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OSCILLATION CRITERIA FOR FORCED NONLINEAR ELLIPTIC EQUATIONS OF ARBITRARY ORDER

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Summary. In the paper sufficient conditions are derived for the oscillation of solutions of the equation

$$\Delta^m u + c(x, u) = f(x), \quad x \in E_{r_0},$$

where Δ^m denotes the m-th iteration of the Laplace operator

$$\Delta = \frac{\partial^2}{\partial x_1^2} + \ldots + \frac{\partial^2}{\partial x_n^2}$$

and E_{r_0} is an exterior domain in an n-dimensional Euclidean space R^n.

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We consider the forced elliptic differential equation of the form

$$(1) \qquad \Delta^m u + c(x, u) = f(x), \quad x \in E_{r_0},$$

where $\Delta^m = (\partial^2/\partial x_1^2 + \ldots + \partial^2/\partial x_n^2)^m$ is the m-metaharmonic operator in an n-dimensional Euclidean space R^n,

$$E_{r_0} = \{(x_1, \ldots, x_n) \in R^n, \ |x| > r_0\}, \quad r_0 > 0$$

$$|x| = (\sum_{i=1}^{n} x_i^2)^{1/2}, \ c \in C(E_{r_0} \times R, R) \quad \text{and} \quad f \in C(E_{r_0}, R).$$

Let $D(E_{r_0})$ denote the set of all functions $u \in C^{2m}(E_{r_0}, R)$ such that $u \not\equiv 0$ in any domain E_r, $r \geqq r_0$, defined analogously as E_{r_0}. Equation (1) will be said to be oscillatory in E_{r_0} if every solution $u \in D(E_{r_0})$ of (1) has arbitrarily large zeros, i.e. the set $\{x \in E_{r_0}: u(x) = 0\}$ is unbounded.

The purpose of this paper is to generalize and improve recent results of Kusano and Naito [6] for the second order case of (1). We note that the unforced case of (1) ($f(x) \equiv 0$) has been studied by Kitamura and Kusano in [4]. Other related results on the oscillation of solutions of the unforced partial differential equations and inequalities can be found in the papers of Kitamura and Kusano [3] and Kulenović [5].

Using the method of spherical means introduced by Noussair and Swanson [8] we reduce the problem of oscillation of the partial differential equation (1) to the problem of oscillation of a certain ordinary differential inequality.

Denote

$$S_r = \{(x_1, \ldots, x_n) \in R^n \colon |x| = r\} .$$

Lemma 1. (Kitamura and Kusano [4].) *If $u \in C^{2m}(E_r, R)$ for some $r \geqq r_0$, then the spherical mean of u over S_r, i.e. the function*

$$U(r) = \frac{1}{\sigma_n r^{n-1}} \int_{S_r} u(x)\, \mathrm{d}S ,$$

where σ_n is the area of the unit sphere S_1, satisfies

$$\left(r^{1-n} \frac{\mathrm{d}}{\mathrm{d}r} r^{n-1} \frac{\mathrm{d}}{\mathrm{d}r}\right)^m U(r) = \frac{1}{\sigma_n r^{n-1}} \int_{S_r} \Delta^m u(x)\, \mathrm{d}S , \quad r \geqq r_0 . \tag{2}$$

Theorem 1. *Suppose that the following condition is satisfied:*
(i) *if $u \neq 0$, then*

$$u[c(x, u) - q(|x|)\, \varphi(u)] \geqq 0$$

for all $x \in E_{r_0}$ where q is continuous and positive on $[r_0, \infty)$, $\varphi \in C(R, R)$ is convex on $[0, \infty)$, concave on $(-\infty, 0)$ and such that $u\, \varphi(u) > 0$ for $u \neq 0$. Moreover, let $F(r)$ be the spherical mean of $f(x)$ over S_r, i.e.

$$F(r) = \frac{1}{\sigma_n r^{n-1}} \int_{S_r} f(x)\, \mathrm{d}S .$$

Then the equation (1) is oscillatory in E_{r_0} if the ordinary differential inequality

$$y\left[\left(r^{1-n} \frac{\mathrm{d}}{\mathrm{d}r} r^{n-1} \frac{\mathrm{d}}{\mathrm{d}r}\right)^m y + q(r)\, \varphi(y) - F(r)\right] \leqq 0 \tag{3}$$

is oscillatory at $r = \infty$ in the sense that every nontrivial solution of (3) has arbitrarily large zeros in $[r_0, \infty)$.

Proof. Suppose that the equation (1) is nonoscillatory, i.e. there exists a nonoscillatory solution $u \in D(E_{r_0})$ of (1).

Let $u(x)$ be positive in E_R for some $R \geqq r_0$. By Lemma 1, the spherical mean $U(r)$ of $u(x)$ over S_r, $r \geqq R$, satisfies (2) and, therefore, from (1) we have

$$\left(r^{1-n} \frac{\mathrm{d}}{\mathrm{d}r} r^{n-1} \frac{\mathrm{d}}{\mathrm{d}r}\right)^m U(r) = -\frac{1}{\sigma_n r^{n-1}} \int_{S_r} c(x, u(x))\, \mathrm{d}S + F(r)$$

for $r \geqq R$. Using (i), we get

$$\left(r^{1-n} \frac{\mathrm{d}}{\mathrm{d}r} r^{n-1} \frac{\mathrm{d}}{\mathrm{d}r}\right)^m U(r) \leqq -\frac{q(r)}{\sigma_n r^{n-1}} \int_{S_r} (u(x))\, \mathrm{d}S + F(r) .$$

Since the function φ is convex on $[0, \infty)$, we can use Jensen's inequality (see for example [9]) and conclude that

$$\left(r^{1-n} \frac{\mathrm{d}}{\mathrm{d}r} r^{n-1} \frac{\mathrm{d}}{\mathrm{d}r}\right)^m U(r) \leqq -q(r)\, \varphi(U(r)) + F(r)\,.$$

But this means that the positive function $U(r)$, $r \geqq R$, satisfies the inequality (3), which contradicts the fact that (3) is oscillatory at $r = \infty$.

Similarly we can prove that the equation (1) cannot have a solution which is negative in E_R for some $R \geqq r_0$.

In the light of Theorem 1 it will be necessary to examine the oscillation properties of the ordinary differential inequality (3). We shall consider a more general inequality of the form

$$(4) \qquad y\left[\frac{1}{p_{2m}(r)} \frac{\mathrm{d}}{\mathrm{d}r} \frac{1}{p_{2m-1}(r)} \frac{\mathrm{d}}{\mathrm{d}r} \cdots \frac{\mathrm{d}}{\mathrm{d}r} \frac{1}{p_1(r)} \frac{\mathrm{d}}{\mathrm{d}r} \frac{y(r)}{p_0(r)} + h(r, y) - F(r)\right] \leqq 0$$

including our inequality (3) as a special case.

We assume that the following conditions hold:

(a) the functions $p_i(r)$ $(0 \leqq i \leqq 2m)$ are continuous and positive on $[r_0, \infty)$ and

$$\int_{r_0}^{\infty} p_i(r)\, \mathrm{d}r = \infty \quad (1 \leqq i \leqq 2m - 1)\,;$$

(b) $h\colon [r_0, \infty) \times R \to R$ is continuous and there exist continuous functions h_1 and h_2 defined on $[r_0, \infty)$ and such that for every $r \geqq r_0$,

$$h(r, y) \geqq h_1(r) \quad \text{for} \quad y > 0$$

and

$$h(r, y) \leqq h_2(r) \quad \text{for} \quad y < 0\,;$$

(c) $F\colon [r_0, \infty) \to R$ is continuous.

We employ the notation

$$D^0 y(r) = \frac{y(r)}{p_0(r)}, \quad D^{j+1} y(r) = \frac{1}{p_{j+1}(r)} \frac{\mathrm{d}}{\mathrm{d}r} D^j y(r)\,, \quad 0 \leqq j \leqq 2m - 1\,;$$

$$P_0(r, s) = p_0(r)\,,$$

$$P_i(r, s) = p_0(r) \int_s^r p_1(s_1) \int_s^{s_1} p_2(s_2) \cdots \int_s^{s_{i-1}} p_i(s_i)\, \mathrm{d}s_i\, \mathrm{d}s_{i-1} \ldots \mathrm{d}s_1\,, \quad 1 \leqq i \leqq 2m - 1\,.$$

The inequality (4) can be rewritten as

$$y[D^{2m} y + h(r, y) - F(r)] \leqq 0\,.$$

Theorem 2. *Let the conditions* (a)–(c) *be satisfied and let for every* $R \geqq r_0$,

(5) $$\liminf_{r\to\infty} \frac{1}{P_{2m-1}(r, R)} \int_R^r P_{2m-1}(r, s)\, p_{2m}(s)\, [F(s) - h_1(s)]\, ds = -\infty$$

and

(6) $$\limsup_{r\to\infty} \frac{1}{P_{2m-1}(r, R)} \int_R^r P_{2m-1}(r, s)\, p_{2m}(s)\, [F(s) - h_2(s)]\, ds = \infty\,.$$

Then every nontrivial solution of (4) *is oscillatory.*

Proof. Suppose that there exists a nonoscillatory solution $y(r)$ of (4) on $[r_0, \infty)$. Then there is $r_1 \geqq r_0$ such that $y(r) \neq 0$ for $r \geqq r_1$. Assume first that $y(r)$ is positive on $[r_1, \infty)$. Then it follows from (4) and (b) that

$$D^{2m}\, y(r) + h_1(r) - F(r) \leqq 0$$

for $r \geqq r_1$. Integrating the above inequality $2m$-times from r_1 to r, we obtain

(7) $$y(r) \leqq \sum_{i=0}^{2m-1} c_i P_i(r, r_1) + \int_{r_1}^r P_{2m-1}(r, s)\, p_{2m}(s)\, [F(s) - h_1(s)]\, ds\,,$$

where c_i $(0 \leqq i \leqq 2m - 1)$ are constants. Since

$$\lim_{r\to\infty} \frac{P_i(r, r_1)}{P_{2m-1}(r, r_1)} = 0\,, \quad i = 0, 1, \ldots, 2m - 2$$

(which can be easily proved with the help of L'Hospital's rule and the condition (a)), dividing (7) by $P_{2m-1}(r, r_1)$ and passing to the lower limit as $r \to \infty$, we get

$$\liminf_{r\to\infty} \frac{y(r)}{P_{2m-1}(r, r_1)} = -\infty$$

which contradicts the positivity of $y(r)$ on $[r_1, \infty)$.

Similarly we get a contradiction

$$\limsup_{r\to\infty} \frac{y(r)}{P_{2m-1}(r, r_1)} = \infty$$

in the case $y(r) < 0$ for $r \geqq r_1$.

In the proof of the next theorem we use the following lemma which is a particular case of Lemma 2 in [10].

Lemma 2. *Let the condition* (a) *be satisfied and let*

$$y(r)\, D^{2m}\, y(r) < 0 \;\; (y(r)\, D^{2m}\, y(r) > 0)$$

on $[r_0, \infty)$. *Then there exist an odd* (*even*) *integer* k $(0 \leqq k \leqq 2m)$ *and* $r_1 \geqq r_0$ *such that either*

(8) $$y(r)\, D^i\, y(r) > 0\,, \quad i = 0, 1, \ldots, 2m\,, \quad r \geqq r_1$$

(*the case* $k = 2m$), *or*

(9) $$y(r)\, D^i\, y(r) > 0\,, \quad i = 0, 1, \ldots, k\,, \quad r \geqq r_1\,,$$
and

(10) $$(-1)^{k+i}\, y(r)\, D^i\, y(r) > 0\,, \quad i = k + 1, \ldots, 2m\,, \quad r \geqq r_1\,,$$
(the case $k < 2m$).

Theorem 3. *Suppose that in addition to* (a) *and* (c) *the following conditions hold:*
(d) *$h: [r_0, \infty) \times R \to R$ is continuous, nondecreasing in the second variable for every $r \geqq r_0$ and such that $yh(r, y) > 0$ for $y \neq 0$ and every $r \geqq r_0$,*
and either
(e) *there exists a continuous oscillatory function $\varrho: [r_0, \infty) \to R$ such that $D^{2m}\, \varrho(r) = F(r)$ and $\lim_{r \to \infty} D^0\, \varrho(r) = 0$,*
or
(e') *there exist a continuous function $\eta: [r_0, \infty) \to R$, constants q_1, q_2 and sequences $\{r'_k\}_{k=1}^{\infty}$ and $\{r''_k\}_{k=1}^{\infty}$ such that*

$$D^{2m}\, \eta(r) = F(r)\,,$$

$\lim_{k \to \infty} r'_k = \lim_{k \to \infty} r''_k = \infty$, $D^0\, \eta(r'_k) = q_1$, $D^0\, \eta(r''_k) = q_2$, $q_1 \leqq D^0\, \eta(r) \leqq q_2$ *for* $r \geqq r_0$.

Let the unforced inequality

(11) $$y[D^{2m} y + h(r, y)] \leqq 0$$

be oscillatory. Then the inequality (4) *is oscillatory, too.*

Proof. Let the inequality (4) have a nonoscillatory solution $y(r)$ defined on $[r_0, \infty)$. Suppose first that this solution is positive for $r \geqq r_1 \geqq r_0$ and that the condition (e) is satisfied. Put $z(r) = y(r) - \varrho(r)$. Then

(12) $$D^{2m}\, z(r) \leqq -h(r, y(r)) < 0$$

for $r \geqq r_1$. Obviously, $D^i\, z(r)$, $i = 2m - 1, 2m - 2, \ldots, 0$, are monotonous and have to be of constant sign for sufficiently large r. If $z(r) < 0$ for $r \geqq r_2 \geqq r_1$, then $y(r) < \varrho(r)$ for $r \geqq r_2$, which contradicts the fact that $\varrho(r)$ is oscillatory. Consequently, $z(r)$ must be positive for $r \geqq r_2$, where r_2 is large enough. Now we can use Lemma 2 and conclude, in particular, that $D^1\, z(r) > 0$ for $r \geqq r_3 \geqq r_2$, i.e. $D^0\, z(r)$ is increasing on $[r_3, \infty)$. Moreover, since $\lim_{r \to \infty} D^0\, \varrho(r) = 0$, there exist constants $r_4 \geqq r_3$ and $\varepsilon > 0$ such that

(13) $$D^0\, z(r) + D^0\, \varrho(r) > D^0\, z(r) - \varepsilon > 0$$

for $r \geqq r_4$. Multiplying (13) by $p_0(r)$ we have

$$z(r) + \varrho(r) > z(r) - p_0(r)\, \varepsilon > 0$$

for $r \geqq r_4$. Put $w(r) = z(r) - \varepsilon\, p_0(r)$. Since the function $h(r, y)$ is nondecreasing in the second variable and $D^i w(r) = D^i z(r)$ for $i = 1, 2, \ldots, 2m$, we get

$$D^{2m} w(r) + h(r, w(r)) \leqq D^{2m} w(r) + h(r, z(r) + \varrho(r)) \leqq 0 .$$

So $w(r)$ is a positive solution of

$$D^{2m} w(r) + h(r, w(r)) \leqq 0 , \quad r \geqq r_4 ,$$

which contradicts the fact that the unforced inequality (11) is oscillatory.

Similarly for $y(r) < 0$, $r \geqq r_1$, we get the inequality

$$D^{2m} w(r) + h(r, w(r)) \geqq 0 ,$$

where $w(r) = z(r) + \varepsilon\, p_0(r) < 0$ for $r \geqq r_4$. This is again a contradiction to the oscillatoricity of (11).

Now, let the condition (e') hold. Put $z(r) = y(r) - \eta(r)$. As in the first part of the proof we conclude for $z(r)$ eventually positive that $D^{2m} z(r) < 0$ on $[r_1, \infty)$.

If $D^0 y(r)$ is unbounded, then $D^0 z(r)$ is unbounded as well and it follows that $\lim_{r \to \infty} D^0 z(r) = \infty$. Thus there exists $r_3 \geqq r_2$ such that

$$D^0 z(r) + D^0 \eta(r) \geqq D^0 z(r) + q_1 > 0$$

for $r \geqq r_3$, i.e.

$$z(r) + \eta(r) \geqq z(r) + q_1\, p_0(r) > 0 , \quad r \geqq r_3 .$$

Therefore, the function $w(r) = z(r) + q_1\, p_0(r)$ is a positive solution of

$$D^{2m} w(r) + h(r, w(r)) \leqq 0 , \quad r \geqq r_3 ,$$

which contradicts the assumption that (11) is oscillatory.

If $D^0 y(r)$ is bounded then $D^0 z(r)$ is also bounded and, by Lemma 2, there exists $r_2 \geqq r_1$ such that $(-1)^i D^i z(r) < 0$ for $r \geqq r_2$, $i = 1, \ldots, 2m$. In particular, $D^1 z(r) > 0$ for $r \geqq r_2$, i.e. the function $D^0 z(r)$ is increasing on $[r_2, \infty)$. We claim that $D^0 z(r) + q_1 > 0$ for sufficiently large r. In fact, there exists $r'_K \in \{r'_k\}_{k=1}^{\infty}$, $r'_K \geqq r_2$, such that

$$\begin{aligned} D^0 z(r) + q_1 &= D^0 y(r) - D^0 \eta(r) + q_1 \geqq \\ &\geqq D^0 y(r'_K) - D^0 \eta(r'_K) + q_1 = D^0 y(r'_K) > 0 \end{aligned}$$

for $r \geqq r'_K$. Thus we again obtain a contradiction to the oscillation of all nontrivial solutions of (11), because the function $w(r) = z(r) + q_1\, p_0(r)$ is an eventually positive solution of (11).

The proof in the case that eventually $y(r) < 0$ is similar.

On the basis of Theorems 1, 2 and 3 we can now establish oscillation criteria for the original partial differential equation (1).

Theorem 4. *Equation* (1) *is oscillatory in an exterior domain* E_{r_0} *in* R^n *if*
(ii) *there exist real-valued continuous functions* c_1 *and* c_2 *defined on* $[r_0, \infty)$

and such that for every $x \in E_{r_0}$,

$$c(x, u) \geqq c_1(|x|) \quad for \quad u > 0$$

and

$$c(x, u) \leqq c_2(|x|) \quad for \quad u < 0\,,$$

(iii) *the conditions* (5) *and* (6) *are satisfied, where* $F(s)$ *is the spherical mean of* $f(x)$ *over* S_s, $s \geqq r_0$, $h_1(s) = c_1(s)$, $h_2(s) = c_2(s)$, *and the coefficients in* $P_i(r, s)$ *are the following ones*:

(I) *if* $n = 2$, *then* $p_0(r) = 1$,
$p_1(r) = p_3(r) = \ldots = p_{2m-1}(r) = r^{-1}$,
$p_2(r) = p_4(r) = \ldots = p_{2m}(r) = r$,

(II) *if* $n > 2$, *then* $p_0(r) = r^{2-n}$, $p_{2m}(r) = r$ *and*
$p_i(r) = p_{2m-i}(r) = r$ *for* $i = 1, 2, \ldots, v - 1$,
$p_i(r) = p_{2m-i}(r) = r^{(-1)^{i-v}(n-2v-1)}$ *for* $i = v, v + 1, \ldots, m$,
where $v = \min\{(m, n - 1)/2]\}$ ([N] *denotes the largest integer not exceeding* N).

Proof. Suppose that the equation (1) is not oscillatory in E_{r_0} in R^n, i.e. there exists a nonoscillatory solution $u(x)$ of (1) defined on E_{r_0}. As in the proof of Theorem 1 we first show that the spherical mean $U(r)$ of $u(x)$ over S_r satisfies the ordinary differential inequality

$$\left(r^{1-n}\frac{\mathrm{d}}{\mathrm{d}r}\,r^{n-1}\,\frac{\mathrm{d}}{\mathrm{d}r}\right)^m U(r) + c_1(r) \leqq F(r) \tag{14}$$

if $u(x)$ is eventually positive, or the inequality

$$\left(r^{1-n}\frac{\mathrm{d}}{\mathrm{d}r}\,r^{n-1}\,\frac{\mathrm{d}}{\mathrm{d}r}\right)^m U(r) + c_2(r) \geqq F(r) \tag{15}$$

if $u(x)$ is eventually negative.

Consider first the case (I). Then the functions $p_i(r)$ $(1 \leqq i \leqq 2m - 1)$ satisfy the condition (a) and we can use Theorem 2 directly. However, in the case (II), i.e. $n > 2$, we cannot apply Theorem 2 directly, because $p_{2i-1}(r)$ $(1 \leqq i \leqq m)$ do not satisfy the condition (a). But on the basis of Trench's theory of canonical forms of disconjugate differential operators [12] the differential operator

$$\left(r^{1-n}\frac{\mathrm{d}}{\mathrm{d}r}\,r^{n-1}\,\frac{\mathrm{d}}{\mathrm{d}r}\right)^m$$

can be rewritten as

$$\frac{1}{p_{2m}(r)}\frac{\mathrm{d}}{\mathrm{d}r}\frac{1}{p_{2m-1}(r)}\frac{\mathrm{d}}{\mathrm{d}r}\cdots\frac{\mathrm{d}}{\mathrm{d}r}\frac{1}{p_1(r)}\frac{\mathrm{d}}{\mathrm{d}r}\frac{\cdot}{p_0(r)}$$

in such a way that the functions $p_i(r)$ $(0 \leqq i \leqq 2m)$ satisfy condition (a). Kitamura and Kusano in [4] evaluated these new coefficients $p_i(r)$ explicitly. This evaluation is given in the case (II) of condition (iii). Therefore we can use Theorem 2 again and conclude that the inequality (14) ((15)) cannot have an eventually positive (negative) solution $U(r)$. Consequently, the solution $u(x)$ of (1) cannot be nonoscillatory in E_{r_0} and the proof is complete.

Applying Theorem 3 to the equation (1) we get the following result.

Theorem 5. *Suppose that the condition* (i) *of Theorem* 1 *with* φ *nondecreasing on* $(-\infty, \infty)$ *is satisfied and either*

(iv) *there exists a continuous oscillatory function* $\varrho: [r_0, \infty) \to R$ *such that* $D^{2m} \varrho(r) = F(r)$, $\lim_{r\to\infty} D^0 \varrho(r) = 0$, *where* $F(r)$ *denotes the spherical mean of* $f(x)$ *over* S_r *and the coefficients in* D^{2m} *are given as in the case* (I) *or* (II) *of Theorem* 4,

or

(v) *there exist a continuous function* $\eta: [r_0, \infty) \to R$, *constants* q_1, q_2 *and sequences* $\{r'_k\}_{k=1}^{\infty}$ *and* $\{r''_k\}_{k=1}^{\infty}$ *such that* $D^{2m} \eta(r) = F(r)$, $\lim_{k\to\infty} r'_k = \lim_{k\to\infty} r''_k = \infty$, $D^0 \eta(r'_k) = q_1$, $D^0 \eta(r''_k) = q_2$, $q_1 \leqq D^0 \eta(r) \leqq q_2$ *for* $r \geqq r_0$, *where* $F(r)$ *and* $p_i(r)$ $(0 \leqq i \leqq 2m)$ *are as in* (iv).

Then the equation (1) *is oscillatory in* E_{r_0}, *if the ordinary differential inequality*

$$y[D^{2m}y + q(r)\,\varphi(y)] \leqq 0 \tag{16}$$

is oscillatory in $[r_0, \infty)$.

Examples 1. Consider the equation

$$\Delta u + \frac{2}{|x|} e^u = |x| \sin(\ln |x|) \tag{17}$$

in $E_1 = \{x \in R^4: |x| \geqq 1\}$. In this case $F(r) = r \sin(\ln r)$, $r \geqq 1$, and it is not difficult to verify that the conditions (5) and (6) with $p_0(r) = r^{-2}$, $p_1(r) = r$, $p_2(r) = r$ and $h_1(r) = h_2(r) = 2/r$, that is

$$\liminf_{r\to\infty} \frac{1}{1 - (R/r)^2} \int_R^r [1 - (s/r)^2]\, s^2 \sin(\ln s)\, ds = -\infty$$

and

$$\limsup_{r\to\infty} \frac{1}{1 - (R/r)^2} \int_R^r [1 - (s/r)^2]\, s^2 \sin(\ln s)\, ds = \infty\,,$$

hold. Therefore, by Theorem 4, all solutions of the above equation are oscillatory in E_1. We note that the unforced equation

$$\Delta u + \frac{2}{|x|} e^u = 0 \tag{18}$$

has a nonoscillatory solution $u(x) = -\ln |x|$.

Example 2. Consider the equation

$$\Delta^2 u + \frac{20}{|x|^4} u = \frac{10}{|x|^5} \sin(\ln |x|) \tag{19}$$

in $E_1 = \{x \in R^3 : |x| \geqq 1\}$. The corresponding ordinary differential inequality

$$y\left[r^{-1}(ry)^{(4)} + \frac{20}{r^4} y\right] \leqq 0$$

is oscillatory and $F(r) = 10/r^5 \sin(\ln r)$ satisfies condition (v) of Theorem 5 with $p_0(r) = r^{-1}$, $p_1(r) = p_2(r) = p_3(r) = 1$, $p_4(r) = r$ and $\eta(r) = -\sin(\ln r)/r$. Consequently, the equation (19) is oscillatory in E_1. One oscillatory solution is $u(x) = \sin(\ln |x|)/|x|$. The homogeneous equation

$$\Delta^2 u + \frac{20}{|x|^4} u = 0 \tag{20}$$

is oscillatory in E_1 (see Müler-Pfeiffer [7]).

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Súhrn

KRITÉRIA OSCILÁCIE PRE NELINEÁRNE ELIPTICKÉ ROVNICE ĽUBOVOĽNÉHO RÁDU S NÚTIACIM ČLENOM

PETER ŠVAŇA

V práci sú odvodené postačujúce podmienky oscilácie riešení rovnice

$$\Delta^m u + c(x, u) = f(x), \quad x \in E_{r_0},$$

kde Δ^m označuje m-tú iteráciu Laplaceovho operátora

$$\Delta = \frac{\partial^2}{\partial x_1^2} + \ldots + \frac{\partial^2}{\partial x_n^2}$$

a E_{r_0} je vonkajšia oblasť v n-rozmernom euklidovskom priestore R^n.

Резюме

ПРИЗНАКИ КОЛЕБЛЕМОСТИ ДЛЯ НЕЛИНЕЙНЫХ ЭЛЛИПТИЧЕСКИХ УРАВНЕНИЙ ЛЮБОГО ПОРЯДКА С ВЫНУЖДАЮЩИМ ЧЛЕНОМ

PETER ŠVAŇA

В работе приведены достаточные условия колеблемости решений уравнения

$$\Delta^m u + c(x, u) = f(x),$$

где Δ^m обозначает m-тую итерацию лапласиана

$$\Delta = \frac{\partial^2}{\partial x_1^2} + \ldots + \frac{\partial^2}{\partial x_n^2},$$

вне некоторой ограниченной области в n-мерном евклидовом пространстве R^n.

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