n$ such that $\mu(\bigcup \mathscr{P}'_n) > 1 - 1/n$. Then $\mu(\bigcup\{(\mathscr{P}'_n): n \in \omega\}) = 1$. Hence $\bigcup\{\bigcup \mathscr{P}'_n: n \in \omega\}$ is dense in X. This means that $\{\mathscr{P}_n: n \in \omega\}$ is not a TS.

To complete the proof we assume that X is coabsolute with a dyadic compact space Y. Since X is extremally disconnected, there exists an irreducible surjection $\varphi: X \to Y$. Since $wX = c$, we have $wY \leqq c$ and thus the dyadic compact space Y is a continuous image of D^c ([1], Ch. 3, 323). But D^c is separable ([3], § 14), hence Y is also separable. This implies that its irreducible preimage X is separable as well, a contradiction.

References

[1] *A. V. Arkhangelskii, V. I. Ponomarev:* Foundations of General Topology through Problems and Exercises (Russian). Moscow, 1974.

[2] *V. I. Malyhin:* On narrow spaces (Russian). Topological spaces and their mappings, 1983, Riga, 70—76.

[3] *R. Sikorski:* Boolean Algebras, Berlin, 1964.

[4] *A. Szymański:* Some applications of tiny sequences. Suppl. Rend. Circolo Matem. Palermo, ser. II, N 3, (1984), 321—328.

[5] *A. I. Veksler:* Dominant sequences and narrow topological spaces (Russian). Topological spaces and their mappings, 1981, Riga, 159—174, 186.

[6] *A. I. Veksler:* Dominant sequences and narrow spaces (Russian). Uspekhi Mat. Nauk, *37* (1), (1982), 143—144.

[7] *A. I. Veksler:* On dominant sequences and narrow spaces (Russian). Topological spaces and their mappings, 1985, Riga, 33—43.

[8] *A. I. Veksler:* Dominant sequences and topological narrow spaces. Colloq. Math. Soc. J. Boloyai, 41, Topology and applications (Eger, 1983), 635—654.

[9] *A. I. Veksler:* Narrow topological spaces and the products of topological spaces (Russian). Applications of Functional Analysis to Approximation Theory, 1985, Kalinin, 31—42.
[10] *A. I. Veksler:* Nettings in zero-dimensional compact spaces. Lecture Notes in Math., 1060, Topology Proceedings (Leningrad, 1982), Berlin, 1984, 105—114.

Souhrn

ÚZKÉ PROSTORY, SOUČINY TOPOLOGICKÝCH PROSTORŮ A „SUPERTINY" POSLOUPNOSTÍ A. SZYMAŃSKÉHO

A. I. VEKSLER

První část práce je věnována úzkým prostorům v souvislosti se součiny topologických prostorů. Jsou uvedeny i příklady a otevřené problémy. Druhá část je věnována „tiny" a „supertiny" posloupnostem ve smyslu A. Szymańského. Je dokázáno, že kompaktní prostor je úzký, právě když obsahuje supertiny posloupnosti, a je podán protipříklad k hypotéze A. Szymańského.

Резюме

УЗКИЕ ПРОСТРАНСТВА, ПРОИЗВЕДЕНИЯ ТОПОЛОГИЧЕСКИХ ПРОСТРАНСТВ И „СУПЕРКРОШЕЧНЫЕ" ПОСЛЕДОВАТЕЛЬНОСТИ А. ШИМАНСКОГО

A. I. VEKSLER

Первая часть работы посвящена узким пространствам в связи с произведениями топологических пространств. Приведены примеры и открытые проблемы. Вторая часть посвящена „крошечным" и „суперкрошечным" последовательностям в смысле А. Шиманского. Доказано, что компактное пространство является узким тогда и только тогда, когда оно содержит „суперкрошечные" псоледовательности, и приведен контрпример к гипотезе А. Шиманского.

Author's address: LITLP im. S. M. Kirova, ul. Gerzena 18, 191065 Leningrad, USSR.