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1. TRIANGLE FREE CONFIGURATION

Let X be a set of $3m$ integers. We identify subsets of size m of X as points. A line corresponds to a partition of X into 3 disjoint subsets and so, on a line lies 3 points. Hence, the total number of lines (M) is the number of ways of partitioning the set X into 3 disjoint subsets; the number of lines (C) through a point is the number of ways of partitioning a set of $2m$ integers into 2 disjoint subsets. Therefore, total number of lines $M = (3m)!\,(m!)^{-3}\,(3!)^{-1}$, total number of points, $N = {}^{3m}C_m$; number of lines through a point $(C) = (2m)!\,(m!)^{-2}\,(2!)^{-1}$, and on a line lies 3 points.

It is easy to see that these points and lines form a configuration [1]. In the configuration any two points represented by disjoint subsets are joined. For any two points represented by disjoint subsets A and B, a third point represented by a subset C, which is disjoint to both A and B, is collinear with them. Hence the configuration is triangle free.

The group G of symmetries of the above configuration which leaves both the set P of ${}^{3m}C_m$ points and the set L of $(3m)!\,(m!)^{-3}\,(3!)^{-1}$ lines invariant is the group of permutations of ${}^{3m}C_m$ points which preserve the set L of lines. G is transitive on the ${}^{3m}C_m$ points and $(3m)!\,(m!)^{-3}\,(3!)^{-1}$ lines. The stabilizers of (i) a point, say $1, 2, \ldots, m$, are a permutation group of $(m!)\,(2m)!$ permutation operations, obtained as product of $(m!)$ permutations of $1, 2, \ldots, m$, and $(2m)!$ of $(m+1), \ldots, 3m$; (ii) a line, say, $(1, 2, \ldots, m, \overline{m+1}, \ldots, 2m, \overline{2m+1}, \ldots, 3m)$ are a permutation group of $(3!)\,(m!)^3$ permutation operations, obtained as product of permutations of $1, 2, \ldots, m$; $\overline{m+1}, \ldots, 2m$, and $2m+1, \ldots, 3m$.

Hence G has order, ${}^{3m}C_m(m)!\,(2m)! = (3m)!\,(m!)^{-3}\,(3!)^{-1}\,.\,(3!)\,(m!)^3 = (3m)!\,.$ If we also allow reciprocity which interchanges P and L we obtain the group G' of order $2(3m)!$.

2. TRIANGULAR PBIB DESIGN

Under the interpretation of points as treatments and lines as blocks, we get a m-associate triangular PBIB design [2], from the above configuration, with the parameters,

$$v = {}^{3m}C_m\,, \quad b = (3m)!\,(m!)^{-3}\,(3!)^{-1}\,,$$

$$r = (2m)!\,(m!)^{-2}\,(2!)^{-1}\,, \quad k = 3\,, \quad \lambda_i = 0\,, \quad i = 1, 2, \ldots, m - 1\,; \quad \lambda_m = 1\,,$$

$$n_i = {}^{2m}C_i\,{}^{m}C_i\,, \quad i = 1, 2, \ldots, m\,;$$

$$p^i_{jk} = \sum_{\omega=0}^{m-i} \binom{m-i}{\omega} \binom{i}{m-k-\omega} \binom{i}{m-j-\omega} \binom{2m-i}{j+k-m+\omega}\,,$$

$$i, j, k = 1, 2, \ldots, m\,.$$

Since no two treatments having i $(i = 1, 2, \ldots, m - 1)$ integers in common lie on a block, therefore

$$\lambda_i = 0\,, \quad i = 1, 2, \ldots, m - 1\,,$$

and, $\lambda_m = 1$.

The def. of m-associate triangular association scheme is given in [3].

3. GENERALIZED QUADRANGLE OF ORDER 2

For $m = 2$, the triangle free (15_3) configuration (Section 1) is a generalized quadrangle [4] of order 2.

The 15 points of the configuration are represented by (ij) same as (ji); $i, j = 1, 2, \ldots, 6$; $i \neq j$. The 15 lines are

(12 34 56)	(13 24 56)	(14 23 56)	(15 23 46)	(16 23 45)
(12 35 46)	(13 25 46)	(14 25 36)	(15 24 36)	(16 24 35)
(12 36 45)	(13 26 45)	(14 26 35)	(15 26 34)	(16 25 34)

On a line, say, (12 34 56) lies three points 12, 34, 56. For $m \geqq 3$, the configuration can not be interpreted as a generalized quadrangle.

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