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Abstract. In the paper we prove a uniqueness theorem for meromorphic functions which provides an answer to a question of H. X. Yi.

1. Introduction and Definitions

Let f be a nonconstant meromorphic function defined on the open complex plane $\mathbb{C}$. Let S be a set of distinct complex numbers and $E_f(S) = \cup_{a\in S}\{z : f(z) - a = 0\}$, where a zero of $f - a$ of multiplicity m is repeated m times in $E_f(S)$.

Gross [3] proved that there exist three finite sets $S_j (j = 1, 2, 3)$ such that any two entire functions f and g satisfying $E_f(S_j) = E_g(S_j)$ for $j = 1, 2, 3$ must be identical.

For meromorphic functions Yi [11, 12] proved the following two theorems.

Theorem A [11]. *Let $S_1 = \{z : z^n - 1 = 0\}$, $S_2 = \{a, b\}$, $S_3 = \{\infty\}$, where $n(\geq 7)$ be a positive integer, a and b be constants such that $ab \neq 0$, $a^n \neq b^n$, $a^{2n} \neq 1$, $b^n \neq 1$ and $a^n b^n \neq 1$. If f and g are nonconstant meromorphic functions satisfying $E_f(S_j) = E_g(S_j)$ for $j = 1, 2, 3$ then $f \equiv g$.*

Theorem B [12]. *Let $S = \{z : z^n + az^{n-m} + b = 0\}$, where n and m are two positive integers such that $m \geq 2$, $n \geq 2m + 7$ with n and m having no common factor, a and b be two nonzero constants such that $z^n + az^{n-m} + b = 0$ has no multiple root. If f and g are nonconstant meromorphic functions satisfying $E_f(S) = E_g(S)$ and $E_f(\{\infty\}) = E_g(\{\infty\})$ then $f \equiv g$.*

One may note that the range set S in Theorem B contains at least eleven elements which corresponds to $m = 2$.

In [12] Yi asked the following question: "What can be said if $m = 1$ in Theorem B?"

To answer this question Yi [12] proved the following theorem.

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Theorem C [12]. *Let $S = \{z : z^n + az^{n-1} + b = 0\}$, where $n(\geq 9)$ be a positive integer and a, b be two nonzero constants such that $z^n + az^{n-1} + b = 0$ has no multiple root. If f and g are two nonconstant meromorphic functions such that $E_f(S) = E_g(S)$ and $E_f(\{\infty\}) = E_g(\{\infty\})$ then either $f \equiv g$ or*

$$f \equiv -\frac{aH(H^{n-1} - 1)}{H^n - 1} \quad \textit{and} \quad g \equiv -\frac{a(H^{n-1} - 1)}{H^n - 1},$$

where H is a nonconstant meromorphic function.

Since one can verify that [12] $H \equiv f/g$, Theorem C is not much significant.

Lahiri [5] proved the following result which provides an answer to the question of Yi.

Theorem D [5]. *Let $S = \{z : z^n + az^{n-1} + b = 0\}$, where $n(\geq 8)$ be a positive integer and a, b be two nonzero constants such that $z^n + az^{n-1} + b = 0$ has no multiple root. If f and g are two nonconstant meromorphic functions having no simple pole such that $E_f(S) = E_g(S)$ and $E_f(\{\infty\}) = E_g(\{\infty\})$ then $f \equiv g$.*

Recently Fang and Lahiri [2] improved Theorem D and proved the following result.

Theorem E [2]. *Let $S = \{z : z^n + az^{n-1} + b = 0\}$, where $n(\geq 7)$ be a positive integer and a, b be two nonzero constants such that $z^n + az^{n-1} + b = 0$ has no multiple root. If f and g are two nonconstant meromorphic functions having no simple pole such that $E_f(S) = E_g(S)$ and $E_f(\{\infty\}) = E_g(\{\infty\})$ then $f \equiv g$.*

Considering $S = \{z : z^7 - z^6 - 1 = 0\}$ and

$$f = \frac{e^z + e^{2z} + \cdots + e^{6z}}{1 + e^z + e^{2z} + \cdots + e^{6z}} \quad \text{and} \quad g = \frac{1 + e^z + e^{2z} + \cdots + e^{5z}}{1 + e^z + e^{2z} + \cdots + e^{6z}}$$

it is verified that for the validity of Theorem E f and g must not have any simple pole. We further note that for these functions $\Theta(\infty; f) = \Theta(\infty; g) = 0$.

If two functions f and g have no simple pole then clearly $\Theta(\infty; f) + \Theta(\infty; g) \geq 1$. In the paper we show that if $\Theta(\infty; f) + \Theta(\infty; g) > 1$ then Theorem E remains valid even if f and g posses simple poles. Also we relax the nature of sharing the sets in Theorem E. To this end we explain the notion of weighted sharing as introduced in [6, 7].

Definition 1. [6, 7] Let k be a nonnegative integer or infinity. For $a \in C \cup \{\infty\}$ we denote by $E_k(a; f)$ the set of all a-points of f where an a-point of multiplicity m is counted m times if $m \leq k$ and $k + 1$ times if $m > k$. If $E_k(a; f) = E_k(a; g)$, we say that f, g share the value a with weight k.

The definition implies that if f, g share a value a with weight k then z_o is a zero of $f - a$ with multiplicity $m(\leq k)$ if and only if it is a zero of $g - a$ with multiplicity $m(\leq k)$ and z_o is a zero of $f - a$ with multiplicity $m(> k)$ if and only if it is a zero of $g - a$ with multiplicity $n(> k)$ where m is not necessarily equal to n.

We write f, g share (a,k) to mean that f, g share the value a with weight k. Clearly if f, g share (a,k) then f, g share (a,p) for all integer p, $0 \leq p < k$. Also we note that f, g share a value a IM or CM if and only if f, g share $(a,0)$ or (a,∞) respectively.

Definition 2. [6] For $S \subset \mathbb{C} \cup \{\infty\}$, we define $E_f(S,k)$ as $E_f(S,k) = \cup_{a\in S} E_k(a;f)$, where k is a nonnegative integer or infinity.

Clearly $E_f(S) = E_f(S,\infty)$.

Definition 3. [6] If s is a positive integer, we denote by $N(r,a;f \mid= s)$ the counting function of those a-points of f whose multiplicity is s, where each a-point is counted according to its multiplicity.

Definition 4. [6] If s is a positive integer, we denote by $\overline{N}(r,a;f \mid\geq s)$ the counting function of those a-points of f whose multiplicities are greater than or equal to s, where each a-point is counted only once.

Definition 5. [1, 6, 8] If s is a nonnegative integer, we denote by $N_s(r,a;f)$ the counting function of a-points of f where an a-point with multiplicity m is counted m times if $m \leq s$ and s times if $m > s$.

We put $N_\infty(r,a;f) \equiv N(r,a;f)$.

Definition 6. [6] Let f,g share a value a IM. We denote by $\overline{N}_*(r,a;f,g)$ the counting function of those a-points of f whose multiplicities are different from multiplicities of the corresponding a-points of g, where each a-point is counted only once.

Clearly $\overline{N}_*(r,a;f,g) \equiv \overline{N}_*(r,a;g,f)$.

In the paper we do not explain the standard notations and definitions of the value distribution theory as those are available in [4, 10]. Unless otherwise stated throughout the paper we denote by f,g two nonconstant meromorphic functions.

Following is the main result of the paper which provides an answer of the question of Yi [12].

Theorem 1. *Let $S = \{z : z^n + az^{n-1} + b = 0\}$, where $n(\geq 7)$ be a positive integer and a, b be two nonzero constants such that $z^n + az^{n-1} + b = 0$ has no multiple root. If $\Theta(\infty;f) + \Theta(\infty;g) > 1$ and $E_f(S,2) = E_g(S,2)$, $E_f(\{\infty\},\infty) = E_g(\{\infty\},\infty)$ then $f \equiv g$.*

2. Lemmas

In this section we discuss some lemmas which will be required in the sequel. Also we denote by H a meromorphic function defined as follows

$$H = \left(\frac{f''}{f'} - \frac{2f'}{f-1}\right) - \left(\frac{g''}{g'} - \frac{2g'}{g-1}\right).$$

Lemma 1. *If f,g share $(1,1)$ and $H \not\equiv 0$ then*

$$N(r,1;f \mid= 1) = N(r,1;g \mid= 1) \leq N(r,H) + S(r,f) + S(r,g).$$

Proof. Since f, g share $(1,1)$, it follows that a simple 1-point of f is a simple 1-point of g and conversely. Let z_o be a simple 1-point of f and g. Then in some neighbourhood on z_o we get by a simple calculation

$$H(z) = (z - z_o)\phi(z)\,,$$

where ϕ is analytic at z_o.

Hence by the first fundamental theorem and Milloux theorem ([4], p. 55) we get

$$N(r,1;f\mid=1) \le N(r,0;H) \le N(r,H) + S(r,f) + S(r,g)\,,$$

from which the lemma follows because $N(r,1;f\mid=1) = N(r,1;g\mid=1)$. This proves the lemma. □

Lemma 2. *Let f, g share $(1,0)$, (∞,∞) and $H \not\equiv 0$. Then*

$$N(r,H) \le \overline{N}(r,0;f\mid\ge 2) + \overline{N}(r,0;g\mid\ge 2) + \overline{N}_*(r,1;f,g) + N_o(r,0;f') + \overline{N}_o(r,0;g')\,,$$

where $\overline{N}_o(r,0;f')$ is the reduced counting function of those zeros of f' which are not the zeros of $f(f-1)$ and $\overline{N}_o(r,0;g')$ is similarly defined.

Proof. One can easily verify that possible poles of H occur at (i) multiple zeros of f, g; (ii) zeros of $f-1$, $g-1$; (iii) zeros of f' which are not the zeros of $f(f-1)$; and (iv) zeros of g' which are not the zeros of $g(g-1)$.

Let z_o be a zero of $f-1$ and $g-1$ with multiplicities m and n respectively. Then in some neighbourhood of z_o we get

$$H(z) = \frac{(m-n)\phi(z)}{z - z_o} + \psi(z)\,,$$

where ϕ, ψ are analytic at z_o and $\phi(z_o) \ne 0$.

This shows that if $m = n$ then z_o is not a pole of H and if $m \ne n$ then z_o is a simple pole of H. Since all the poles of H are simple, the lemma is proved. □

Lemma 3. *If f, g share $(1,2)$ then*

$$\overline{N}_o(r,0;g') + \overline{N}(r,1;g\mid\ge 2) + \overline{N}_*(r,1;f,g) \le \overline{N}(r,\infty;g) + \overline{N}(r,0;g) + S(r,g)\,.$$

Proof. Remembering the definition of $\overline{N}_o(r,0;g')$ and noting that $\overline{N}_*(r,1;f,g) \le \overline{N}(r,1;g\mid\ge 3)$ because f, g share (1,2), we get

$$(1)\qquad \begin{aligned}\overline{N}_o(r,0;g') &+ \overline{N}(r,1;g\mid\ge 2) + \overline{N}_*(r,1;f,g) + N(r,0;g) - \overline{N}(r,0;g)\\ &\le \overline{N}_o(r,0;g') + \overline{N}(r,1;g\mid\ge 2) + \overline{N}(r,1;g\mid\ge 3)\\ &\quad + N(r,0;g) - \overline{N}(r,0;g)\\ &\le N(r,0;g')\,.\end{aligned}$$

By the first fundamental theorem and Milloux theorem ([4], p. 55)

$$
\begin{aligned}
(2) \qquad N(r,0;g') &\le N(r,0;\frac{g'}{g}) + N(r,0;g) - \overline{N}(r,0;g)\\
&\le N(r,\frac{g'}{g}) + N(r,0;g) - \overline{N}(r,0;g) + S(r,g)\\
&= \overline{N}(r,\infty;g) + \overline{N}(r,0;g) + N(r,0;g) - \overline{N}(r,0;g) + S(r,g)\\
&= \overline{N}(r,\infty;g) + N(r,0;g) + S(r,g)\,.
\end{aligned}
$$

Now the lemma follows from (1) and (2). This proves the lemma. □

Lemma 4. [9] *Let* $P(f) = \sum_{j=0}^{n} a_j f^j$, *where* $a_o, a_1, \dots, a_n (\not\equiv 0)$ *are such that* $T(r,a_j) = S(r,f)$ *for* $j = 0,1,2,\dots,n$. *Then*

$$T(r,P(f)) = nT(r,f) + S(r,f)\,.$$

Lemma 5. *If* f, g *share* $(\infty,0)$ *then for* $n \ge 2$

$$f^{n-1}(f+a)g^{n-1}(g+a) \not\equiv b^2\,,$$

where a, b *are finite nonzero numbers.*

Proof. If possible let

$$f^{n-1}(f+a)g^{n-1}(g+a) \equiv b^2\,. \tag{3}$$

If f and g have no pole, from (3) it follows that f has no zero and $-a$-point, which is impossible.

If z_o is a pole of f, by (3) it follows that z_o is either a zero or an $-a$-point of g and this contradicts the fact that f, g share $(\infty,0)$. This proves the lemma. □

Lemma 6. *If* $\Theta(\infty;f) + \Theta(\infty;g) > 1$ *then for* $n \ge 6$

$$f^{n-1}(f+a) \equiv g^{n-1}(g+a)$$

implies $f \equiv g$, *where* a *is a finite nonzero number.*

Proof. Let

$$f^{n-1}(f-1) \equiv g^{n-1}(g-1)\,. \tag{4}$$

and suppose $f \not\equiv g$. We consider two cases:

(a) $y = g/f$ is a constant. Then from (4) it follows that $y \ne 1$, $y^{n-1} \ne 1$, $y^n \ne 1$ and

$$f \equiv -a\frac{1-y^{n-1}}{1-y^n} = \text{constant},$$

which leads to a contradiction.

(b) $y = g/f$ is not a constant. We can rewrite $f \equiv -a\frac{1-y^{n-1}}{1-y^n}$ in the form

$$f \equiv a\left(\frac{y^{n-1}}{1+y+y^2+\dots+y^{n-1}} - 1\right)\,. \tag{5}$$

From (5) we get by the first fundamental theorem and Lemma 4

$$\begin{aligned} T(r,f) &= T(r,\sum_{j=0}^{n-1}\frac{1}{y^j}) + S(r,y) \\ &= (n-1)T(r,\frac{1}{y}) + S(r,y) \\ &= (n-1)T(r,y) + S(r,y)\,. \end{aligned}$$

Now we note that any pole of y does not contribute any pole of $\{y^{n-1}/\sum_{j=1}^{n-1} y^j\}-1$. So from (5) it follows that

$$\sum_{k=1}^{n-1}\overline{N}(r,u_k;y) \le \overline{N}(r,\infty;f)\,,$$

where $u_k = \exp(\frac{2k\pi i}{n})$, for $k = 1, 2, \ldots, n-1$.

By the second fundamental theorem we get

$$\begin{aligned} (n-3)T(r,y) &\le \sum_{k=1}^{n-1}\overline{N}(r,u_k;y) + S(r,y) \\ &\le \overline{N}(r,\infty;f) + S(r,y) \\ &< (1-\Theta(\infty;f)+\varepsilon)T(r,f) + S(r,y) \\ &= (n-1)(1-\Theta(\infty;f)+\varepsilon)T(r,y) + S(r,y)\,, \end{aligned} \tag{6}$$

where $\varepsilon(>0)$.

Again putting $y_1 = \frac{1}{y}$, noting that $T(r,y) = T(r,y_1) + O(1)$ and proceeding as above we get

$$(n-3)T(r,y) \le (n-1)(1-\Theta(\infty;g)+\varepsilon)T(r,y) + S(r,y)\,, \tag{7}$$

where $\varepsilon(>0)$.

From (6) and (7) we get in view of the given condition

$$\begin{aligned} 2(n-3)T(r,y) &\le (n-1)(2-\Theta(\infty;f)-\Theta(\infty;g)+2\varepsilon)T(r,y) + S(r,y) \\ &< (n-1)(1+2\varepsilon)T(r,y) + S(r,y)\,, \end{aligned}$$

which implies a contradiction for all sufficiently small $\varepsilon(>0)$ because $n \ge 6$.

Hence $f \equiv g$ and this completes the proof of the lemma. □

3. Proof of Theorem 1

Let $F = -\frac{1}{b}f^{n-1}(f+a)$ and $G = -\frac{1}{b}g^{n-1}(g+a)$. We first show that following inequality does not hold:

$$\begin{aligned} T(r) \le\ & N_2(r,0;F) + N_2(r,0;G) + \overline{N}(r,\infty;F) + \overline{N}(r,\infty;G) \\ & + S(r,F) + S(r,G)\,, \end{aligned} \tag{8}$$

where $T(r) = \max\{T(r,F), T(r,G)\}$.

By Lemma 4 we see that

$$(9) \quad T(r,F)=nT(r,f)+S(r,f) \quad \text{and} \quad T(r,G)=nT(r,g)+S(r,g)\,.$$

Now

$$\begin{aligned} &N_2(r,0;F)+N_2(r,0;G)+\overline{N}(r,\infty;F)+\overline{N}(r,\infty;G)+S(r,F)+S(r,G)\\ &\quad\le 2\overline{N}(r,0;f)+N_2(r,0;f+a)+2\overline{N}(r,0;g)+N_2(r,0;g+a)\\ &\qquad+\overline{N}(r,\infty;f)+\overline{N}(r,\infty;g)+S(r,F)+S(r,G)\\ &\quad< 3T(r,f)+3T(r,g)+\{1-\Theta(\infty;f)+\varepsilon\}T(r,f)\\ &\qquad+\{1-\Theta(\infty;g)+\varepsilon\}T(r,g)+S(r,F)+S(r,G)\,, \end{aligned}$$

where $\varepsilon(>0)$ is given.

In view of (9) and the given condition we get

$$\begin{aligned} &N_2(r,0;F)+N_2(r,0;G)+\overline{N}(r,\infty;F)+\overline{N}(r,\infty;G)+S(r,F)+S(r,G)\\ &\quad< \frac{1}{7}\{8-\Theta(\infty;f)-\Theta(\infty;g)+2\varepsilon\}T(r)+S(r,F)+S(r,G)\\ &\quad= (1-\alpha)T(r)+S(r,F)+S(r,G)\,, \end{aligned}$$

where $\alpha(>0)$ and $\varepsilon(>0)$ are so chosen that $7\alpha=\Theta(\infty;f)+\Theta(\infty;g)-1-2\varepsilon>0$. This shows that (8) does not hold. Let

$$H=\left(\frac{F''}{F'}-\frac{2F'}{F-1}\right)-\left(\frac{G''}{G'}-\frac{2G'}{G-1}\right).$$

We note that F, G share $(1,2)$ and (∞,∞) because $E_f(S,2)=E_g(S,2)$ and $E_f(\{\infty\},\infty)=E_g(\{\infty\},\infty)$.

Let $H\not\equiv 0$. Then by Lemma 1, Lemma 2 and Lemma 3 we obtain

$$\begin{aligned}(10)\qquad N(r,1;F\mid=1)&\le \overline{N}(r,0;F\mid\ge 2)+\overline{N}(r,0;G\mid\ge 2)+\overline{N}(r,\infty;G)\\ &\quad+\overline{N}(r,0;G)-\overline{N}(r,1;G\mid\ge 2)+\overline{N}_o(r,0;F')+S(r,G)\,.\end{aligned}$$

By the second fundamental theorem we get

$$\begin{aligned}(11)\qquad T(r,F)&\le \overline{N}(r,\infty;F)+\overline{N}(r,0;F)+\overline{N}(r,1;F)\\ &\quad-N(r,0;F')+S(r,F)\,.\end{aligned}$$

Since F, G share $(1,2)$ we see that

$$\begin{aligned}(12)\qquad \overline{N}(r,1;F)&=N(r,1;F\mid=1)+\overline{N}(r,1;F\mid\ge 2)\\ &=N(r,1;F\mid=1)+\overline{N}(r,1;G\mid\ge 2)\,.\end{aligned}$$

From (10), (11) and (12) we get

$$\begin{aligned}(13)\qquad T(r,F)&\le N_2(r,0;F)+N_2(r,0;G)+\overline{N}(r,\infty;F)\\ &\quad+\overline{N}(r,\infty;G)+S(r,F)+S(r,G)\,.\end{aligned}$$

Similarly we obtain

$$\begin{aligned}(14)\qquad T(r,G)&\le N_2(r,0;F)+N_2(r,0;G)+\overline{N}(r,\infty;F)\\ &\quad+\overline{N}(r,\infty;G)+S(r,F)+S(r,G)\,.\end{aligned}$$

We see that (13) and (14) together imply (8) which does not hold. Hence $H \equiv 0$ and so

$$\frac{F''}{F'} - \frac{2F'}{F-1} \equiv \frac{G''}{G'} - \frac{2G'}{G-1}$$

i.e.,

$$(\log F')' - (2\log(F-1))' \equiv (\log G')' - (2\log(G-1))' .$$

From this equation we get

$$F \equiv \frac{AG+B}{CG+D}, \tag{15}$$

where A,B,C,D are complex numbers such that $AD - BC \neq 0$.

From (15) it follows that

$$T(r,F) = T(r,G) + O(1) . \tag{16}$$

We now consider the following cases.

Case I Let $AC \neq 0$. Then

$$F - \frac{A}{C} \equiv \frac{B - \frac{AD}{C}}{CG+D}$$

and so by the second fundamental theorem we get

$$\begin{aligned} T(r,F) &\leq \overline{N}(r,0;F) + \overline{N}(r,\infty;F) + \overline{N}(r,A/C;F) + S(r,F) \\ &= \overline{N}(r,0;F) + \overline{N}(r,\infty;F) + \overline{N}(r,\infty;G) + S(r,F) . \end{aligned}$$

This by (16) implies (8) which does not hold.

Case II Let $AC = 0$. Since $AD - BC \neq 0$, it follows that A and C are not simultaneously zero.

Let $A = 0$. Then from (15) we get

$$G + \frac{D}{C} \equiv \frac{B}{CF}, \tag{17}$$

where $BC \neq 0$.

If $D \neq 0$, from (17) we get by the second fundamental theorem

$$\begin{aligned} T(r,G) &\leq \overline{N}(r,0;G) + \overline{N}(r,\infty;F) + \overline{N}(r,-D/C;G) + S(r,G) \\ &= \overline{N}(r,0;G) + \overline{N}(r,\infty;G) + \overline{N}(r,\infty;F) + S(r,G) . \end{aligned}$$

This by (16) implies (8) which does not hold.

Let $D = 0$. Then from (17) we get

$$FG \equiv \frac{B}{C} . \tag{18}$$

Since F, G share (∞,∞), it follows from (18) that F has no zero and pole. Hence there exists $z_o \in \mathbb{C}$ such that $F(z_o) = G(z_o) = 1$ because F, G share $(1,2)$. So from (18) we get $\frac{B}{C} = 1$ and so $FG \equiv 1$ i.e.

$$f^{n-1}(f+a)g^{n-1}(g+a) \equiv b^2$$

which is impossible by Lemma 5.

Let $C = 0$. Then from (15) we get

$$F \equiv \frac{A}{D}G + \frac{B}{D}\,, \tag{19}$$

where $AD \neq 0$.

If $B \neq 0$, from (19) we get by the second fundamental theorem

$$\begin{aligned} T(r,F) &\leq \overline{N}(r,0;F) + \overline{N}(r,\infty;F) + \overline{N}(r,B/D;F) + S(r,F) \\ &= \overline{N}(r,0;F) + \overline{N}(r,\infty;F) + \overline{N}(r,0;G) + S(r,F)\,. \end{aligned}$$

This by (16) implies (8) which does not hold.

Let $B = 0$. Then from (19) we get

$$F \equiv \frac{AG}{D}\,. \tag{20}$$

If F has no 1-point, by the second fundamental theorem we get

$$T(r,F) \leq \overline{N}(r,0;F) + \overline{N}(r,\infty;F) + S(r,F)\,.$$

This by (16) implies (8) which does not hold.

Let $F(z_o) = 1$ for some $z_o \in \mathbb{C}$. Since F, G share $(1,2)$, we get $G(z_o) = 1$ and so from (20) it follows that $\frac{A}{D} = 1$. Therefore $F \equiv G$ i.e.

$$f^{n-1}(f+a) \equiv g^{n-1}(g+a)$$

which implies by Lemma 6 that $f \equiv g$. This proves the theorem. □

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