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ON THE G-STRUCTURE OF HIGHER ORDER

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To the G-structure of order r, defined in [2], [5], there is found an associated M-valued tensor and a canonical representation of a subgroup G of the group L_n^r (group of all invertible holonomic r-jets of R^n into R^n, with source and target 0) in a vector space M.

1. We give here the fundamental definitions of a fibre bundle and principal fibre bundle from the standpoint used throughout this paper.

Definition. A space $E(B, F, G, p, H)$ is called a *differentiable bundle*, if

a) E, B, F are differentiable manifolds. The space B is called the base and F is called the fibre. The Lie group G is a left effective transformation group on a manifold F, so that a mapping $\eta : G \times F \to F$ has the following properties:

(i) η is a differentiable mapping,
(ii) $\eta(e, y) = y$, e the unit of G,
(iii) $\eta(g_2 g_1, y) = \eta(g_2, \eta(g_1 y))$; $g_1, g_2 \in G$, $y \in F$. The group G is called the structural group.

b) There is an equivalence relation R defined on E such that $B = E/R$. The natural projection $p : E \to B$ is a differentiable mapping. Each space $F_x = p^{-1}(x)$ is called a fiber over $x \in B$.

c) For an arbitrary neighborhood U on B there exists a mapping $\Phi_U : U \times F \to p^{-1}(U)$ (a differentiable homeomorphism) such that $(p \circ \Phi_U)(x, y) = x$. If U, V are neighborhoods on B with $U \cap V \neq \emptyset$ then $\Phi_V^{-1} \Phi_U \in G$. For an arbitrary point $x \in B$, let Φ_U^x be a mapping $\Phi_U : \{x\} \times F \to p^{-1}(x)$, and H_x the set of all mappings Φ_U^x with fixed $x \in B$; $H = \bigcup_{x \in B} H_x$.

Definition. A space $H(B, G, G, p, H)$ is called a principal differentiable fibre bundle if H is a fibre bundle with a fibre G. The group G is then a group of transformations onto itself.

Definition. Let $E(B, F, G, p, H)$ be a fibre budle. A frame of E at a point $x \in B$ is a differentiable homeomorphism of the fibre onto; i.e., an element $h = \Phi_U^x \circ g$, where Φ_U^x is a differentiable homeomorphism of $\{x\} \times F$ onto $p^{-1}(x)$ and $g \in G$.

The definition of a frame is evidently independent of the choice of the neighborhood U of $x \in B$. Let $\hat{E}$ be the set of all frames of E at points $x \in B$. Define now the differentiable homeomorphisms

$$\Phi_U : U \times G \to \hat{E}\,, \quad \Phi_U(x, g) = \Phi_U^x \circ g\,, \quad x \in B\,, \quad g \in G\,.$$

It is evident that $\hat{E}$ is a principal differentiable fibre bundle. We shall speak about the associated principal fibre bundle.

2. Suppose V_n and V_m are two differentiable manifolds of the dimension n and m respectively. Let f be a C^∞ mapping of a neighborhood of a point $x \in V_n$ into V_m. Let $C_x^r(V_n, V_m)$ be the set of points (f, x), f being a C^∞ mapping of a neighborhood of the point $x \in V_n$ into V_m. Two points (f, x) and (g, x) are said to be r-equivalent if the functions f_i and g_i determining the mappings f and g in the coordinates, have equal partial derivatives of order $s(1 \leqq s \leqq r)$ at $x \in V_n$. The set of all these r-equivalence classes of the elements $C_x^r(V_n, V_m)$ will be denoted by $J_x^r(V_n, V_m)$. The class $j_x^r f$ determined by an element $(f, x) \in C_x^r$ is called an r-jet. Set $J^r(V_n, V_m) = \bigcup_{x \in V_n} J_x^r(V_n, V_m)$. Let $H^r(V_n)$ be the set of all invertible r-jets of R^n into V_n with source $0 \in R^n$. The set of r-frames $H^r(V_n)$ of the manifold V_n is a fibre bundle, and is called a principal prolongation of order r of the manifold V_n. The structural group of $H^r(V_n)$ is a group L_n^r (group of all invertible r-jets of R^n into R^n with source and target $0 \in R^n$). A fibre bundle associated [9] with the principal bundle $H^r(V_n)$ is said to be a prolongation of order r of the manifold V_n.

For each point $x_0 \in V_n$ let $\mathscr{A}^+(x_0)$ be the system of C^∞ functions whose domain is an open subset of V_n containing x_0. Let $\mathscr{A}^C(x_0)$ be the system of all functions of $\mathscr{A}^+(x_0)$ which are constant on some neighborhood of x_0. Finally, let $\mathscr{A}(x_0)$ be the subsystem of $\mathscr{A}^+(x_0)$ consisting of functions which vanish at x_0. It is evident that every function $f^+ \in \mathscr{A}^+(x_0)$ can be expressed uniquely in the form $f^+ = f^C + f$, $f^C \in \mathscr{A}^C(x_0)$, $f \in \mathscr{A}(x_0)$. Let $\mathscr{A}^{r+1}(x_0)$ be the system of all sums of products of $r + 1$ elements from $\mathscr{A}(x_0)$.

Definition. A tangent vector of order r at a point x_0 of the manifold V_n is a linear mapping $X : \mathscr{A}^+(x_0) \to R$ which vanishes on $\mathscr{A}^C(x_0)$ and on $\mathscr{A}^{r+1}(x_0)$.

Let $(x^1, \ldots, x^n)$ be a coordinate system on V_n at a neighborhood of a point $x_0 = (x_0^1, \ldots, x_0^n)$. Then each tangent vector of order rX at x_0 can be written in the form

$$X = \sum_{j=1}^{r} \sum_{\substack{k_1 + \ldots + k_n = j \\ k_1 \geqq 0, \ldots, k_n \geqq 0}} \frac{1}{k_1! \ldots k_n!} X\{(x^1 - x_0^1)^{k_1} \ldots (x^n - x_0^n)^{k_n}\} \frac{\partial^j}{(\partial x^1)^{k_1} \ldots (\partial x^n)^{k_n}}\,.$$

Let $T_x^r = T_x^r(V_n)$ be the system of all tangent vectors of order r at a point x of the manifold V_n. Set $T_0^r(R^n) = F^r$. If $(t^1, \ldots, t^n)$ is a coordinate system at $0 \in R^n$ then evidently

$$\frac{1}{k_1! \ldots k_n!} \left(\frac{\partial^j}{(\partial t^1)^{k_1} \ldots (\partial t^n)^{k_n}} \right)_0$$

are linearly independent vectors. The points

$$\partial_{\alpha_1\ldots\alpha_k} = \frac{1}{k!}\left(\frac{\partial^k}{\partial t^{\alpha_1}\ldots\partial t^{\alpha_k}}\right)_0 ; \quad k = 1, 2, \ldots, r ,$$

are not linearly independent, but it is possible to choose from them a basis of F^r. We now wish to obtain the coordinate expression for the transformation of the vectors $\partial_{\alpha_1\ldots\alpha_k}$ if the coordinate are transformed as follows: $t^{\alpha'} = h^{\alpha'}(t^1, \ldots, t^n)$, $0 = h^{\alpha'}(0, \ldots, 0)$. There results the following transformation

$$(2.1) \qquad \partial_{\alpha_1\ldots\alpha_i} = \sum_{k=1}^{i} \partial_{\alpha_1'\ldots\alpha_k'} \sum_{j_1+\ldots+j_k=i} h^{\alpha_1'}_{\alpha_1\ldots\alpha_{j_1}} \cdots h^{\alpha'_k}_{\alpha_{j_1}+\ldots+j_{k-1}+1\ldots,\alpha_i};$$

$$i = 1, 2, \ldots, r ,$$

where

$$h^{\alpha'}_{\beta_1\ldots\beta_s} = \frac{1}{s!}\left(\frac{\partial^s h^{\alpha'}}{\partial t^{\beta_1} \ldots \partial t^{\beta_s}}\right)_0 .$$

It may be verified that $\{(\partial^s h^{\alpha'}/\partial t^{\beta_1} \ldots \partial t^{\beta_s})_0\}$ is an element of the left transformation effective group L^r_n on F^r. Then the following proposition holds.

Proposition. *An* $E^r = \bigcup_{x\in V_n} T^r_x$ *has the structure of a fibre bundle with the basis* V_n, *structural group* L^r_n *and fibre* F^r. *The space* $H^r(V_n)$ *is a principal bundle associated with* E^r.

Proof. Let $(t^1, \ldots, t^n)$ be a coordinate system in the neighborhood V of the point 0 on R^n and let $(x^1, \ldots, x^n)$ be a coordinate system in a neighborhood U of x_0 on V_n. Let $x^\alpha = f^\alpha(t^1, \ldots, t^n)$; $\alpha = 1, 2, \ldots, n$; $x^\alpha_0 = f^\alpha(0)$ be a mapping f of V into U. We have then

$$(2.2) \qquad \partial_{\alpha_1\ldots\alpha_i} = \sum_{k=1}^{i} \partial_{\beta_1\ldots\beta_k} \sum_{j_1+\ldots+j_k=i} f^{\beta_1}_{\alpha_1\ldots\alpha_{j_1}} \cdots f^{\beta_k}_{\ldots\alpha_i} ;$$

$$i = 1, 2, \ldots, r ; \quad f^{\beta}_{\alpha_1\ldots\alpha_s} = \frac{1}{s!}\left(\frac{\partial^s f^\beta}{\partial t^{\alpha_1} \ldots \partial t^{\alpha_s}}\right)_0 .$$

But $z = \{(\partial^s f^\beta/\partial t^{\alpha_1} \ldots \partial t^{\alpha_s})_0\}$; $1 \leqq s \leqq r$; $\alpha, \beta = 1, 2, \ldots, n$ is an element of $H^r(V_n)$ over a point $x_0 = (x^1_0, \ldots, x^0_j)$. From (2.2) follows that z is a mapping $z : F^r \to \to E^r$, $z : \eta \to z\eta$ and that $(za)\,\eta = z(a\eta) = za\eta \in E^r$, $a \in L^r_n$.

Each tangent vector space F^s of order s; $1 \leqq s \leqq r$, is a subspace of F^{s+1} and is invariant under the transformations of L^r_n on F^r. But each point of F^s is not invariant under this transformation. Let N_s be a subgroup of L^r_n leaving each point of F^s fixed. Then we may identify $G^s = L^r_n/N_s$ with L^s_n. Let H^r/N_s be the coset space by the subgroup N_s. We now consider two fibre spaces $E^{r,1}(V_n, F^1, L^1_n, H^r/N_1)$ and $E^1(V_n, F^1, L^1_n, H^1)$.

Proposition. *The fibre bundles $E^{r,1}$ and E^1 are equivalent.*

Proof. The associated principal bundles H^1 and H^r/N_1 are equivalent by [4]. Then the bundles $E^{r,1}$ and E^1 are equivalent.

We shall now define an s_r-form on the manifold as an element of the dual space to $\overset{s}{\bigwedge} T^r_x$.

Definition. A differential s-form ω of order r on a manifold V_n (abbreviated to s_r-form) is, for each $x \in V_n$, a linear mapping of a vector space $\overset{s}{\bigwedge} T^r_x$ into R such that

a) $\omega(\underset{a(1)}{X} \wedge \ldots \wedge \underset{a(s)}{X}) = \operatorname{sgn} a\, \omega(\underset{1}{X} \wedge \ldots \wedge \underset{s}{X}),\ \underset{1}{X}, \underset{2}{X}, \ldots, \underset{s}{X} \in T^r_x$;

b) $\omega(a^{i_1}_1 \underset{i_1}{X} \wedge \ldots \wedge a^{i_s}_s \underset{i_s}{X}) = a^{i_1}_1 \ldots a^{i_s}_s \omega(\underset{i_1}{X} \wedge \ldots \wedge \underset{i_s}{X})$;

c) ω depends differentiably on $x \in V_n$.

An 0_r-form is a differentiable function on V_n. It is clear that in natural manner one may define the sum of s_r-forms and the product $f\omega$ with a differentiable function f on V_n. The exterior product $\omega_1 \wedge \omega_2$ of a u_r-form ω_1 and a v_r-form ω_2 is a $(u+v)_r$-form defined by the formula

$$\omega_1 \wedge \omega_2(\underset{1}{X} \wedge \ldots \wedge \underset{u}{X} \wedge \underset{u+1}{X} \wedge \ldots \wedge \underset{u+v}{X}) =$$

$$= \sum_a \frac{\operatorname{sgn} a}{(u+v)!}\, \omega_1(\underset{a(1)}{X} \wedge \ldots \wedge \underset{a(u)}{X})\, \omega_2(\underset{a(u+1)}{X} \wedge \ldots \wedge \underset{a(u+v)}{X}).$$

Let $(x^1, \ldots, x^n)$ be the coordinates of a point x in the neighborhood U on V_n. We now have linearly independent vectors $X^{(j)}_{k_1 \ldots k_n} = \partial^j/(\partial x^1)^{k_1} \ldots (\partial x^n)^{k_n}$ at x. Denote by $X^{(j)}_{k_1 \ldots k_n}$ the vector field which assigns to each point x the vector $X^{(j)}_{k_1 \ldots k_n}$. Then define linear operators $a^{h_1 \ldots h_n}_{(j)}$, $h_1 + \ldots + h_n = i$, by

$$a^{h_1 \ldots h_n}_{(i)} X^{(j)}_{k_1 \ldots k_n} = \delta^{h_1 \ldots h_n}_{k_1 \ldots k_n}\,; \quad 1 \leqq i, j \leqq r\,; \quad k_1 + \ldots + k_n = j\,.$$

The 1_r-form ω can then be written in U in the form $\omega = \sum_{j=1}^{r} \Phi^{(j)}_{h_1 \ldots h_n} a^{h_1 \ldots h_n}_{(j)}$, $\Phi^{(j)}_{h_1 \ldots h_n}$ being the functions on U, $\omega(X^{(j)}_{h_1 \ldots h_n}) = \Phi^{(j)}_{h_1 \ldots h_n}$. If M is a vector space, define an M-valued s_r-form to be a linear mapping of $\overset{s}{\bigwedge} T^r_x$ into M such that the above mentioned conditions are satisfied. It is clear that the operations defined for the s_r-forms may also be defined for the M-valued s_r-forms.

Note. s_1-forms are called s-forms. For such forms the exterior differential is defined.

3. At this point we wish to consider the tensor associated with the M-valued s_1-form ω defined on $H(V_n, G)$. We know that $H^r(V_n)$ is a set of isomorphisms of

$T_0^r(R^n)$ onto $T_x^r(V_n)$ for each $x \in V_n$. If we consider vectors of the first order only, we see that $H^r(V_n)$ is a set of isomorphisms of $T_0^1(R^n)$ onto $T_x^1(V_n)$. We know that $H^1(V_n)$ is a set of isomorphisms of $T_0^1(R^n)$ onto $T_x^1(V_n)$, we have then an equivalence relation on $H^r(V_n)$. The coset space H^r/N_1 is then equivalent with H^1. We shall identify H^1 with H^r/N_1. One can then define a fundamental 1-form on $H^r(V_n)$ [2]. It is not difficult to prove the

Theorem. *Let G be a subgroup of L_n^r. Let $H(V_n, G)$ be a principal fibre bundle, a subbundle of $H^r(V_n)$, and let ω be a fundamental Λ-form on H. The M-valued s-forms Λ on H of type $\mathscr{S}(G)$ are one-to-one correspondence with the tensors $t\Lambda$ on H with values in $M \otimes \overset{s}{\bigwedge} R^{n*}$ of type $\varrho(G)$, where $\varrho(g) = \mathscr{S}(g) \otimes \overset{s}{\bigwedge} \mathscr{R}(g^{-1})$. The tensor associated to the form Λ is defined by $\Lambda = (t\Lambda)(\overset{s}{\bigwedge}\omega)$. $\mathscr{S}$ is a representation of G on the vector space M, and $\mathscr{R}$ is a representation of L_n^r on the vector space R^n.*

Let γ be a canonical projection of L_n^r onto $L_n^r/N_1 = L_n$, and $\mathfrak{B}$ a canonical representation of L_n on R^n; then $\mathscr{R} = \mathfrak{B} \circ \gamma$ is a canonical representation of L_n^r on R^n. Let $\mathscr{R}$ be a canonical representation of the Lie algebra $\mathbf{L}_n^r$ of L_n^r on $\mathscr{L}(R^n)$ given by the reprecentation $\mathscr{R}$.

A special affine connection of order r on a manifold V_n is an infinitesimal connection on the principal fibre bundle $H^r(V_n)$ [6]. Suppose π to be an $\mathbf{L}_n^r$-valued Λ-form of the connection on $H^r(V_n)$. Let ω be a fundamental Λ-form on H^r, i.e. an R^n-valued 1-form ω defined by the formula $\omega(\tau_z) = z^{-1} \,.\, p\tau_z \in R^n$, τ_z being the tangent vector to H^r at a point $z \in H^r$. The 1-form ω is a tensorial form.

Note. Let M and P be two vector spaces. Let Φ (or φ) be an $\mathscr{L}(M, P) = P \otimes M^*$ (or M)-valued vector form on V_n. The P-valued form $\Phi \,.\, \varphi$ is defined by the formula $\Phi \,.\, \varphi = \sum_{\alpha, A} \Phi^\alpha \wedge \varphi^A \otimes f_\alpha(e_A)$, $\Phi = \Phi^\alpha \otimes f_\alpha$, $\varphi = \varphi^A \otimes e_A$.

The torsion form of the special affine connection of order r is a 2-form $\Sigma = \nabla\omega$. On the basis of the note mentioned above we can write Σ in the form $\Sigma = d\omega + \tilde{\mathscr{R}}(\pi)\,\omega$.

4. In this part we shall study in detail the subspace of $H^r(V_n)$.

Defiinition. Let G be a subgroup of L_n^r. A G-structure of the order r is the set $H(V_n, G)$ of all the r-frames of the manifold V_n.

In the case $r = 1$ we obtain the well known G-structure [2]. We shall prove that the G-structure of order r on V_n gives rise to an invariant tensor on a principal fibre bundle H with values in certain vector space, and that a canonical representation of L_n^r on this vector space can be defined.

Let π be a form of the infinitesimal connection on a principal fibre bundle H. Because $\tilde{\mathscr{R}}$ is a representation of the Lie algebra $\mathbf{L}_n^r$ on a vector space $\mathscr{L}(R^n)$ we have an $R^n \otimes R^{n*}$ – valued 1-form $\tilde{\mathscr{R}}(\pi)$ on H. Let $\{\varepsilon_\varrho\}$ be a basis of **G** and $\{e_i\}$ a basis of R^n. Then the torsion form Σ can be written as

$$\Sigma = \mathrm{d}\omega + \tilde{\mathscr{R}}(\pi)\,\omega = \mathrm{d}\omega + \left(\pi^\varrho \otimes \tilde{\mathscr{R}}(\varepsilon_\varrho)\right)\left(\omega^i \otimes e_i\right) = \left(\tilde{\mathscr{R}}(\varepsilon_\varrho)\, e_i\right) \otimes \pi^\varrho \wedge \omega^i\,,$$

ω being the fundamental 1-form on H. Then Σ is an R^n-valued 2-form on H. If $\mathscr{S}$ is a representation of the group L_n^r on P, $\mathscr{S}(l) = \mathscr{R}(l) \otimes \overset{2}{\bigwedge}\mathscr{R}(l^{-1})$, $l \in L_n^r$, $t\Sigma$ is a tensor (associated to the form Σ) with values in $P = R^n \otimes \overset{2}{\bigwedge} R^{n*}$ of type $\mathscr{S}(G)$.

Let two connections π', π on H be given. Let Σ', Σ be their torsion forms. The 1-form $u = \pi' - \pi$ is a **G**-valued 1-form on H of type adj. The tensor $tu = \xi$ associated with the form u is defined on H and has values in the vector space $N = \mathbf{G} \otimes R^{n*}$. It is of type $\mathfrak{B}(G)$, where $\mathfrak{B}$ is a representation of L_n^r on $Q = \mathbf{L}_n^r \otimes R^{n*}$, $\mathfrak{B}(l) = \mathrm{adj}\,(l) \otimes \mathscr{R}(l^{-1})$, $l \in L_n^r$. Let us consider the vector space $K = R^n \otimes R^{n*} \otimes R^{n*}$ and a mapping $\mathscr{B}$ of Q into K defined as follows $\mathscr{B} : \mathfrak{g} \otimes \alpha \to \tilde{\mathscr{R}}(\mathfrak{g}) \otimes \alpha$, $\mathbf{L}_n^r \ni \mathfrak{g}$, $\alpha \in R^{n*}$. Further, let $\mathscr{V}$ be a representation of the group L_n^r on K, $\mathscr{V}(l) = \mathrm{adj}\,\mathscr{R}(l) \otimes \mathscr{R}(l^{-1})$, $l \in L_n^r$. It is easy to see that $\mathscr{B} \circ \mathfrak{B}(l) = \mathscr{V}(l) \circ \mathscr{B}$, $l \in L_n^r$.

In chosen local bases of **G** and R^n we can write $u = u^\varrho \otimes \varepsilon_\varrho$, $u^\varrho = (tu^\varrho)_i\, \omega^i = \xi_i^\varrho \omega^i$ and then $u = \xi_i^\varrho \varepsilon_\varrho \otimes \omega^i$. We have further $\mathscr{B}(u) = \tilde{\mathscr{R}}(\varepsilon_\varrho)\, \xi_i^\varrho \otimes \omega^i = a_{k\varrho}^j \xi_i^\varrho e_j \otimes \omega^k \otimes \omega^i$, if $\tilde{\mathscr{R}}(\varepsilon_\varrho) = a_{k\varrho}^j e_j \otimes \omega^k$. $\mathscr{B}(u)$ is an element of the vector space $W = \mathscr{B}(N)$. W is invariant under the transformations of $\mathscr{V}(G)$, but not pointwise. Now let us consider the representation $\mathscr{S}$ of L_n^r on P. If $\{e_i\}$ is the basis for R^n, let $\{\omega^i\}$ be the dual basis of R^{n*}. A mapping $\mathscr{A} : K \to P$ is defined by $\mathscr{A} : \lambda_{jK}^i e_i \otimes \omega^j \otimes \oplus\, \omega^K \to \frac{1}{2}(\lambda_{Kj}^i - \lambda_{jK}^i)\, e_i \otimes \omega^j \wedge \omega^K$ so that $\mathscr{A} \circ \mathscr{V}(l) = \mathscr{S}(l) \circ \mathscr{A}$, $l \in L_n^r$. As W is invariant under the transformations of $\mathscr{V}(G)$, we have $\mathscr{A} \circ \mathscr{V}(g) = \mathscr{S}(g) \circ \mathscr{A}$, $g \in G$. The space $V = \mathscr{A}(W)$ is then invariant under the transformations of $\mathscr{S}(G)$. Then we have the R^n-valued 2-form $\mathscr{A}\mathscr{B}(u)$ of type $\mathscr{R}(G)$. It is an element of the vector space $V = \mathscr{A}(W)$ and the equality $\Sigma' - \Sigma = \mathscr{A}\mathscr{B}(u)$ holds.

Let $M = P/V$ be a vector space and α the canonical projection $P \to P/N$. Let ϱ be a representation of G on M defined by $\varrho(g) \circ \alpha = \alpha \circ \mathscr{S}(g)$, $g \in G$. Now we have the M-valued function $ts = \alpha \circ t\Sigma$ on H. But we know that $\alpha \circ t\Sigma' = \alpha \circ t\Sigma$. The function is then independent on the choice of the infinitesimal connection on H. We have also $t_s(zg) = \varrho(g^{-1})\, t_s(z)$. Then t_s is an M-valued tensor on H of type $\varrho(G)$. All these results are included in the

Theorem. *Let G be a Lie group, a subgroup of L_n^r. The representation ϱ defined by the relation $\varrho(g) \circ \alpha = \alpha \circ \mathscr{S}(g)$, $g \in G$, is a canonical representation of G on a vector space M.*

To the G-structure of the order r on V_n a tensor t_s on H with values in M of type $\varrho(G)$ is assigned. This tensor is called the G-structure tensor.

It is easy to verify that the tensor t_s defined above is, in the case $r = 1$, the structure tensor defined in [2].

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Výtah

TENSOR G-STRUKTURY r-TÉHO ŘÁDU

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Buď L_n^r rozšíření r-tého řádu lineární grupy L_n. Buď G lieova podgrupa grupy L_n^r. Fibrovaný podprostor $H(V_n, G)$ hlavního prodloužení r-tého řádu H_r variety V_n nazýváme G-strukturou r-tého řádu na varietě V_n. K takto definované struktuře na varietě V_n je jednoznačně přiřazen vektorový prostor M a nalezena kanonická representace ϱ grupy G v M. Ke G-struktuře je nalezen tenzor t_s na H s hodnotami v M typu $\varrho(G)$.

Резюме

ТЕНЗОР G-СТРУКТУРЫ r-ГО ПОРЯДКА

БОГУМИЛ ЦЕНКЛ (Bohumil Cenkl), Прага

Пусть L_n^r – расширение r-го порядка линейной группы L_n. Пусть G – подгруппа Ли группы L_n^r. Расслоенное подпространство $H(V_n, G)$ главного продолжения r-го порядка H^r многообразия V_n мы называем G-структурой r-го порядка на многообразии V_n. Определенной таким образом структуре на многообразии V_n ставится в однозначное соответствие векторное пространство M и найдено каноническое представление ϱ группы G в M. Для G-структуры найден тензор t_s на H с значениями в M типа $\varrho(G)$.