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REMARK ON A THEOREM OF K. M. SLIPENČUK IN THE THEORY OF SUMMABILITY OF SERIES

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In the paper [1] K. M. SLIPENČUK proved the following Tauberian theorem:

Let $T = (a_{nk})$ be a regular matrix which fulfils the following condition:

(*) *there exist $M_1, M_2 > 0$ such that for each $n = 1, 2, \ldots$ we have*

$$\sum_{k=1}^{n} |a_{nk} - 1| < M_1, \quad \sum_{k=n+1}^{\infty} |a_{nk}| < M_2 .$$

If

$$u_n = o(1) \quad (n \to \infty), \tag{1}$$

then from the T-summability of the series $\sum_{n=1}^{\infty} u_n$ to S the convergence of this series follows and $\sum_{n=1}^{\infty} u_n = S$.

Let us remark that it is not clear from the print of the paper [1] whether small o or capital O appears in the condition (1). But it is obvious from the proof of the theorem that the small o should be in that condition.

In the review of the paper of K. M. Slipenčuk in Math. Rev. (cf. [2]) the mentioned theorem is stated with the condition $u_n = O(1)$ instead of $u_n = o(1)$. The last formulation of the mentioned theorem is false as it can be easily deduced from the following result.

Theorem. *Let $K = \sum_{k=1}^{\infty} |b_k| < +\infty$, $\sum_{k=1}^{\infty} (-1)^k b_k = -\frac{1}{2}$* [1]*. Let us put $a_{nk} = 1$ for $k = 1, 2, \ldots, n$ and $a_{nn+s} = b_s$ $(s = 1, 2, 3, \ldots)$ for each $n = 1, 2, \ldots$ Then the matrix $T = (a_{nk})$ is regular, fulfils the condition (*) and the series $\sum_{k=1}^{\infty} (-1)^k$ is T-summable to $-\frac{1}{2}$.*

[1]) We can choose $b_k = (-1)^{k+1} (1/2^{k+1})$ $(k = 1, 2, \ldots)$.

Proof. Obviously $\lim_{n\to\infty} a_{nk} = 1$ for each fixed k. Further for each $n = 1, 2, 3, \ldots$ we have $\sum_{k=1}^{\infty} |a_{nk} - a_{nk+1}| = |1 - b_1| + |b_1 - b_2| + \ldots \leqq 1 + 2K < +\infty$. Therefore T is a regular matrix (cf. [3] p. 83–84).

From the definition of T we get for each $n = 1, 2, \ldots \sum_{k=1}^{n} |a_{nk} - 1| = 0$, $\sum_{k=n+1}^{\infty} |a_{nk}| = \sum_{i=1}^{\infty} |b_i| < +\infty$, so T fulfils the condition (*).

Further for each even n we have

$$\sigma_n = \sum_{k=1}^{\infty} a_{nk}(-1)^k = (-a_{n1} + a_{n2}) + \ldots + (-a_{nn-1} + a_{nn}) - \\ - b_1 + b_2 - b_3 + \ldots = -\tfrac{1}{2},$$

while for the odd n's we have

$$\sigma_n = \sum_{k=1}^{\infty} a_{nk}(-1)^k = (-a_{n1} + a_{n2}) + \ldots + (-a_{nn-2} + a_{nn-1}) - \\ - a_{nn} + b_1 - b_2 + b_3 - \ldots = -1 + \tfrac{1}{2} = -\tfrac{1}{2}.$$

Then $\sigma_n = -\frac{1}{2}$ $(n = 1, 2, \ldots)$ so that the series $\sum_{k=1}^{\infty} (-1)^k$ is T-summable to $-\frac{1}{2}$.

References

[1] *К. М. Сліпенчук:* Про одну теорему Тауберового типу для підсумовування рядів. Доповіді АН УССР (1966), 32—35.

[2] Math. Rev. vol. *33*, No 4 (1967), No of review 4528.

[3] *R. G. Cooke:* Infinite matrices and sequence spaces (Russian translation), Moscow, 1960.

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