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ON THE DEGREE OF CONVERGENCE OF BOREL AND EULER MEANS OF TRIGONOMETRIC SERIES

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Summary. The rates of pointwise convergence for Borel and Euler means of Fourier series of 2π-periodic functions of bounded variation are estimated. Some analogous estimates concerning the convergence of these means of conjugate series are also presented.

Keywords: trigonometric series, Borel means, Euler means, rate of convergence.

1. INTRODUCTION

Let L be the class of all 2π-periodic complex-valued functions Lebesgue-integrable in the interval $\langle -\pi, \pi \rangle$ and let BV be the set of all 2π-periodic functions of bounded variation on $\langle -\pi, \pi \rangle$. Denote by $V(f; a, b)$ the total variation of $f \in BV$ on $\langle a, b \rangle$.

Given any function $f \in L$, let $S_n[f]$ be the n-th partial sum of its Fourier series. Introduce the Borel and Euler means if this series ([3], pp. 488, 525):

$$B_r[f](x) = e^{-r} \sum_{k=0}^{\infty} \frac{r^k}{k!} S_k[f](x) \quad (r > 0), \tag{1}$$

$$E_n[f](x) = \frac{1}{2^n} \sum_{k=0}^{n} \binom{n}{k} S_k[f](x) \quad (n = 0, 1, 2, \ldots). \tag{2}$$

In [1] Bojanic has proved that if $f \in BV$, then, at every point x,

$$|S_n[f](x) - \tfrac{1}{2}(f(x+) + f(x-))| \leqq \frac{3}{n} \sum_{k=1}^{n} V\left(\varphi_x; 0, \frac{\pi}{k}\right) \quad (n = 1, 2, \ldots),$$

where $f(x+)$ and $f(x-)$ denote the one-sided limits of f at x and φ_x is a 2π-periodic function such that

$$\varphi_x(t) = \begin{cases} f(x+t) + f(x-t) - f(x-) - f(x+) & \text{when } 0 < |t| \leqq \pi, \\ 0 & \text{when } t = 0. \end{cases} \tag{3}$$

The aim of this note is to give the analogous inequalities concerning the rate of convergence of Borel and Euler means of Fourier series for $f \in BV$. Some estimates for the deviation of Borel and Euler means of conjugate series from the conjugate function will be also presented.

2. PROPERTIES OF BOREL MEANS

A simple calculation shows, that the Borel means defined by (1) can be represented in the form

$$B_r[f](x) = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x+t)\, K_r(t)\, \mathrm{d}t\,,$$

where

$$K_r(t) = \mathrm{e}^{-2r\sin^2\frac{1}{2}t}\, \frac{\sin\left(r\sin t + \frac{1}{2}t\right)}{2\sin\frac{1}{2}t}$$

(see [4], p. 364).

The following lemma will be needed in the proof of Theorem 1.

Lemma. *Let* $0 < x < \delta \leqq \pi$ *and* $r > 0$. *Then*

$$\left|\int_x^\delta K_r(t)\, \mathrm{d}t\right| \leqq \frac{10}{rx}\,.$$

Proof. Clearly,

$$\int_x^\delta K_r(t)\, \mathrm{d}t = \int_x^\delta \mathrm{e}^{-2r\sin^2\frac{1}{2}t}\, \frac{\sin(r\sin t)\cos\frac{1}{2}t}{2\sin\frac{1}{2}t}\cos t\, \mathrm{d}t +$$

$$+ \int_x^\delta \mathrm{e}^{-2r\sin^2\frac{1}{2}t}\sin(r\sin t)\cos\tfrac{1}{2}t\sin\tfrac{1}{2}t\, \mathrm{d}t + \tfrac{1}{2}\int_x^\delta \mathrm{e}^{-2r\sin^2\frac{1}{2}t}\cos(r\sin t)\, \mathrm{d}t =$$

$$= I_1 + I_2 + I_3\,, \quad \text{say}\,.$$

It is easy to see that

$$|I_2| \leqq \frac{\pi^2}{2rx} \quad \text{and} \quad |I_3| \leqq \frac{\pi^2}{4rx}\,.$$

Using the second mean-value theorem for the integral I_1 we obtain

$$|I_1| \leqq \tfrac{1}{2}\mathrm{e}^{-2r\sin^2\frac{1}{2}x}\cot\tfrac{1}{2}x \max_{x\leqq\xi\leqq\delta}\left|\int_x^\xi \sin(r\sin t)\cos t\, \mathrm{d}t\right| \leqq \frac{1}{r}\,\mathrm{e}^{-2r\sin^2\frac{1}{2}t}\cot\frac{x}{2} \leqq$$

$$\leqq \frac{1}{r}\cot\frac{x}{2} \leqq \frac{2}{rx}\,.$$

Hence, we get the desired assertion.

Theorem 1. *Let* $f \in L$ *and let there exist a positive number* $\delta \leqq \pi$ *such that* f *is of bounded variation on the interval* $\langle x - \delta, x + \delta\rangle$. *Then, for* $r \geqq 2$,

(4)

$$|B_r[f](x) - \tfrac{1}{2}(f(x+) + f(x-))| \leqq \left(2\delta^2 + \frac{20}{\pi}\right)\frac{1}{r\delta}\sum_{k=1}^{[r]} V\left(\varphi_x; 0, \frac{\delta}{k}\right) + \frac{1}{4\delta}\,\mathrm{e}^{-2r\delta^2}\,\|\varphi_x\|,$$

where $[r]$ *is the integral part of* r *and* $\|\ \|$ *denotes the norm in the space* L.

Proof. Obviously,

$$B_r[f](x) - \tfrac{1}{2}(f(x+) + f(x-)) =$$

$$= \frac{1}{\pi}\left(\int_0^{\delta/r} + \int_{\delta/r}^{\delta} + \int_{\delta}^{\pi}\right) \varphi_x(t)\, K_r(t)\, \mathrm{d}t = P_r + Q_r + R_r\,, \quad \text{say}\,.$$

Since, for $t \in (0, \delta/r)$,

$$|K_r(t)| \leqq \frac{r \sin t + \frac{1}{2}t}{2 \sin \frac{1}{2}t} \leqq r\pi \quad \text{and} \quad \varphi_x(0) = 0\,,$$

we have

$$|P_r| \leqq \frac{1}{\pi}\int_0^{\delta/r} |\varphi_x(t) - \varphi_x(0)|\, r\pi\, \mathrm{d}t \leqq \delta V\left(\varphi_x; 0, \frac{\delta}{r}\right).$$

To estimate Q_r let us put

$$\Lambda_r(x) = \int_x^{\delta} K_r(t)\, \mathrm{d}t \quad \text{for} \quad x \in \langle 0, \delta \rangle\,.$$

Integrating by parts, we get

$$|Q_r| \leqq \frac{1}{\pi}\left|\varphi_x\left(\frac{\delta}{r}\right)\right| \left|\Lambda_x\left(\frac{\delta}{r}\right)\right| + \frac{1}{\pi}\left|\int_{\delta/r}^{\delta} \Lambda_r(t)\, \mathrm{d}\varphi_x(t)\right|.$$

In view of Lemma,

$$|Q_r| \leqq \frac{10}{\delta\pi} V\left(\varphi_x; 0, \frac{\delta}{r}\right) + \frac{10}{\pi r}\int_{\delta/r}^{\delta} \frac{1}{t}\, \mathrm{d}V(\varphi_x; 0, t) \leqq$$

$$\leqq \frac{10}{r\delta\pi} V(\varphi_x; 0, \delta) + \frac{10}{r\pi}\int_{\delta/r}^{\delta} \frac{1}{t^2} V(\varphi_x; 0, t)\, \mathrm{d}t\,.$$

The inequality

(5) $$\frac{1}{r}\int_{\delta/r}^{\delta} \frac{1}{t^2} V(\varphi_x; 0, t)\, \mathrm{d}t \geqq \frac{1}{2\delta} V\left(\varphi_x; 0, \frac{\delta}{r}\right) \quad (r \geqq 2)$$

implies

$$|P_r| + |Q_r| \leqq \left(2\delta^2 + \frac{10}{\pi}\right)\frac{1}{r}\int_{\delta/r}^{\delta} \frac{1}{t^2} V(\varphi_x; 0, t)\, \mathrm{d}t + \frac{10}{r\delta\pi} V(\varphi_x; 0, \delta)\,.$$

It is easily seen that

$$|R_r| \leqq \frac{1}{\pi} \mathrm{e}^{-2(\delta/\pi)^2} \int_{\delta}^{\pi} |\varphi_x(t)|\, \frac{\pi}{2t}\, \mathrm{d}t \leqq \frac{1}{4\delta} \mathrm{e}^{-2(\delta/\pi)^2} \|\varphi_x\|\,.$$

The desired result follows, because

(6) $$\int_{\delta/r}^{\delta} \frac{1}{t^2} V(\varphi_x; 0, t)\, \mathrm{d}t = \frac{1}{\delta}\int_1^r V\left(\varphi_x; 0, \frac{\delta}{t}\right) \mathrm{d}t \leqq \frac{1}{\delta}\sum_{k=1}^{[r]} V\left(\varphi_x; 0, \frac{\delta}{k}\right).$$

Corollary 1. *In particular, if* $f \in BV$,

$$(7)\qquad |B_r[f](x) - \tfrac{1}{2}(f(x+) + f(x-))| \leqq \frac{28}{r\pi}\sum_{k=1}^{[r]} V\left(\varphi_x; 0, \frac{\pi}{k}\right) \quad (r \geqq 2)$$

at every point x.

Remark. Since the function φ_x is of bounded variation on $\langle 0, \delta\rangle$, continuous at the point $t = 0$, the total variation $V(\varphi_x; 0, y)$, $y \in \langle 0, \delta\rangle$ is a continuous function at $y = 0$ and $\lim_{k\to\infty} V(\varphi_x; 0, \delta/k) = 0$. Consequently, the right-hand sides of the inequalities (4) and (7) converge to zero as $r \to \infty$.

Consider now the Borel means $\tilde{B}_r[f]$ of the trigonometric series conjugate to the Fourier series of $f \in BV$. It can be easily verified that

$$\tilde{B}_r[f](x) = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x+t)\,\tilde{K}_r(t)\,\mathrm{d}t$$

with

$$\tilde{K}_r(t) = \mathrm{e}^{-2r\sin^2\frac{1}{2}t}\,\frac{\cos(r\sin t + \frac{1}{2}t)}{2\sin\frac{1}{2}t} - \tfrac{1}{2}\cot\tfrac{1}{2}t\,.$$

Theorem 2. *For any $f \in BV$ and all $r \geqq \frac{5}{2}$ we have*

$$(8)\qquad \left|\tilde{B}_r[f](x) + \frac{1}{\pi}\int_{\pi/r}^{\pi} \frac{\psi_x(t)}{2\tan\frac{1}{2}t}\,\mathrm{d}t\right| \leqq \frac{10}{r}\sum_{k=1}^{[r]} V\left(\psi_x; 0, \frac{\pi}{k}\right),$$

where

$$(9)\qquad \psi_x(t) = f(x+t) - f(x-t) \quad (-\infty < t < +\infty)\,.$$

Proof. We can easily observe that

$$\tilde{B}_r[f](x) + \frac{1}{\pi}\int_{\pi/r}^{\pi} \frac{\psi_x(t)}{2\tan\frac{1}{2}t}\,\mathrm{d}t = \frac{1}{\pi}\int_0^{\pi/r} \psi_x(t)\,\tilde{K}_r(t)\,\mathrm{d}t + \frac{1}{\pi}\int_{\pi/r}^{\pi} \psi_x(t)\,(\tilde{K}_r(t) + \tfrac{1}{2}\cot\tfrac{1}{2}t)\,\mathrm{d}t \equiv M_r + N_r\,, \quad \text{say}\,.$$

Analogously as in the proof of our Lemma,

$$|\Lambda_r^*(x)| \equiv \left|\int_x^{\pi}(\tilde{K}_r(t) + \tfrac{1}{2}\cot\tfrac{1}{2}t)\,\mathrm{d}t\right| \leqq \left|\int_x^{\pi} \mathrm{e}^{-2r\sin^2\frac{1}{2}t}\,\frac{\cos(r\sin t)}{2\sin\frac{1}{2}t}\cos t\,\mathrm{d}t\right| + \left|\int_x^{\pi} \mathrm{e}^{-2r\sin^2\frac{1}{2}t}\,\frac{\cos(r\sin t)}{2\sin\frac{1}{2}t}(\cos\tfrac{1}{2}t - \cos t)\,\mathrm{d}t\right| + \left|\int_x^{\pi} \mathrm{e}^{-2r\sin^2\frac{1}{2}t}\,\frac{\sin(r\sin t)}{2}\,\mathrm{d}t\right| \leqq$$

$$\leqq \frac{1}{r\sin\frac{1}{2}x}\,\mathrm{e}^{-2r\sin^2\frac{1}{2}x} + \int_x^{\pi} \frac{\sin\frac{3t}{4}\sin\frac{t}{4}}{2r\sin^3\frac{1}{2}t}\,\mathrm{d}t + \frac{1}{4r}\int_x^{\pi}\frac{\mathrm{d}t}{\sin^2\frac{1}{2}t} \leqq$$

$$\leqq \frac{\pi}{rx} + \frac{3\pi^4}{32rx} + \frac{\pi^2}{4rx} \leqq \frac{5\pi}{rx}\,.$$

Hence

$$|\Lambda_r^*(x)| \leqq \frac{5\pi}{rx} \quad (0 < x < \pi,\ r > 0)\,. \tag{10}$$

For N_r we use the estimate (10), and after the partial integration we find that

$$|N_r| \leqq \frac{1}{\pi}\left|\psi_x\left(\frac{\pi}{r}\right)\Lambda_r^*\left(\frac{\pi}{r}\right)\right| + \frac{1}{\pi}\left|\int_{\pi/r}^{\pi} \Lambda_r^*(t)\,\mathrm{d}\psi_x(t)\right| \leqq \frac{5}{\pi}V\left(\psi_x; 0, \frac{\pi}{r}\right) +$$
$$+ \frac{5}{r}\int_{\pi/r}^{\pi}\frac{1}{t}\,\mathrm{d}V(\psi_x; 0, t) \leqq \frac{5}{r\pi}\,V(\psi_x; 0, \pi) + \frac{5}{r}\int_{\pi/r}^{\pi}\frac{1}{t^2}\,V(\psi_x; 0, t)\,\mathrm{d}t\,.$$

Observing that, for $t \in (0, \pi/r\rangle$ and $r \geqq 5/2$,

$$|\tilde{K}_r(t)| \leqq \left|\tfrac{1}{2}\cot\tfrac{1}{2}t\left[\mathrm{e}^{-2r\sin^2\frac{1}{2}t}\cos(r\sin t) - 1\right]\right| + \left|\mathrm{e}^{-2r\sin^2\frac{1}{2}t}\,\frac{\sin(r\sin t)}{2}\right| \leqq$$
$$\leqq \left|\tfrac{1}{2}\cot\tfrac{1}{2}t\,\mathrm{e}^{-2r\sin^2\frac{1}{2}t}\,2\sin^2\left(\tfrac{1}{2}r\sin t\right)\right| + \left|\tfrac{1}{2}\cot\tfrac{1}{2}t\left(\mathrm{e}^{-2r\sin^2\frac{1}{2}t} - 1\right)\right| + \tfrac{1}{2} \leqq$$
$$\leqq r + \left|\tfrac{1}{2}\cot\tfrac{1}{2}t\,.\,2r\sin^2\tfrac{1}{2}t\right| + \tfrac{1}{2} \leqq 3r$$

and applying the estimate analogous to (5) we get

$$|M_r| \leqq 3V\left(\psi_x; 0, \frac{\pi}{r}\right) \leqq \frac{6\pi}{r}\int_{\pi/r}^{\pi}\frac{1}{t^2}\,V(\psi_x; 0, t)\,\mathrm{d}t\,.$$

Consequently,

$$\left|\tilde{B}_r[f](x) + \frac{1}{\pi}\int_{\pi/r}^{\pi}\frac{\psi_x(t)}{2\tan\frac{1}{2}t}\,\mathrm{d}t\right| \leqq \frac{8\pi}{r}\int_{\pi/r}^{\pi}\frac{1}{t^2}\,V(\psi_x; 0, t)\,\mathrm{d}t + \frac{5}{r\pi}\,V(\psi_x; 0, \pi)$$

and our estimate is now evident, by an inequality similar to (6).

Remark. The right-hand side of the inequality (8) converges to zero as $r \to \infty$, provided $f(x+) = f(x-)$. If, at the point x,

$$\int_0^{\pi}\frac{|\psi_x(t)|}{t}\,\mathrm{d}t < \infty\,,$$

then there exists a finite limit

$$\tilde{f}(x) = \lim_{\varepsilon\to 0+}\left(-\frac{1}{\pi}\int_{\varepsilon}^{\pi}\frac{\psi_x(t)}{2\tan\frac{1}{2}t}\,\mathrm{d}t\right)$$

and

$$|\tilde{B}_r[f](x) - \tilde{f}(x)| \leqq \tfrac{1}{2}\int_0^{\pi/r}\frac{|\psi_x(t)|}{t}\,\mathrm{d}t + \frac{10}{r}\sum_{k=1}^{[r]}V\left(\psi_x; 0, \frac{\pi}{k}\right) \quad (r \geqq \tfrac{5}{2})\,;$$

hence $\lim_{r\to\infty}\tilde{B}_r[f](x) = \tilde{f}(x)$.

Direct calculation shows that, for the operator (2), the following equation is true:

$$E_n[f](x) = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x+t)\, U_n(t)\,\mathrm{d}t \quad \text{with} \quad U_n(t) = \frac{\cos^n \frac{1}{2}t \sin(n+1)\frac{1}{2}t}{2\sin\frac{1}{2}t}$$

(cf. [2]). Reasoning analogously as in the proof of Theorem 1, we obtain

Theorem 3. *Let $f \in L$ and let there exist a positive number $\delta \leqq \pi$ such that f is of bounded variation on the interval $\langle x-\delta, x+\delta\rangle$. Then, for $n \geqq 1$,*

$$|E_n[f](x) - \tfrac{1}{2}(f(x+) + f(x-))| \leqq \left(\frac{\delta^2}{\pi} + 4\right)\frac{1}{n\delta}\sum_{k=1}^{n} V\left(\varphi_x; 0, \frac{\delta}{k}\right) + \frac{1}{2\delta}\cos^n \tfrac{1}{2}\delta \|\varphi_x\|,$$

where the function φ_x is defined by (3). *Consequently,*

$$\lim_{n\to\infty} E_n[f](x) = \tfrac{1}{2}(f(x+) + f(x-)).$$

Corollary 2. *In particular, if $f \in BV$,*

$$|E_n[f](x) - \tfrac{1}{2}(f(x+) + f(x-))| \leqq \frac{7}{3n}\sum_{k=1}^{n} V\left(\varphi_x; 0, \frac{\pi}{k}\right)$$

for every point x and all $n \geqq 1$.

Finally, let us denote by $\tilde{E}_n[f]$ the Euler means of the trigonometric series conjugate to the Fourier series of $f \in BV$. This operator can be written in the form

$$\tilde{E}_n[f](x) = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x+t)\, \tilde{U}_n(t)\,\mathrm{d}t \quad \text{with} \quad \tilde{U}_n(t) = \frac{\cos^n \frac{1}{2}t \cos(n+1)\frac{1}{2}t}{2\sin\frac{1}{2}t} + $$

$$- \tfrac{1}{2}\cot\tfrac{1}{2}t.$$

Retaining the symbol (9) we present

Theorem 4. *If $f \in BV$ and if $n \geqq 2$, then*

$$\left|\tilde{E}_n[f](x) + \frac{1}{\pi}\int_{\pi/n}^{\pi} \frac{\psi_x(t)}{2\tan\frac{1}{2}t}\,\mathrm{d}t\right| \leqq \frac{6}{n}\sum_{k=1}^{n} V\left(\psi_x; 0, \frac{\pi}{k}\right).$$

Proof. Evidently,

$$\tilde{E}_n[f](x) + \frac{1}{\pi}\int_{\pi/n}^{\pi} \frac{\psi_x(t)}{2\tan\frac{1}{2}t}\,\mathrm{d}t =$$

$$= \frac{1}{\pi}\int_0^{\pi/n} \psi_x(t)\,\tilde{U}_n(t)\,\mathrm{d}t + \frac{1}{\pi}\int_{\pi/n}^{\pi} \psi_x(t)\frac{\cos^n\frac{1}{2}t\cos(n+1)\frac{1}{2}t}{2\sin\frac{1}{2}t}\,\mathrm{d}t =$$

$$= G_n + H_n, \quad \text{say}.$$

Let us observe that, for $t \in (0, \pi/n)$ and $n \geqq 1$,

$$|\tilde{U}_n(t)| \leqq |\tfrac{1}{2} \cot \tfrac{1}{2}t(\cos^n \tfrac{1}{2}t \cos \tfrac{1}{2}nt - 1) + |\tfrac{1}{2} \cos^n \tfrac{1}{2}t \sin \tfrac{1}{2}nt| \leqq$$

$$\leqq \tfrac{1}{2} + \left|\tfrac{1}{2} \cot \tfrac{1}{2}t \left\{\cos \tfrac{1}{2}nt \sum_{k=0}^{n} (-1)^k \binom{n}{k} (2 \sin^2 \tfrac{1}{4}t)^k - 1\right\}\right| \leqq$$

$$\leqq \tfrac{1}{2} + |\cot \tfrac{1}{2}t \sin^2 \tfrac{1}{4}nt| + \left|\tfrac{1}{2} \cot \tfrac{1}{2}t \cos \tfrac{1}{2}nt \sum_{k=1}^{n} (-1)^k \binom{n}{k} (2 \sin^2 \tfrac{1}{4}t)^k\right| \leqq$$

$$\leqq \tfrac{1}{2} + \tfrac{1}{2}n + |\cot \tfrac{1}{2}t \; n \sin^2 \tfrac{1}{4}t| \leqq \tfrac{1}{2} + \frac{3n}{2} \leqq 2n .$$

Consequently,

$$|G_n| \leqq 2V\left(\psi_x; 0, \frac{\pi}{n}\right).$$

Next, an argument similar to that of the proof of Theorem 2 leads to

$$|H_n| \leqq \frac{2}{n\pi} V(\psi_x; 0, \pi) + \frac{2}{n} \int_{\pi/n}^{\pi} \frac{1}{t^2} V(\psi_x; 0, t) \, dt .$$

Because, for $n \geqq 2$,

$$\frac{1}{n} \int_{\pi/n}^{\pi} \frac{1}{t^2} V(\psi_x; 0, t) \, dt \geqq \frac{1}{2\pi} V\left(\psi_x; 0, \frac{\pi}{n}\right)$$

and

$$\int_{\pi/n}^{\pi} \frac{1}{t^2} V(\psi_x; 0, t) \, dt \leqq \frac{1}{\pi} \sum_{k=1}^{n} V\left(\psi_x; 0, \frac{\pi}{k}\right),$$

the desired result is established.

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Souhrn

O STUPNI KONVERGENCE BORELOVÝCH A EULEROVÝCH PRŮMĚRŮ TRIGONOMETRICKÝCH ŘAD

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Je odhadnuta rychlost bodové konvergence Borelových a Eulerových průměrů Fourierových řad 2π-periodických funkcí s konečnou variací. Dále jsou uvedeny analogické odhady konvergence těchto průměrů pro konjugované řady.

Резюме

О ПОРЯДКЕ СХОДИМОСТИ СРЕДНИХ БОРЕЛЯ И ЭЙЛЕРА ТРИГОНОМЕТРИЧЕСКИХ РЯДОВ

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Оценивается скорость поточечной сходимости средних Бореля и Эйлера рядов Фурье 2π-периодических функций ограниченной вариации. Представлены также аналогические результаты об этих средних сопряженных рядов.

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