Časopis pro pěstování matematiky

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Časopis pro pěstování matematiky, Vol. 90 (1965), No. 4, 442--450

Persistent URL: http://dml.cz/dmlcz/108654

Časopis pro pěstování matematiky, roč. 90 (1965), Praha

ON $\mathscr{K}$-POSITIVE ELEMENTS OF THE SPECTRAL RESOLUTION OF A $\mathscr{K}$-POSITIVE OPERATOR

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(Received August 29, 1964)

This paper deals with the Laurent development of the resolvent of a $\mathscr{K}$-positive operator in a neighbourhood of the maximal positive eigenvalue. It is shown that some terms of this development are $\mathscr{K}$-positive, eventually u_0-positive, strongly $\mathscr{K}$-positive, absolutely $\mathscr{K}$-positive operators. Further it is shown how these properties can be used for the approximative construction of eigenelements of the investigated operator using Kellogg's iterative method.

1. INTRODUCTION

The famous Perron-Frobenius theorem (cf. [2] p. 323) on non negative indecomposable matrices, which has many applications in the theory of probability and in physics, was generalized in several directions. The positivity of an operator as a notion was generalized in the paper [6] and the assumption that the corresponding spaces are finite dimensional was replaced by the compactness of the investigated operators in Banach spaces (cf. [4], [5], [6]) or in locally convex linear topological spaces (cf. [13]). The generalization of the Perron theorem on the matrices with positive element in the Banach space of continuous functions on a compact set is well known as the Jentzsch's theorem (cf. [3]). The compactness of the investigated operator was at last replaced with some weaker demands (cf. [1], [7], [11], [12]). The concept indecomposable matrix was generalized by introducing the notions u_0-positive (cf. [4], [5]) uniformly positive (cf. [1]), absolutely $\mathscr{K}$-positive (cf. [12]) and quasiinterior (cf. [15]) operator. As results of investigations of these notions we get several assertions which guarante on the one hand the existence of one eigenvalue at least and the corresponding eigenvector which lies in the cone of non negative elements (analogously to the case of a decomposable matrix) and on the second hand the unicity of the $\mathscr{K}$-positive eigenvector (analogously to the case of an indecomposable matrix).

In this paper we shall investigate the following question. Let a linear $\mathscr{K}$-positive operator T be given, which maps a real Banach space $\mathscr{Y}$ into itself, where $\mathscr{K}$ denotes a cone in $\mathscr{Y}$. Let $R(\lambda, \tilde{T}) = (\lambda I - \tilde{T})$ be the resolvent of the complex extension $\tilde{T}$ of the operator T. Let the Laurent development

$$R(\lambda, T) = \sum_{k=0}^{\infty} A_k(\lambda - \mu_0)^k + \sum_{k=1}^{\infty} B_k(\lambda - \mu_0)^{-k}$$

in a neighbourhood of an isolated singularity μ_0 be given. We ask if there is some operator B_s $\mathscr{K}$-positive in the sequence $\{B_k\}$, where $B_{k+1} = (\tilde{T} - \mu_0 I) B_k$, $B_1 = (2\pi i)^{-1} \int_{C_0} R(\lambda, \tilde{T}) \, d\lambda$. Besides of this question we shall study also some its modifications which are concerned with further properties of the investigated operator and the mentioned Laurent development of $R(\lambda, \tilde{T})$ as with the u_0-positivity, strong $\mathscr{K}$-positivity in the spaces with volume-type cones or with the absolute $\mathscr{K}$-positivity in the space of vector-functions, components of which are integrable with the square in some region of an Eucleidean space.

The problems just mentioned occur in the approximate construction of the eigenelements mentioned above by using the iterative methods (cf. [8], [9], [10], [12]).

The investigations of the $\mathscr{K}$-positivity of some elements of the Laurent development of $R(\lambda, \tilde{T})$ as a rule is a part of the studying of the spectral properties of $\mathscr{K}$-positive operators (cf. [14]).

Some similar results can be obtained also by insetting stronger assumptions on the cone $\mathscr{K}$. It is natural that then the assumptions on the operator T or some its functions $f(\tilde{T})$ can be weekened (cf. [14], [15]).

This paper contains two parts besides the introduction. In the first part the needed definitions and notation are given. In the second one several assertions are proved on the $\mathscr{K}$-positivity of some terms of the Laurent development of the resolvent $R(\lambda, \tilde{T})$ in a neighbourhood of the pole to which a positive eigenvector correponds. It is shown that the proved theorems can be used by applications of the Kellogg's and other similar iterative methods.

2. NOTATION AND DEFINITIONS

Some definitions and denotations are taken over from the monographies [4,] [5], [16] and from the paper [6].

Let $\mathscr{Y}$ be a real Banach space, $\mathscr{K}$ a cone of positive elements of $\mathscr{Y}$ (cf. [6]). The cone $\mathscr{K}$ is assumed to be such that every $x \in \mathscr{Y}$ can be written in the form

$$x = \lim_{n \to \infty} c_n(u_n - v_n),$$

where $u_n, v_n \in \mathscr{K}$ and c_n are real numbers. The cone $\mathscr{K}$ having this property is called the base of the space $\mathscr{Y}$. The symbols $\mathscr{Y}'$,$[\mathscr{Y}]$ denote the Banach spaces of continuous linear forms on $\mathscr{Y}$ with the usual norm

$$\|x'\| = \sup_{\|x\|=1} |x'(x)|, \quad x \in \mathscr{Y}, \quad x' \in \mathscr{Y}'$$

and the space of linear bounded transformations of the space into itself with the norm

$$\|T\| = \sup_{\|x\|=1} \|Tx\| , \quad x \in \mathcal{Y} , \quad T \in [\mathcal{Y}]$$

respectively.

Let $\mathcal{X}$ denote the complex extension of the space $\mathcal{Y}$ i.e. the set of pairs $z = x + iy$, where $x \in \mathcal{Y}$, $y \in \mathcal{Y}$, $i^2 = -1$, with the norm

$$\|z\| = \sup_{0 \leq \vartheta \leq 2\pi} \|x \cos \vartheta + y \sin \vartheta\|$$

or with some equivalent norm. Similarly $\mathcal{X}'$, $[\mathcal{X}]$ denote corresponding Banach spaces of continuous linear forms on $\mathcal{X}$ and the space of linear bounded transformations of $\mathcal{X}$ into itself with the norms

$$\|x'\| = \sup_{\|x\|=1} |x'(x)| , \quad x \in \mathcal{X} , \quad x' \in \mathcal{X}' ,$$

or

$$\|T\| = \sup_{\|x\|=1} \|Tx\| , \quad x \in \mathcal{X} , \quad T \in [\mathcal{X}] ,$$

respectively.

The complex extension $\tilde{T}$ of T can be defined by the formula

$$\tilde{T}z = Tx + i\, Ty ,$$

where $z = x + iy$. The symbol T' denotes the adjoint operator to the operator T, i.e. $y' = T'x' \Leftrightarrow y'(x) = x'(Tx)$ for every $x \in \mathcal{Y}$. The condition $T \in [\mathcal{Y}]$ implies $\tilde{T} \in [\mathcal{X}]$. It is well known (cf. [16] p. 262) that there exists the limit

$$\lim_{n \to \infty} \sqrt[n]{[\|T^n\|]} = r(T)$$

which is called the spectral radius of the operator T. The set of all complex λ for which the resolvent $R(\lambda, \tilde{T})$ lies in $[\mathcal{X}]$ is called the resolvent set of T and is denoted by $\varrho(T)$. The complement of $\varrho(T)$ in the complex plane is called the spectrum of the operator T and it is denoted by $\sigma(T)$.

Let f be a complex function of the complex variable and let $\Delta(f)$ be the domain of f. The symbol $\mathfrak{A}(T)$ will be denoted the set of all functions such that for corresponding $\Delta(f) \supset \sigma(T)$ and such that f is holomorph on every component of $\Delta(f)$.

The operator $T \in [\mathcal{X}]$ is called Radon-Nicolski operator if T can be expressed in the form $T = U + V$, where $U, V \in [\mathcal{X}]$, U is a compact operator and for the spectral radii $r(V)$, $r(T)$ the inequality $r(T) > r(V)$ holds.

The operator $T \in [\mathcal{Y}]$ is called $\mathcal{K}$-positive (cf. [6]) if $x \in \mathcal{K}$ implies $Tx \in \mathcal{K}$. If $y - x \in \mathcal{K}$ then we write $y \succ x$ or $x \prec y$. In this way we get the partial ordering of the space $\mathcal{Y}$. The cone $\mathcal{K}$ is called volume-type cone (cf. [6]) if it has interior points. The operator $T \in [\mathcal{Y}]$ is called strongly $\mathcal{K}$-positive if $\mathcal{K}$ is a volume-type cone and there is a positive number $p = p(x)$ such that $T^p x$ is an interior point of $\mathcal{K}$ (cf. [6]).

The symbol 0 denotes the zero-elements in both spaces $\mathscr{Y}$, $\mathscr{X}$.

The operator $T \in [\mathscr{Y}]$ is called u_0-positive (cf. [4] p. 262, [5] p. 60) if there is an element $u_0 \in \mathscr{K}$, $\|u_0\| = 1$, such that for every $x \in \mathscr{K}$, $x \neq 0$ there are positive numbers α, β, p such that the relations

$$\alpha u_0 \prec T^p x \prec \beta u_0$$

hold. The u_0-positive operator T is called strongly u_0-positive, if for every $x \in \mathscr{Y}$ there are numbers γ, q, γ – positive, such that

$$\gamma T^q x \prec u_0 .$$

If $\mathscr{K}$ is a cone in the space $\mathscr{Y}$ then the set of all forms $y' \in \mathscr{Y}'$ for which $y'(x) \geqq 0$ for $x \in \mathscr{K}$ generates a cone of $\mathscr{Y}'$ which is called the adjoint cone to the cone $\mathscr{K}$ (cf. [6]). A linear form $x' \in \mathscr{Y}'$ is called strongly positive, if $x'(x) > 0$ for $x \in \mathscr{K}$, $x \neq 0$.

Let I denote the identity-transformation of both spaces $\mathscr{Y}$, $\mathscr{X}$. Assuming the value $\mu_0 \in \sigma(T)$ to be an isolated singularity of the resolvent $R(\lambda, \tilde{T})$, $R(\lambda, \tilde{T})$ can be developed in the Laurent series in the neighbourhood of the point μ_0 (cf. [16] p. 305)

$$(2.1) \qquad R(\lambda, \tilde{T}) = \sum_{k=0}^{\infty} A_k(\lambda - \mu_0)^k + \sum_{k=1}^{\infty} B_k(\lambda - \mu_0)^{-k} ,$$

where $A_k \in [\mathscr{X}]$, $k = 0, 1, \ldots$ and

$$(2.2) \qquad B_1 = \frac{1}{2\pi i} \int_{C_0} R(\lambda, \tilde{T})\, \mathrm{d}\lambda , \qquad B_{k+1} = (\tilde{T} - \mu_0 I)\, B_k , \quad k = 1, 2, \ldots$$

where C_0 is a circle with the centre μ_0 and with the radius ϱ_0 such that the relation $\Gamma_0 \cap \sigma(T) = \{\mu_0\}$ holds for the set $\Gamma_0 = \{\lambda |\, |\lambda - \mu_0| \leqq \varrho_0\}$.

3. $\mathscr{K}$-POSITIVE PROJECTIONS

In this paragraph we give a few propositions concerning some elements of the Laurent expansion of the resolvent $R(\lambda, \tilde{T})$ of a $\mathscr{K}$-positive operator $T \in [\mathscr{Y}]$.

Let the cone $\mathscr{K}$ be a base of the space $\mathscr{Y}$. If ν is a pole of $R(\lambda, \tilde{T})$ then $p(\nu)$ denotes the multiplicity of ν.

Theorem 1. *Let us assume that*

(a) *$T \in [\mathscr{Y}]$ is a $\mathscr{K}$-positive operator.*

(b) *There is a function $f \in \mathfrak{A}(T)$ such that $f(\tilde{T})$ is a Radon-Nicolski operator and $|f(\lambda)| > r(V)$ for $|\lambda| = r(V)$.*

Then the following assertions are valid:

1. *Every point* $\nu \in \sigma(T)$, $|\nu| = r(T)$ *is a pole of* $R(\lambda, \tilde{T})$.
2. *There is a point* $\mu_0 \in \sigma(T)$ *such that*

$$|\nu| \leqq \mu_0 \quad \text{for all} \quad \nu \in \sigma(T)\,. \tag{3.1}$$

3. *If* $p(\mu_0) \geqq p(\nu)$ *for* $\nu \in \sigma(T)$, $|\nu| = \mu_0$, *then in the expansion*

$$R(\lambda, \tilde{T}) = \sum_{k=0}^{\infty} A_k(\lambda - \mu_0)^k + \sum_{k=1}^{q} B_k(\lambda - \mu_0)^k, \quad q = p(\mu_0),$$

in a neighbourhood of μ_0 *the operator* B_q *is* $\mathscr{K}$*-positive.*

Remark. If we assume that the cone $\mathscr{K}$ is normal (i.e. the inequality $\|x + y\| \geqq \|y\|$ holds for every couple $x, y \in \mathscr{K}$ then the relation $p(\mu_0) \geqq p(\nu)$ for $|\nu| = \mu_0$ is valid for every $\mathscr{K}$-positive operator T for which $\mu_0 = r(T)$ is a pole of $R(\lambda, \tilde{T})$ (cf. [15]).

Proof. From (b) it follows according to [11] theorem 2.3 that every point $\nu \in \sigma(T)$, $|\nu| = r(T)$ is a pole of the resolvent $R(\lambda, \tilde{T})$ and that there is a finite number of $\nu_1, \ldots, \nu_l, \nu_j \in \sigma(T)$ for which $|\nu_j| = r(T)$. If T is a $\mathscr{K}$-positive operator then similarly as in [11] theorem 3.2 there exists a point $\mu_0 \in \sigma(T)$ such that the relations (3.1) are valid. Thus $\mu_0 = r(T)$ and we have

$$R(\lambda, \tilde{T}) = \sum_{k=0}^{\infty} \lambda^{-k} T^k = \sum_{k=0}^{\infty} A_k(\lambda - \mu_0)^k + \sum_{k=1}^{q} B_k(\lambda - \mu_0)^{-k} \tag{3.3}$$

for real $\lambda > r(T)$ and therefore

$$(\lambda - \mu_0)^q R(\lambda, \tilde{T}) \to B_q \quad \text{if} \quad \lambda \to \mu_0 \quad (q = p(\mu_0))\,.$$

The $\mathscr{K}$-positivity of $W_\lambda = (\lambda - \mu_0)^q R(\lambda, \tilde{T}) = \sum_{k=0}^{\infty} (\lambda - \mu_0)^q \lambda^{-k} T^k$ implies the positivity of B_q since $\mathscr{K}$ is closed.

Theorem 2. Let

(α) $T \in [\mathscr{Y}]$ *be an* u_0*-positive operator.*

(β) *There is a function* $f \in \mathfrak{A}(T)$ *such that* $f(\tilde{T}) = U + V$ *is a Radon-Nicolski operator and* $|f(\lambda)| > r(V)$ *for* $|\lambda| = r(T)$.

(γ) *The points* $\nu \in \sigma(T)$, $|\nu| = r(T)$ *are simple poles of the resolvent* $R(\lambda, \tilde{T})$.

Then the projection B_1 *in the Laurent development of* $R(\lambda, \tilde{T})$ *in a neighbourhood of* μ_0 *is* u_0*-positive.*

Proof. First, we prove that $B_1 x \neq 0$ for arbitrary vector $x \in \mathscr{K}$, $x \neq 0$. Since B_1 is non zero-operator there exists a vector y_0 in the base $\mathscr{K}$ of the space $\mathscr{Y}$ such that

$B_1 y_0 \neq 0$. By theorem 1, we have $B_1 y_0 \in \mathscr{K}$. It is easy to see that $B_1 y_0$ is an eigenvector of the operator T corresponding to the eigenvalue μ_0: $TB_1 y_0 = \mu_0 B_1 y_0$. Let us put

$$\frac{B_1 y_0}{\|B_1 y_0\|} = v_0 ,$$

hence $\|v_0\| = 1$. According to [4] lemma 2.2 p. 262 the operator T is also v_0-positive and therefore for every vector $x \in \mathscr{K}$, $x \neq 0$ there are positive numbers α, β, p such that

$$\alpha v_0 \prec T^p x \prec \beta v_0 . \tag{3.4}$$

It is well known ([16] p. 299) that $B_1^2 = B_1$ which implies $B_1 v_0 = v_0$. According to the preceding theorem B_1 is a $\mathscr{K}$-positive operator and thus we get from (3.4) the relations

$$\alpha v_0 \prec B_1 T^p x \prec \beta v_0 . \tag{3.5}$$

From the commutativity of T and B_1 and from (3.5) it follows that $B_1 x \neq 0$.

Now we shall prove the u_0-positivity of B_1. The u_0-positivity of T guarantes the existence of positive numbers α, β, p such that the relations

$$\alpha u_0 \prec T^p B_1 x \prec \beta u_0 \tag{3.6}$$

hold for $x \in \mathscr{K}$, $x \neq 0$. But it is evident that $T^p B_1 x = \mu_0^p B_1 x$, consequently the relations (3.6) are equivalent to the relations

$$\alpha \mu_0^{-p} u_0 \prec B_1 x \prec \beta \mu_0^{-p} u_0$$

which show the u_0-positivity of the operator B_1.

Proposition. *Let us replace the assumption* (α) *in the theorem 2 by one of the following assumptions:*

(α') *The operator* $T \in [\mathscr{Y}]$ *is strongly* u_0*-positive operator.*

(a'') *The operator* $T \in [\mathscr{Y}]$ *is strongly* $\mathscr{K}$*-positive operator, where* $\mathscr{K}$ *is a volume-type cone.*

(α''') *The operator* $T \in [\mathscr{Y}]$ *is absolutely* $\mathscr{K}$*-positive operator in the space* $\mathscr{Y} = \mathscr{L}_2(\Gamma) \times \ldots \mathscr{L}_2(\Gamma)$, *where* Γ *is a region in some Eucleidean space* $\mathscr{E}_1$ (*cf.* [12]).

Then the assumption (γ) *of the theorem 2 can be ommited and the projection* B_1 *is an* u_0*-positive, strongly* $\mathscr{K}$*-positive, absolutely* $\mathscr{K}$*-positive operator respectively.*

Proof. The fact that the conditions $\nu \in \sigma(T)$, $\nu \neq r(T)$, imply

$$|\nu| < \mu_0 \tag{3.7}$$

for $v \in \sigma(T)$, $v \neq \mu_0$, where $\mu_0 = r(T)$ and that μ_0 is a simple pole of the resolvent $R(\lambda, \tilde{T})$, follow for (α') from [7] theorem 1, for (α'') from [11] theorem 3.3 and for (α''') from [12] lemma 1.

We shall show how the assertions proved above can be used for the approximate construction of the eigenvalue μ_0 and the corresponding eigenvector by using the Kellogg's iterative procedure. It is proved in [8] that for convergence of the iterative process

$$x_{(n)} = \frac{T^n x_{(0)}}{x'_n(T^n x_{(0)})}, \quad \mu_{(n)} = \frac{z'_n(T x_{(n)})}{y'_n(x_{(n)})}$$

to the eigenvector x_0 corresponding to the eigenvalue μ_0 of the operator T, the following properties of the linear functionals are needed:

$$x'(x) = \lim_{n \to \infty} x'_n(x), \quad y'(x) = \lim_{n \to \infty} y'_n(x) = \lim_{n \to \infty} z'_n(x) \tag{3.8}$$

for $x \in \mathscr{X}$ and there exists a vector $x_{(0)} \in \mathscr{X}$ such that

$$x'(B_s x_{(0)}) \neq 0, \quad y'(B_s x_{(0)}) \neq 0, \tag{3.9}$$

where B_s, $1 \leqq s \leqq p(\mu_0)$, is some term of the Laurent development of $R(\lambda, \tilde{T})$ in a neighbourhood of μ_0.

Theorem 3. *Let us asume that*

(i) $T \in [\mathscr{Y}]$.

(ii) *There exists a function* $f \in \mathfrak{A}(T)$ *such that* $f(\tilde{T}) = U + V$ *is a Radon-Nicolski operator and* $|f(\lambda)| > r(V)$ *if* $|\lambda| = r(T)$.

(iii) *One of the conditions* (α'), (α''), (α''') *is fulfilled for the operator* T.

(iv) *Let* $x_{(0)} \in \mathscr{K}$, $x_{(0)} \neq 0$.

(v) *Let the forms* $x', y' \in \mathscr{Y}'$ *be strongly positive.*

Then the inequalities

$$x'(B_1 x_{(0)}) > 0, \quad y'(B_1 x_{(0)}) > 0$$

are valid.

Let us investigate similar conditions for the iterative process

$$y_{(n+1)} = \lambda_{(n)} T y_{(n)}, \quad y_{(0)} = x_{(0)}, \tag{3.10}$$

$$\lambda_{(n)} = \frac{y'_n(y_{(n)})}{z'_n(T y_{(n)})}, \tag{3.11}$$

where the forms $y'_n \in \mathscr{X}'$, $n = 0, 1, \ldots$ are elements of the iterative sequence

$$y'_{n+1} = \lambda_{(n)} T' y'_n, \quad y'_0 \in \mathscr{Y}'. \tag{3.12}$$

This process is optimal in the sense described in the paper [9].

Theorem 4. *Besides the assumptions* (i)–(iv) *of the theorem* 3 *let the following condition be fulfilled*:

(vi) *The form* y'_0 *is strongly positive.*

Then the inequality

$$y'(B_1 x_{(0)}) > 0$$

holds and therefore the processes (3.10), (3.11), (3.12) *converge*:

$$y_{(n)} \to x_0\,, \quad \lambda_{(n)} \to \lambda_0 = \mu_0^{-1}, \quad y'_n \to x'_0\,,$$

where

$$Tx_0 = \mu_0 x_0\,, \quad T'x'_0 = \mu_0 x'_0\,.$$

Added in proof. After this paper had been prepared the author became acquainted with the following papers: *D. W. Sasser*:" Quasi-positive operators", Pacif. J. Math. *14* (1964), 1029-1037, and *I. Sawashima:* "On spectral properties of some positive operators," Natural Science Report of the Ochanomizu University *15* (1964), 53-64. There the concepts of $\mathscr{K}$-positive operators, indecomposable operators and primitive operators in Banach spaces were generalized, by introducing quasi-positive, strictly quasi-positive. strongly quasi positive, semi - non - support, non - support and strict non - support operators. Using these concepts, results similar to those of the present paper can be obtained.

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Výtah

O $\mathscr{K}$-KLADNÝCH PRVCÍCH SPEKTRÁLNÍHO ROZKLADU $\mathscr{K}$-KLADNÉHO OPERÁTORU

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V práci jsou vyšetřovány některé postačující podmínky zaručující $\mathscr{K}$-kladnost určitých členů Laurentova rozvoje resolventy $R(\lambda, T)$ $\mathscr{K}$-kladného operátoru T v okolí bodu $\mu_0 = r(T)$, kde $r(T)$ je spektrální poloměr operátoru T, za předpokladu, že bod μ_0 je pólem resolventy $R(\lambda, T)$. Jsou zavedeny některé speciální třídy $\mathscr{K}$-kladných operátorů, pro něž jsou odpovídající projektory, vyskytující se ve zmíněném rozvoji $\mathscr{K}$-kladné, resp. silně $\mathscr{K}$-kladné, resp. u_0-kladné, resp. absolutně $\mathscr{K}$-kladné. Vyšetřování se provádějí za předpokladu, že kužel $\mathscr{K}$ tvoří basi reálné komponenty Banachova prostoru $\mathscr{X}$ a operátor T je takový, že pro některou vhodnou funkci f je $f(T)$ operátorem Radona-Nikolského.

Резюме

О $\mathscr{K}$-ПОЛОЖИТЕЛЬНЫХ ЭЛЕМЕНТАХ СПЕКТРАЛЬНОГО РАЗЛОЖЕНИЯ $\mathscr{K}$-ПОЛОЖИТЕЛЬНОГО ОПЕРАТОРА

ИВО МАРЕК (Ivo Marek), Прага

В статье исследованы некоторые достаточные условия для $\mathscr{K}$-положительности определенных членов разложения Лорана резольвенты $R(\lambda, T)$ $\mathscr{K}$-положительного оператора T в окрестности точки $\mu_0 = r(T)$, где $r(T)$-спектральный радиус оператора T, в предположении, что эта точка является полюсом резольвенты $R(\lambda, T)$. Выделены некоторые специальные классы $\mathscr{K}$-положительных операторов, для которых соотбетствующие проекционные операторы появляющиеся в названном разложении соответственно $\mathscr{K}$-положительны, сильно $\mathscr{K}$-положительны, u_0-положительны, абсолютно $\mathscr{K}$-положительны. Исследования ведены в предположении, что конус $\mathscr{K}$ является базой вещественной компоненты банахова пространства $\mathscr{X}$ и оператор T такой, что для некоторой функции f оператор $f(T)$ является оператором Радона-Никольского.