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# ON CERTAIN EXTENSIONS OF INTERVALS IN GRAPHS

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*Summary.* Let $G$ be a connected graph. If $u, v \in V(G)$, then we denote by $(u, v)^{\#}$ the set of all $w \in V(G)$ such that either (i) $w = u$ or (ii) there exists $w^* \in V(G)$ such that $ww^* \in E(G)$, $w^*$ belongs to a shortest $w - u$ path but does not belong to any shortest $w - v$ path. If $w_1, w_2 \in V(G)$, then we define $(w_1, w_2)^{\cap} = (w_1, w_2)^{\#} \cap (w_2, w_1)^{\#}$ and $(w_1, w_2)^{\cup} = (w_1, w_2)^{\#} \cup (w_2, w_1)^{\#}$. Using functions $(\ldots, \ldots)^{\cap}$ and $(\ldots, \ldots)^{\cup}$ we characterize some classes of connected graphs.

*Keywords:* Shortest paths, intervals, bipartite graphs, induced subgraphs.

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Let $F$ be a graph (in the sense of [1], for example); we denote by $V(F)$, $E(F)$ and $\bar{F}$ the vertex set of $F$, the edge set of $F$ and the complement of $F$, respectively; if $u \in V(F)$, then we denote by $N_F(u)$ the set of all vertices adjacent to $u$ in $F$; if $v, w \in V(F)$, then we denote by $d_F(v, w)$ the distance between $v$ and $w$ in $F$. If $G$ is a graph, then instead of $V(G)$, $E(G)$, $N_G(u)$ and $d_G(v, w)$ we shall write $V$, $E$, $N(u)$ and $d(v, w)$, respectively. If $F_1$ and $F_2$ are graphs, then the expression $F_1 + F_2$ will denote the join of the graphs $F_1$ and $F_2$ in the sense of [1].

Let $G$ be a connected graph. If $u_1, u_2 \in V$, then – similarly as in [2] – by the interval $I(u_1, u_2)$ we mean the set

$$\{u_0 \in V;\ d(u_1, u_0) + d(u_0, u_2) = d(u_1, u_2)\}\ .$$

Let $u, v \in V$. We denote by $(u, v)^{\#}$ the set of all $w \in V$ with the property that either $w = u$ or

$$N(w) \cap (I(w, u) - I(w, v)) \neq \emptyset\ .$$

In other words, $(u, v)^{\#}$ is the set of all $w \in V$ such that either (i) $w = u$ or (ii) there exists $w^* \in V$ such that $ww^* \in E$, $w^*$ belongs to a shortest $w - u$ path but does not belong to any shortest $w - v$ path. Since $I(u, v) \subseteq (u, v)^{\#}$, we may say that $(u, v)^{\#}$ is a certain extension of $I(u, v)$.

The following two propositions can be easily derived from the definition.

**Proposition 1.** *Let $G$ be a connected graph, and let $u, v, w \in V$. Then*

$$(u, w)^{\#} \subseteq (u, v)^{\#} \cup (v, w)^{\#}\ .$$

**Proposition 2.** *Let $G$ be a nontrivial connected graph. Then*
(a) *$G$ is a tree if and only if $(u, v)^{\#} = \{u, v\}$ for any adjacent $u, v \in V$, and*
(b) *$G$ is a complete if and only if $(u, v)^{\#} = V$ for any adjacent $u, v \in V$.*

Let $G$ be a connected graph, and let $u, v \in V$. We define

$$(u, v)^{\cap} = (u, v)^{\#} \cap (v, u)^{\#} \quad \text{and} \quad (u, v)^{\cup} = (u, v)^{\#} \cup (v, u)^{\#} .$$

Since $I(u, v) \subseteq (u, v)^{\cap} \subseteq (u, v)^{\cup}$, we may assume that $(u, v)^{\cap}$ and $(u, v)^{\cup}$ are also extensions of $I(u, v)$.

Let $G$ be a connected graph, and let $k$ be a positive integer. We shall say that $G$ fulfils condition $\mathscr{C}_k^{\cap}$ if

$$(u, v)^{\cap} = I(u, v) \quad \text{for any} \quad u, v \in V \quad \text{such that} \quad d(u, v) = k .$$

Similarly, we say that $G$ fulfils condition $\mathscr{C}_k^{\cup}$ if

$$(u, v)^{\cup} = V \quad \text{for any} \quad u, v \in V \quad \text{such that} \quad d(u, v) = k .$$

**Proposition 3.** *Let $G$ be a nontrivial connected graph. Then $G$ fulfils $C_1^{\cap}$ and only if $G$ is bipartite.*

Proof. (I) Assume that $G$ is not bipartite. It is not difficult to see that there exists an odd cycle $C$ in $G$ such that

$$d_C(r, s) = d(r, s) \quad \text{for any} \quad r, s \in V(C) .$$

Consider $u, v \in V(C)$ such that $uv \in E(C)$. Then there exists $w \in V(C)$ such that $d(u, w) = d(v, w)$. This means that $u \neq w \neq v$. There exist $u_0, v_0 \in V(C)$ such that $u_0 w, v_0 w \in E(C)$ and

$$d(u, u_0) = d(u, w) - 1 = d(v, v_0) .$$

It is clear that $d(u_0, v) = d(u, w)$. Similarly, $d(v_0, u) = d(v, w)$. Hence $u_0 \notin I(v, w)$ and $v_0 \notin I(u, w)$. This means that $w \in (u, v)^{\cap}$, and therefore, $G$ does not fulfil $\mathscr{C}_1^{\cap}$.

(II) Assume that $G$ does not fulfil $\mathscr{C}_1^{\cap}$. There exist $u, v, w \in V$ such that $uv \in E$, $u \neq w \neq v$, and $w \in (u, v)^{\cap}$. Without loss of generality, let $d(u, w) \leqq d(v, w)$. If $d(u, w) < d(v, w)$, then $d(u, v) = d(v, w) - 1$, and thus $w \notin (u, v)^{\#}$, which is a contradiction. Let $d(u, v) = d(v, w)$. Since $uv \in E$, it is easy to see that $G$ contains an odd cycle. Thus, $G$ is not bipartite, which completes the proof of the proposition.

Remark 1. If $G$ is an even cycle of length $\geqq 6$, then $G$ fulfils $\mathscr{C}_1^{\cap}$ and does not fulfil $\mathscr{C}_2^{\cap}$. If $G$ is isomorphic to $K_m + \bar{K}_n$, where $m \geqq 2$, $n \geqq 1$, then $G$ fulfils $\mathscr{C}_2^{\cap}$ and does not fulfil $\mathscr{C}_1^{\cap}$.

In the present paper we shall characterize the connected graphs which fulfil $\mathscr{C}_1^{\cup}$ and $\mathscr{C}_2^{\cup}$, and the connected graphs which fulfil $\mathscr{C}_2^{\cup}$ and $\mathscr{C}_2^{\cap}$.

If $n$ is a positive integer, then we denote by $P_n$ a path with exactly $n$ vertices. This implies that $K_1 + \bar{P}_3$ is a connected graph which has exactly two blocks: a triangle and a bridge.

**Lemma 1.** *Let $G$ be a connected graph. Assume that $G$ fulfils $\mathscr{C}_2^{\cap}$. Then $G$ contains no induced $K_1 + \bar{P}_3$.*

Proof. To the contrary, assume that $G$ contains an induced $K_1 + \bar{P}_3$. Then there exist distinct $u_1, u_2, u_3, u_4 \in V$ such that $u_1u_2, u_2u_3, u_2u_4, u_3u_4 \in E$ and $u_1u_3, u_1u_4 \notin$ $\notin E$. Obviously, $u_4 \notin I(u_1, u_3)$ and $u_4 \in (u_1, u_3)^{\cap}$, which is a contradiction. Thus, the lemma is proved.

**Lemma 2.** *Let $G$ be a connected graph. Assume that $G$ fulfils $\mathscr{C}_2^{\cap}$ and at least one of the conditions $\mathscr{C}_1^{\cup}$ and $\mathscr{C}_2^{\cup}$. Then $G$ contains no induced $P_4$.*

Proof. To the contrary, we assume that $G$ contains an induced $P_4$. Then there exist distinct $u_1, u_2, u_3, u_4 \in V$ such that

$$u_1u_2, u_2u_3, u_3u_4 \in E \quad \text{and} \quad u_1u_3, u_1u_4, u_2u_4 \notin E .$$

Obviously, $2 \leqq d(u_1, u_4) \leqq 3$.

First, let $d(u_1, u_4) = 2$. Since $u_1u_4 \notin E$, it is clear that $u_4 \notin I(u_1, u_3)$. According to $\mathscr{C}_2^{\cap}$, $u_4 \notin (u_1, u_3)^{\cap}$. Thus $u_1u_3 \in E$, which is a contradiction.

Let now $d(u_1, u_4) = 3$. Since $G$ fulfils $\mathscr{C}_1^{\cup}$ or $\mathscr{C}_2^{\cup}$, we have that $u_4 \in (u_1, u_2)^{\cup} \cup$ $\cup (u_1, u_3)^{\cup}$. There exists $u_5 \in V$ such that $u_4 \neq u_5 \neq u_1$, $u_5 \neq u_3$, $u_4u_5 \in E$, $d(u_1, u_5) = 2$, and

$$\text{if } G \text{ does not fulfil } \mathscr{C}_2^{\cup}, \text{ then } d(u_2, u_5) \geqq 2 .$$

Obviously, $u_1 \neq u_5$. We distinguish two cases:

*Case 1.* Assume that $u_2u_5 \notin E$. If $u_3u_5 \in E$, then the subgraph of $G$ induced by $\{u_2, u_3, u_4, u_5\}$ is isomorphic to $K_1 + \bar{P}_3$, and thus – according to Lemma 1 – $G$ does not fulfil $\mathscr{C}_2^{\cap}$, which is a contradiction. Let $u_3u_5 \notin E$. Then the subgraph of $G$ induced by $\{u_1, u_2, u_3, u_4, u_5\}$ is isomorphic to $P_5$. Since $d(u_1, u_5) = 2$, there exists $u_6 \in V$ such that $u_1 \neq u_6 \neq u_5$ and $u_5u_6, u_6u_1 \in E$. Since $u_2u_5 \notin E$, it is clear that the vertices $u_1, \ldots, u_6$ are mutually distinct. Since $d(u_1, u_4) = 3$, we have that $u_4u_6 \notin E$. Then $u_6 \notin I(u_2, u_4)$.

If $u_2u_6 \in E$, then the subgraph of $G$ induced by $\{u_1, u_2, u_5, u_6\}$ is isomorphic to $K_1 + \bar{P}_3$, which is a contradiction. Let $u_2u_6 \notin E$. Since $u_6 \notin I(u_2, u_4)$, it follows from $\mathscr{C}_2^{\cap}$ that $u_6 \notin (u_2, u_4)^{\cap}$. Hence $u_1u_4 \in E$ or $u_2u_5 \in E$, which is a contradiction.

*Case 2.* Assume that $u_2u_5 \in E$. Then $G$ fulfils $\mathscr{C}_2^{\cup}$. It follows from Lemma 1 that $u_3u_5 \notin E$. Recall that

$$u_1u_2, u_2u_3, u_2u_5, u_3u_4, u_4u_5 \in E \quad \text{and} \quad u_1u_3, u_1u_4, u_1u_5 ,$$
$$u_2u_4, u_3u_5 \notin E .$$

According to $\mathscr{C}_2^{\cup}$, $u_1 \in (u_3, u_5)^{\cup}$. There exists $v \in V$ such that $v \notin \{u_1, u_2, u_3, u_5\}$, $u_1v \in E$, and either (a) $u_3v \in E$ and $u_5v \notin E$

or (b) $u_5v \in E$ and $u_3v \notin E$.

Obviously, $v \neq u_4$. Since $d(u_1, u_4) = 3$, we have that $u_4v \notin E$. Without loss of generality we assume (a). It is clear that $u_3 \notin I(u_1, u_5)$. According to $\mathscr{C}_2^{\cap}$, $u_3 \notin (u_1, u_5)^{\cap}$. This implies that either $u_1u_4 \in E$ or $u_5v \in E$, which is a contradiction.

Thus, we have proved that $G$ contains no induced $P_4$, which completes the proof of the lemma.

**Lemma 3.** *Let $G$ be a connected graph. Assume that $G$ contains no induced $P_4$ and no induced $K_1 + \bar{P}_3$. Then $G$ fulfils $\mathscr{C}_2^{\cap}$.*

Proof. To the contrary, we assume that there exist $u, v, w \in V$ such that $d(u, v) = 2$, $w \notin I(u, v)$ and $w \in (u, v)^{\cap}$. Without loss of generality we assume that $d(u, w) \leqq d(v, w)$. Since $G$ contains no induced $P_4$, we have that $d(v, w) \leqq 2$. If $u = w$ or $vw \in E$, then $w \in I(u, v)$, which is a contradiction. Let $u \neq w$ and $vw \notin E$. Since $d(u, v) = 2$, there exists $w_0 \in V$ such that $u \neq w_0 \neq v$ and $uw_0, w_0v \in E$. Since $vw \notin E$, we have that $w \neq w_0$.

First we assume that $uw \in E$. If $ww_0 \notin E$ or $ww_0 \in E$, then the subgraph of $G$ induced by $\{u, v, w, w_0\}$ is isomorphic to $P_4$ or to $K_1 + \bar{P}_3$, respectively, which is a contradiction.

We now assume that $uw \notin E$. Since $w \in (u, v)^{\cap}$, there exist distinct $u_0, v_0 \in V$ such that

$$uu_0, u_0w, vv_0, v_0w \in E \quad \text{and} \quad uv_0, u_0v \notin E.$$

Since the subgraph of $G$ induced by $\{u, u_0, v_0, w\}$ is not isomorphic to $P_4$, we have that $u_0v_0 \in E$. Then the subgraph of $G$ induced by $\{u, u_0, v, v_0\}$ is isomorphic to $P_4$, which is a contradiction.

Thus, we have proved that $G$ fulfils $\mathscr{C}_2^{\cap}$, which completes the proof of the lemma.

Remark 2. The graph obtained from $K(3, 3)$ by deleting exactly one edge is an example of a connected graph which contains $P_4$ and fulfils $\mathscr{C}_2^{\cap}$.

**Theorem 1.** *Let $G$ be a nontrivial connected graph. Then the following statements are equivalent:*

(a) *$G$ fulfils $\mathscr{C}_1^{\cup}$ and $\mathscr{C}_2^{\cap}$;*

(b) *$G$ is a block and contains no induced $P_4$ or $K_1 + \bar{P}_3$.*

Proof. (I) Let (a) holds. Assume that $G$ is not a block. Then there exist distinct $u_1, u_2, u_3 \in V$ such that $u_1u_2, u_2u_3 \in E$, and $u_1$ and $u_3$ belong to distinct blocks of $G$. Obviously, $u_2$ is a cut-vertex of $G$. We can see that $u_3 \notin (u_1, u_2)^{\cup}$, which is a contradiction. Thus, $G$ is a block. According to Lemma 1, $G$ contains no induced $K_1 + \bar{P}_3$. According to Lemma 2, $G$ contains no induced $P_4$. This implies that (b) holds.

(II) We now wish to show that if (b) holds, then (a) holds. To the contrary, we assume that (b) holds but (a) does not hold. Since(b) holds, it follows from Lemma 3 that $G$ fulfils $\mathscr{C}_2^{\cap}$. Since (a) does not hold, we have that $G$ does not fulfil $\mathscr{C}_1^{\cup}$. Then there exist $v_1, v_2, v_3 \in V$ such that $v_1v_2 \in E$ and $v_3 \notin (v_1, v_2)^{\cup}$. Without loss of generality we assume that $d(v_1, v_3) \leqq d(v_2, v_3)$. If $v_1 = v_3$, then $v_3 \in (v_1, v_2)^{\cup}$, which is a contradiction. Let $v_1 \neq v_3$.

First we assume that $v_1v_3 \in E$. If $v_2v_3 \in E$, then $v_3 \in (v_1, v_2)^{\cup}$, which is a contradiction. Let $v_2v_3 \notin E$. Since $G$ is a block, there exists an induced $v_2 - v_3$ path in $G$ which does not contain $v_1$. Since $G$ contains no induced $P_4$ and $v_2v_3 \notin E$, we have that there exists $v_4 \in V$ such that $v_4 \notin \{v_1, v_2, v_3\}$, and $v_2v_4, v_3v_4 \in E$. Since $v_1v_2$, $v_1v_3 \in E$, we can easily see that $v_3 \in (v_1, v_2)^{\cup}$, which is a contradiction.

We now assume that $v_1v_3 \notin E$. Then $d(v_1, v_3) = 2$. There exists $v_0 \in V$ such that $v_1 \neq v_0 \neq v_3$ and $v_0v_1, v_0v_3 \in E$. Since $d(v_1, v_3) \leqq d(v_2, v_3)$, it is obvious that $v_2v_3 \notin E$. If $v_0v_2 \in E$ or $v_0v_2 \notin E$, then the subgraph of $G$ induced by $\{v_0, v_1, v_2, v_3\}$ is isomorphic to $K_1 + \bar{P}_3$ or to $P_4$, respectively, which is a contradiction.

Thus, (b) implies (a), which completes the proof of the theorem.

Combining Theorem 1 and Proposition 3 we get the following result:

**Corollary 1.** *Let $G$ be a nontrivial connected graph. Then $G$ fulfils $\mathscr{C}_1^{\cap}$, $\mathscr{C}_2^{\cap}$ and $\mathscr{C}_1^{\cup}$ if and only if $G$ is a complete bipartite graph different from a star.*

**Theorem 2.** *Let $G$ be a connected graph. Then the following statements are equivalent:*

(a) *$G$ fulfils $\mathscr{C}_2^{\cup}$ aqd $\mathscr{C}_2^{\cap}$;*

(b) *$G$ contains no induced $P_4$, $K_1 + \bar{P}_3$, or $K(1, 3)$.*

Proof. (I) Let $G$ fulfil $\mathscr{C}_2^{\cup}$ and $\mathscr{C}_2^{\cap}$. According to Lemma 2, $G$ contains no induced $P_4$, and according to Lemma 1, $G$ contains no induced $K_1 + \bar{P}_3$.

Assume that $G$ contains an induced $K(1, 3)$. Then there exist distinct $u, u_1, u_2, u_3 \in$ $\in V$ such that

$$uu_1, uu_2, uu_3 \in E \quad \text{and} \quad u_1u_2, u_2u_3, u_1u_3 \notin E .$$

Since $d(u_1, u_2) = 2$, it follows from $\mathscr{C}_2^{\cup}$ that $u_3 \in (u_1, u_2)^{\cup}$. Since $d(u_1, u_3) = 2 =$ $= d(u_2, u_3)$, there exists $v \in V$ such that $v \notin \{u_1, u_2, u_3\}$, $u_3v \in E$, and either (i) $u_1v \in E$ and $u_2v \notin E$ or (ii) $u_2v \in E$ and $u_1v \notin E$. Without loss of generality we assume that (i). If $uv \in E$ or $uv \notin E$, then the subgraph of $G$ induced by $\{u_1, u_2, u, v\}$ is isomorphic to $K_1 + \bar{P}_3$ or to $P_4$, respectively, which is a contradiction.

Thus, $G$ contains no induced $K(1, 3)$.

(II) Let (b) hold. It follows from Lemma 3 that $G$ fulfils $\mathscr{C}_2^{\cap}$.

Assume that $G$ does not fulfil $\mathscr{C}_2^{\cup}$. Then there exist $u, v, w \in V$ such that $d(u, v) = 2$ and $w \notin (u, v)^{\cup}$. Without loss of generality we assume that $d(u, w) \leqq d(v, w)$. If $u = w$ or $vw \in E$, then $w \in (u, v)^{\cup}$, which is a contradiction. Let $u \neq w$ and $vw \notin E$. Since $G$ contains no induced $P_4$, it is obvious that $d(v, w) = 2$.

First, let $uw \in E$. Since $d(v, w) = 2 = d(u, v)$, we have that $w \in (u, v)^{\cup}$, which is a contradiction.

Let now $uw \notin E$. Then there exists $u_0 \in V$ such that $u \neq u_0 \neq w$ and $uu_0, u_0w \in E$. Obviously, $u_0 \neq v$. Since $w \notin (u, v)^{\cup}$, we have that $u_0v \in E$. Then the subgraph of $G$ induced by $\{u, u_0, v, w\}$ is isomorphic to $K(1, 3)$, which is a contradiction.

Thus, $G$ fulfils $\mathscr{C}_2^{\cup}$, which completes the proof of the theorem.

It is obvious that if $G$ is isomorphic to $P_3$, then $G$ fulfils $\mathscr{C}_2^{\cap}$ and $\mathscr{C}_2^{\cup}$ but does not fulfil $\mathscr{C}_1^{\cup}$. From Theorems 1 and 2 the following corollary can be derived:

**Corollary 2.** *Let $G$ be a nontrivial connected graph. Assume that $G$ fulfils $\mathscr{C}_2^{\cap}$ and is not isomorphic to $P_3$. Then $G$ fulfils $\mathscr{C}_2^{\cup}$ if and only if $G$ fulfils $\mathscr{C}_1^{\cup}$ and contains no induced $K(1, 3)$.*

Proof. First, let $G$ fulfil $\mathscr{C}_2^{\cup}$. As follows from Theorem 2, $G$ contains no induced $P_4$, $K_1 + \bar{P}_3$, or $K(1, 3)$. First we assume that $G$ is not a block. Since $G$ contains no induced $P_4$ or $K(1, 3)$, we can easily see that $G$ has exactly two blocks. Since $G$ is not isomorphic to $P_3$, at least one of the blocks of $G$ is cyclic. Thus, $G$ contains an induced $K(1, 3)$ or $K_1 + \bar{P}_3$, which is a contradiction. We now assume that $G$ is a block. According to Theorem 1, $G$ fulfils $\mathscr{C}_1^{\cup}$.

Conversely, let $G$ fulfil $\mathscr{C}_1^{\cup}$ and let it $G$ contain no induced $K(1, 2)$. As follows from Theorem 1, $G$ contains no induced $P_4$ or $K_1 + \bar{P}_3$. Thus – according to Theorem 2 – $G$ fulfils $\mathscr{C}_2^{\cup}$.

Remark 3. If $G$ is a cycle of length 5 or 6, then $G$ fulfils $\mathscr{C}_2^{\cup}$ but does not fulfil $\mathscr{C}_1^{\cup}$. Combining Corollary 2, Theorem 2 and Proposition 3 we get the following result:

**Corollary 3.** *Let $G$ be a connected graph. Then $G$ fulfils $\mathscr{C}_1^{\cap}, \mathscr{C}_2^{\cap}, \mathscr{C}_1^{\cup}$ and $\mathscr{C}_2^{\cup}$ if and only if $G$ is isomorphic to $K_1, K_2$, or $K(2, 2)$.*

**Problems.** Characterize the connected graphs which fulfil $\mathscr{C}_1^{\cap}$ and $\mathscr{C}_1^{\cup}$. Characterize the connected graphs which fulfil $\mathscr{C}_1^{\cap}$ and $\mathscr{C}_2^{\cup}$.

Remark 4. The subject of the paper has its origin in the author's study of mathematical models in semiotics.

*References*

[1] *M. Behzad, G. Chartrand, L. Lesniak-Foster:* Graphs & Digraphs. Prindle, Weber & Schmidt, Boston 1979.

[2] *H. M. Mulder:* The Interval Function of a Graph. Mathematisch Centrum, Amsterdam 1980.

## O JISTÝCH ROZŠÍŘENÍCH INTERVALŮ V GRAFECH

LADISLAV NEBESKÝ

Nechť $G$ je souvislý graf. Když $u$ a $v$ jsou uzly grafu $G$, tak jako $(u, v)^{\#}$ označíme množinu všech uzlů $w$ grafu $G$ takových, že buď (i) $w = u$ nebo (ii) existuje uzel $w^*$ grafu $G$ takový, že $ww^*$ je hrana, $w^*$ leží na nějaké nejkratší $w - u$ cestě, ale neleží na žádné nejkratší $w - v$ cestě. Když $w_1$ a $w_2$ jsou uzly grafu $G$, tak definujeme $(w_1, w_2)^{\cap} = (w_1, w_2)^{\#} \cap (w_2, w_1)^{\#}$ a $(w_1, w_2)^{\cup} = (w_1, w_2)^{\#} \cup (w_2, w_1)^{\#}$. S využitím funkcí $(\ldots, \ldots)^{\cap}$ a $(\ldots, \ldots)^{\cup}$ jsou v článku charakterizovány některé třídy souvislých grafů.

Резюме

## О ПРОДОЛЖЕНИЯХ ИНТЕРВАЛОВ В ГРАФАХ

LADISLAV NEBESKÝ

Пусть $G$ — связный граф. Для вершин $u$ и $v$ графа $G$ пусть $(u, v)^{\#}$ обозначает множество всех вершин $w$ графа $G$, для которых либо (i) $w = u$, либо (ii) существует такая вершина $w^*$ графа $G$, что $ww^*$ — ребро и $w^*$ лежит на некотором кратчейшем $w - u$ пути, но не лежит ни на каком кратчайшем $w - v$ пути. Далее, для вершин $w_1, w_2$ графа $G$ пусть $(w_1, w_2)^{\cap} = (w_1, w_2)^{\#} \cap (w_2, w_1{}^{\#})$ и $(w_1, w_2)^{\cup} = (w_1, w_2)^{\#} \cup (w_2, w_1)^{\#}$. В статье при помощи функций $(\ldots, \ldots)^{\cap}$ и $(\ldots, \ldots)^{\cup}$ характеризуются некоторые классы связных графов.

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