Časopis pro pěstování matematiky

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Časopis pro pěstování matematiky, Vol. 110 (1985), No. 3, 294--301

Persistent URL: http://dml.cz/dmlcz/118236

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TWO EDGE-DISJOINT HAMILTONIAN CYCLES OF POWERS OF A GRAPH

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(Received January 30, 1984)

If G is a graph (in the sense of the books [1] or [3]) and n is a positive integer, then by the n-th power G^n of G we mean the graph with

$$V(G^n) = V(G) \quad \text{and} \quad E(G^n) = \{vv';\ v, v' \in V(G) \text{ and } 1 \leqq d_G(v, v') \leqq n\}\,.$$

The expressions $V(H)$, $E(H)$, and $d_H(v_1, v_2)$ denote the vertex set of a graph H, the edge set of H, and the distance between vertices v_1 and v_2 in H, respectively.

A number of results concerning powers of graphs is known. We now mention four results concerning low powers (the number of vertices of a graph G is called the order of G):

Theorem A ([2] and [7]). *For every connected graph G of even order, G^2 has a 1-factor.*

Theorem B. *For every connected graph G of order $\geqq 3$, G^3 has a hamiltonian cycle.*

Theorem C ([4]). *For every connected graph G of even order $\geqq 4$, G^4 has three mutually edge-disjoint 1-factors.*

Theorem D. *For every connected graph G of order $\geqq 5$, G^5 has a 4-factor.*

Theorem B is an immediate consequence of Sekanina's result in [6]; he proved that for every connected graph G, G^3 is hamiltonian-connected. Theorem D is a special case of Theorem 2 in [5] (for $n = 4$). In the present paper we shall prove the following theorem, which improves both Theorem B and Theorem D:

Theorem 1. *Let G be a connected graph of order $\geqq 5$. Then there exist a hamiltonian cycle C of G^3 and a hamiltonian cycle C' of G^5 such that C and C' are edge-disjoint.*

The following corollary is an immediate consequence of Theorem 1:

Corollary. *If G is a connected graph of order* $\geqq 5$, *then* G^5 *has two edge-disjoint hamiltonian cycles.*

Note that it has been shown in [5] that there exists an infinite set of nonisomorphic trees T such that T^4 has no 4-factor.

Theorem 1 will be derived from three lemmas. Before stating the first of them we shall introduce some useful notions.

We say that an ordered pair (T', r') is a rooted tree if T' is a tree and $r' \in V(T')$. We say that rooted trees (T', r') and (T'', r'') are isomorphic if T' and T'' are isomorphic and there exists an isomorphism from T' onto T'' which maps r' onto r''. Let T be a tree. By a terminal subtree of T we shall mean a rooted tree (T', r') with the properties that T' is a subtree of T and for each $v \in V(T' - r')$, $\deg_{T'} v = \deg_T v$ (where $\deg_H w$ denotes the degree of a vertex w in a graph H).

Let $m \geqq 0$ and $n \geqq 1$ be integers, and let $u_0, \ldots u_m, w_1, \ldots, w_n$ be mutually distinct vertices. We denote by A_n the path with

$$V(A_n) = \{w_1, \ldots, w_n\} \quad \text{and} \quad E(A_n) = \{w_i w_{i+1};\ 1 \leqq i \leqq n-1\}\,.$$

Similarly, we denote by B_{mn} the path with

$$V(B_{mn}) = \{u_m, \ldots, u_0,\ w_1, \ldots, w_n\} \quad \text{and}$$

$$E(B_{mn}) = \{u_j u_{j-1};\ m \geqq j > 0\} \cup \{u_0 w_1\} \cup \{w_k w_{k+1};\ 1 \leqq k \leqq n-1\}\,.$$

Moreover, we define

$$A_{n*} = A_n - w_{n-1}w_n + w_{n-2}w_n \quad \text{for} \quad n \geqq 3\,, \quad \text{and}$$

$$A_{*n*} = A_{n*} - w_1 w_2 + w_1 w_3 \quad \text{for} \quad n \geqq 4\,.$$

Finally, we define the following rooted trees:

$$D_{mn} = (B_{mn}, u_0)\,;$$

$$D_{mn*} = (B_{mn} - w_{n-1}w_n + w_{n-2}w_n, u_0) \quad \text{for} \quad n \geqq 3\,;$$

$$D_{*mn} = (B_{mn} - u_{m-1}u_m + u_{m-2}u_m, u_0) \text{ for } m \geqq 2; \text{ and}$$

$$D_{*mn*} = (B_{mn} - u_m u_{m-1} - w_{n-1}w_n + u_m u_{m-2} + w_{n-2}w_n, u_0)$$

$$\text{for} \quad m \geqq 2 \quad \text{and} \quad n \geqq 3\,.$$

Lemma 1. *Let T be a tree of order* $\geqq 6$. *Then there exists a terminal subtree of T which is isomorphic to one of the twenty three rooted trees that follow:*

D_{*21},

D_{21}, D_{22},

D_{*31}, D_{31}, D_{*32}, D_{32}, D_{*33*}, D_{33*}, D_{33},

D_{*41}, D_{41}, D_{*42}, D_{42}, D_{*34*}, D_{*34}, D_{34*}, D_{34}, D_{*44*}, D_{44*}, D_{44},

D_{05*}, D_{05}.

Proof. Let δ denote the diameter of T. Obviously, there exists a terminal subtree (T_0, r_0) of T with the properties that

$$d_{T_0}(r_0, v) \leqq 5 \quad \text{for every} \quad v \in V(T_0), \quad \text{and}$$

$$d_{T_0}(r_0, \bar{v}) = \min(5, \delta) \quad \text{for at least one} \quad \bar{v} \in V(T_0).$$

It is easy to see that there exists a terminal subtree (T', r') of T such that $V(T') \subseteq \subseteq V(T_0)$ and (T', r') is isomorphic to one of the 23 rooted trees mentioned in the statement of the lemma.

If G is a graph, then we denote by $\mathscr{H}(G)$ the set of hamiltonian cycles of G.

Lemma 2. *Let $n \geqq 5$, and let T be one of the trees A_n, A_{n*}, and A_{*n*}. Then there exist $C \in \mathscr{H}(T^3)$ and $C' \in \mathscr{H}(T^5)$ such that $E(C) \cap E(C') = \emptyset$, $w_1w_2 \in E(C)$ and $w_1w_3 \in E(C')$.*

Proof. We determine $E(C)$ and $E(C')$. If $n = 5$, we put

$$E(C) = \{w_1w_2, w_1w_4, w_2w_3, w_3w_5, w_4w_5\} \quad \text{and}$$
$$E(C') = \{w_1w_3, w_1w_5, w_2w_4, w_2w_5, w_3w_4\}.$$

If $n = 6$, we put

$$E(C) = \{w_1w_2, w_1w_4, w_2w_3, w_3w_5, w_4w_6, w_5w_6\} \quad \text{and}$$
$$E(C') = \{w_1w_3, w_1w_6, w_2w_5, w_2w_6, w_3w_4, w_4w_5\}.$$

Let $n \geqq 7$. Then we put

$$E(C) = \{w_1w_2, w_1w_4, w_2w_5, w_3w_4, w_3w_6, w_{n-1}w_n\} \cup \{w_iw_{i+2};\ 5 \leqq i \leqq n-2\}.$$

If $n = 7$, we put

$$E(C') = \{w_1w_3, w_1w_6, w_2w_3, w_2w_7, w_4w_5, w_4w_7, w_5w_6\}.$$

If $n = 8$, we put

$$E(C') = \{w_1w_3, w_1w_6, w_2w_3, w_2w_7, w_4w_7, w_4w_8, w_5w_6, w_5w_8\}.$$

If $n \geqq 9$, we put

$$E(C') = \{w_1w_3, w_1w_6, w_2w_3, w_2w_7, w_4w_5, w_{n-5}w_n, w_{n-4}w_{n-1},$$
$$w_{n-3}w_n, w_{n-2}w_{n-1}\} \cup \{w_iw_{i+4};\ 4 \leqq i \leqq 6\}.$$

It is clear that C and C' have the desired properties.

Lemma 3. *Let T be a tree of order $p \geqq 5$. Then there exist $C \in \mathscr{H}(T^3)$ and $C' \in \mathscr{H}(T^5)$ such that $E(C) \cap E(C') = \emptyset$.*

Proof. The case when $p = 5$ follows immediately from Lemma 2. Let $p \geqq 6$.

Assume that for every tree T_0 of order p_0, where $5 \leqq p_0 < p$, it is proved that there exist $C_{(0)} \in \mathscr{H}(T_0^3)$ and $C'_{(0)} \in \mathscr{H}(T_0^5)$ such that $E(C_{(0)}) \cap E(C'_{(0)}) = \emptyset$. If T is isomorphic to one of the trees A_p, A_{p*}, ane A_{*p*}, then the result follows from Lemma 2. We shall assume that T is isomorphic to none of the trees A_p, A_{p*}, and A_{*p*}. Let $\mathscr{D}$ denote the set of the 23 rooted trees mentioend in Lemma 1. Moreover, we denote $\mathscr{D}_1 = \mathscr{D} - \{D_{05}, D_{05*}\}$. We now distinguish two cases and several subcases.

1. Assume that T has a terminal subtree isomorphic to one of the elements of $\mathscr{D}_1$. Consider such a terminal subtree (T_1, r_1) that (T_1, r_1) is isomorphic to one of the elements of $\mathscr{D}_1$ and that for every terminal subtree (T', r') of T which is isomorphic to one of the elements of $\mathscr{D}_1$, $|V(T_1)| \leqq |V(T')|$. For the sake of simplicity we shall assume that $(T_1, r_1) \in \mathscr{D}_1$. Then $r_1 = u_0$ and there exist $m \geqq 2$ and $n \geqq 1$ such that $V(T_1) = \{u_m, \ldots, u_0, w_1, \ldots, w_n\}$. Denote $S = T - w_1 - \ldots - w_n$. It is clear that $5 \leqq m + 3 \leqq |V(S)| < p$. It follows from the induction hypothesis that there exist $F \in \mathscr{H}(S^3)$ and $F' \in \mathscr{H}(S^5)$ such that $E(F) \cap E(F') = \emptyset$.

The following convention will be useful for us: $1^\#$ means 2 and $2^\#$ means 1.

1.1. Assume that $(T_1, u_0) \in \{D_{*21}, D_{21}, D_{22}, D_{*32}\}$. Then $n \leqq 2$. There exist $v_1, v_2, v'_1, v'_2 \in V(S)$ such that $u_1v_1, u_1v_2 \in E(F)$ and $u_2v'_1, u_2v'_2 \in E(F')$. Since $E(F) \cap \cap E(F') = \emptyset$.

$$(1) \qquad |\{u_1, u_2\} \cap \{v_1, v_2, v'_1, v'_2\}| \leqq 1\,.$$

1.1.1. Assume that $n = 1$. We define

$$C^{(i)} = F - u_1v_i + u_1w_1 + v_iw_1 \quad \text{for} \quad i = 1, 2\,;$$

obviously, $C^{(1)}, C^{(2)} \in \mathscr{H}(T^3)$. Similarly, we define

$$C'^{(j)} = F' - u_2v'_j + u_2w_1 + v'_jw_1 \quad \text{for} \quad j = 1, 2\,;$$

obviously, $C'^{(1)}, C'^{(2)} \in \mathscr{H}(T^5)$. Let $i, j \in \{1, 2\}$; if $E(C^{(i)}) \cap E(C'^{(j)}) \neq \emptyset$, then

$$u_1 = v'_j \quad \text{or} \quad u_2 = v_i \quad \text{or} \quad v_i = v'_j\,.$$

It follows from (1) that there exist $g, h \in \{1, 2\}$ such that $E(C^{(g)}) \cap E(C'^{(h)}) = \emptyset$.

1.1.2. Assume that $n = 2$. We define

$$C^{(i)} = F - u_1v_i + u_1w_2 + v_iw_1 + w_1w_2 \quad \text{for} \quad i = 1, 2\,;$$

obviously, $C^{(1)}, C^{(2)} \in \mathscr{H}(T^3)$. Similarly, we define

$$C'^{(j)} = F' - u_2v'_j - u_2v'_{j\#} + u_2w_1 + u_2w_2 + v'_jw_1 + v'_{j\#}w_2 \quad \text{for} \quad j = 1, 2\,;$$

obviously, $C'^{(1)}, C'^{(2)} \in \mathscr{H}(T^5)$. Let $i, j \in \{1, 2\}$; if $E(C^{(i)}) \cap E(C'^{(j)}) \neq \emptyset$, then

$$u_1 = v'_{j\#} \quad \text{or} \quad u_2 = v_i \quad \text{or} \quad v_i = v'_j\,.$$

It follows from (1) that there exist $g, h \in \{1, 2\}$ such that $E(C^{(g)}) \cap E(C'^{(h)}) = \emptyset$.

1.2. Assume that $(T_1, u_0) \notin \{D_{*21}, D_{21}, D_{22}, D_{*32}\}$. Then $m \geqq 3$. Since $\deg_S u_0 \geqq 2$, there exist $u^* \in \{u_1, \ldots, u_m\}$ and $u \in V(S) - \{u_0, \ldots, u_m\}$ such that $uu^* \in E(F)$. Since $F \in \mathscr{H}(S^3)$, there exists $g \in \{1, 2\}$ such that $u_g = u^*$. Thus, $uu_g \in E(F)$. Denote $h = g^*$.

1.2.1. Let $n = 1$. Then $(T_1, u_0) \in \{D_{*31}, D_{31}, D_{*41}, D_{41}\}$. Since $|V(S)| \geqq 5$, there exist $i \in \{h, 3\}$ and $v \in V(S) - \{u_i\}$ such that $v \notin \{u, u_g\}$ and $u_i v \in E(F')$. Define

$$C = F - uu_g + uw_1 + u_g w_1 \quad \text{and} \quad C' = F' - u_i v + u_i w_1 + v w_1 .$$

Obviously, $C \in \mathscr{H}(T^3)$, $C' \in \mathscr{H}(T^5)$ and $E(C) \cap E(C') = \emptyset$.

1.2.2. Let $n = 2$. Then $(T_1, u_0) \in \{D_{32}, D_{*42}, D_{42}\}$. Since $|V(S)| \geqq m + 3$, there exist distinct $v_1, v_2, v_3 \in V(S)$ such that $u_1 v_1, u_2 v_2, u_3 v_3$ are distinct edges of F' and if $m = 4$, then $u_4 \notin \{v_1, v_2, v_3\}$. Since $E(F) \cap E(F') = \emptyset$, $v_g \neq u$. Define

$$C = F - uu_g + uw_g + u_g w_h + w_1 w_2 ;$$

obviously, $C \in \mathscr{H}(T^3)$. First, let $v_3 \neq u_g$; define

$$C' = F' - u_g v_g - u_3 v_3 + u_g w_g + u_3 w_h + v_g w_g + v_3 w_h ;$$

we have that $C' \in \mathscr{H}(T^5)$ and $E(C) \cap E(C') = \emptyset$. Let now $v_3 = u_g$; then $v_h \neq u_g$; define

$$C'' = F' - u_h v_h - u_3 v_3 + u_h w_h + u_3 w_g + v_h w_h + v_3 w_g ;$$

we have that $C'' \in \mathscr{H}(T^5)$ and $E(C) \cap E(C'') = \emptyset$.

1.2.3. Let $n = 3$. Then $(T_1, u_0) \in \{D_{*33*}, D_{33*}, D_{33}\}$. Define

$$C = F - uu_g + uw_g + u_g w_h + w_1 w_3 + w_2 w_3 ;$$

obviously, $C \in \mathscr{H}(T^3)$. There exist distinct $v_2, v_3 \in V(S) - \{u_2, u_3\}$ such that $u_2 v_2, u_3 v_3 \in E(F')$ and $F - u_2 v_2 - u_3 v_3 + u_2 v_3 + u_3 v_2$ is also a cycle. First, let $v_2 w_1 \notin E(C)$; define

$$C' = F' - u_2 v_2 - u_3 v_3 + u_2 w_3 + u_3 w_2 + v_2 w_1 + v_3 w_3 + w_1 w_2 ;$$

we have that $C' \in \mathscr{H}(T^5)$ and $E(C) \cap E(C') = \emptyset$. Let now $v_2 w_1 \in E(C)$; then $d_T(u_0, v_2) \leqq 2$ and $v_3 w_1 \notin E(C)$; define

$$C'' = F' - u_2 v_2 - u_3 v_3 + u_2 w_3 + u_3 w_2 + v_2 w_3 + v_3 w_1 + w_1 w_2 ;$$

we have that $C'' \in \mathscr{H}(T^5)$ and $E(C) \cap E(C'') = \emptyset$.

1.2.4. Let $n = 4$. Then $(T_1, u_0) \in \{D_{*34*}, D_{*34}, D_{34*}, D_{34}, D_{*44*}, D_{44*}, D_{44}\}$. Define

$$C = F - uu_g + uw_g + u_g w_h + w_1 w_3 + w_2 w_4 + w_3 w_4 ;$$

obviously, $C \in \mathscr{H}(T^3)$. First we assume that $u_1 u_2 \in E(F')$. Define

$$C' = F' - u_1 u_2 + u_1 w_4 + u_2 w_3 + w_1 w_2 + w_1 w_4 + w_2 w_3 .$$

Clearly, $C' \in \mathscr{H}(T^5)$ and $E(C) \cap E(C') = \emptyset$.

Now we assume that $u_1 u_2 \notin E(F')$. There exist $v_1, v_2 \in V(S) - \{u_1, u_2\}$ with the properties that $u_1 v_1, u_2 v_2$ are distinct edges of F', $v_1 w_1 \notin E(C)$, and $v_2 \neq u_m$. Let $v_2 w_2 \in E(C)$; if $v_2 = u$, then $g = 2$ and $u_2 v_2 = u_g u$, and therefore, $E(F) \cap E(F') \neq \emptyset$, which is a contradiction; if $v_2 = u_g$, then $g = 1$, and thus $u_1 u_2 = v_2 u_2 \in E(F')$, which is a contradiction. Hence, $v_2 w_2 \notin E(C)$. Define

$$C'' = F' - u_1 v_1 - u_2 v_2 + u_1 w_4 + u_2 w_3 + v_1 w_1 + v_2 w_2 + w_1 w_4 + w_2 w_3 ;$$

we have that $C'' \in \mathscr{H}(T^5)$ and $E(C) \cap E(C'') = \emptyset$.

2. Assume that T contains no terminal subtree isomorphic to an element of $\mathscr{D}_1$. It follows from Lemma 1 that there exist $n \geqq 5$ and a terminal subtree (T_2, r_2) of T with the properties that (T_2, r_2) is isomorphic either to D_{0n*} or to D_{0n} and $\deg_T r_2 \geqq 3$. For the sake of simplicity we shall assume that $(T_2, r_2) = D_{0n*}$ or D_{0n}; thus $r_2 = u_0$ and $V(T_2 - u_0) = \{w_1, \ldots, w_n\}$. As follows from Lemma 2, there exist $J \in \mathscr{H}((T_2 - u_0)^3)$ and $J' \in \mathscr{H}((T_0 - u_0)^5)$ with the properties that $E(J) \cap E(J') = \emptyset$, $w_1 w_2 \in E(J)$ and $w_1 w_3 \in E(J')$. Denote $S = T - w_1 - \ldots - w_n$. Since T is isomorphic to none of the trees A_p, A_{p*}, and A_{*p*}, $|V(S)| > 4$. Since $|V(S)| < p$, it follows from the induction hypothesis that there exist $F \in \mathscr{H}(S^3)$ and $F' \in \mathscr{H}(S^5)$ such that $E(F) \cap E(F') = \emptyset$. Since $\deg_S u_0 \geqq 2$, there exist $v_1, v_2, v_1', v_2' \in V(S) - \{u_0\}$ such that $v_1 v_2 \in E(F)$, $v_1' v_2' \in E(F')$,

$$d_S(u_0, v_1) + d_S(u_0, v_2) \leqq 3 \quad \text{and} \quad d_S(u_0, v_1') + d_S(u_0, v_2') \leqq 5 .$$

We shall find $e_1, e_2 \in E(T^3)$ and $e_1', e_2' \in E(T^5)$ such that

$$C = ((F - v_1 v_2) \cup (J - w_1 w_2)) + e_1 + e_2 \in \mathscr{H}(T^3),$$
$$C' = (F' - v_1' v_2') \cup (J' - w_1 w_3)) + e_1' + e_2' \in \mathscr{H}(T^5),$$

and $E(C) \cap E(C') = \emptyset$.

2.1. Assume that $\{v_1, v_2\} \cap \{v_1', v_2'\} = \emptyset$. Without loss of generality we assume that $d_T(u_0, v_1) \leqq d_T(u_0, v_2)$, and $d_T(u_0, v_1') \leqq d_T(u_0, v_2')$. We put $e_1 = v_1 w_2$, $e_2 = v_2 w_1$, $e_1' = v_1' w_3$ and $e_2' = v_2' w_1$.

2.2. Assume that $\{v_1, v_2\} \cap \{v_1', v_2'\} \neq \emptyset$. Since $E(F) \cap E(F') = \emptyset$, $|\{v_1, v_2\} \cap \{v_1', v_2'\}| = 1$. Without loss of generality we assume that $v_2' = v_2$. If $u_0 v_2 \in E(T)$, then we put $e_1 = v_1w_1$, $e_2 = v_2w_2$, $e'_1 = v'_1w_1$ and $e'_2 = v_2w_3$. If $u_0v_2 \notin E(T)$, then $d_T(u_0, v_2) = 2$, and we put $e_1 = v_1w_2$, $e_2 = v_2w_1$, $e'_1 = v'_1w_1$ and $e'_2 = v_2w_3$.

Thus the proof of Lemma 3 is complete.

Theorem 1 immediately follows from Lemma 3.

As follows from Theorem B, if G is a connected graph of even order $p \geqq 4$, then G^3 has two edge-disjoint 1-factors, which is an analogue to our Corollary. It is natural to ask whether there exists a similar analogue to Theorem 1. The following proposition gives a negative answer.

Proposition. *There exists an infinite set of mutually nonisomorphic trees T such that for every 1-factor F of T^2 and every 1-factor F' of T^3, $E(F) \cap E(F') \neq \emptyset$.*

Proof. Let $n \geqq 5$ be an odd integer, let $v, v_{11}, v_{12}, v_{13}, \ldots, v_{n1}, v_{n2}, v_{n3}$ be distinct vertices, and let T be a tree defined as follows:

$$V(T) = \{v, v_{11}, v_{12}, v_{13}, \ldots, v_{n1}, v_{n2}, v_{n3}\} \quad \text{and}$$

$$E(T) = \{vv_{11}, v_{11}v_{12}, v_{12}v_{13}, \ldots, vv_{n1}, v_{n1}v_{n2}, v_{n2}v_{n3}\}.$$

Assume that there exist a 1-factor F of T^2 and a 1-factor F' of T^3 such that $E(F) \cap \cap E(F') = \emptyset$. Without loss of generality we may assume that

$$(2) \qquad vv_{k1}, vv_{k2} \notin E(F) \quad \text{and} \quad vv_{k1}, vv_{k2}, vv_{k3} \notin E(F') \quad \text{for every} \quad k \in \{1, \ldots, n-2\}.$$

Since F is a 1-factor of T^2, it follows from (2) that

$$(3) \qquad v_{k2}v_{k3} \in E(F), \quad \text{for every} \quad k \in \{1, \ldots, n-2\}.$$

Since F' is a 1-factor of T^3 and $E(F) \cap E(F') = \emptyset$, it follows from (2) and (3) that

$$(4) \qquad v_{k1}v_{k3} \in E(F'), \quad \text{for every} \quad k \in \{1, \ldots, n-2\}.$$

Since $n - 2 \geqq 3$, it follows from (2) and (4) that F' is not a 1-factor in T^3, which is a contradiction. Thus, the proposition is proved.

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