Commentationes Mathematicae Universitatis Carolinae

Vladimír Puš

Algebraic and combinatorial properties of products [Abstract of thesis]

Commentationes Mathematicae Universitatis Carolinae, Vol. 30 (1989), No. 1, 200

Persistent URL: http://dml.cz/dmlcz/106732

In Chapter 3 we construct universal regular Stone spaces of given weight. As an application, we obtain universal regular topological spaces with bases whose each linked subsystem is filtered. We also solve Rosický's problem whether regular locales form an epireflective hull of regular spaces in the negative.

In Chapter 4 we construct (hyper) completion of uniform locales by means of generators and defining relations.

The Chapter 5 is devoted to connectedness. We present a short proof of a result of Moerdijk and Wraith that connected locally connected locales are arcwise connected. However, the main result of the chapter is to construct an example of two connected locales, whose product is not connected. This strengthens our joint result with A.Pultr.

In Chapter 6 we characterize LT–groups (a result from my earlier paper) and construct a non–trivial L–group with a single point.

The Chapter 7 is an appendix containing some constructions of technical nature.

ALGEBRAIC AND COMBINATORIAL PROPERTIES OF PRODUCTS

V.PUŠ, Výzkumný ústav lesního hospodářství a myslivosti, Jíloviště–Strnady
(24.8.1988, supervisors M.Vlach, J.Nešetřil)

In 1964, P.Erdös proved the following theorem (T)

(T) Suppose that $M \subseteq N$ is a set of positive integers such that the following condition holds: every positive integer can be expressed as the product $x \cdot y$ where $x, y \in M$. Then for every positive integer p there exists a positive integer n which can be expressed as the product of two elements of M in at least p different ways.

Erdös's proof of Theorem (T) was very complicated and had a purely number–theoretical character. Thus it gave no possibility to generalize Theorem (T) to other commutative semigroups besides the semigroup $(N, .)$. But, in 1985, J.Nešetřil and V.Rödl gave another proof of Theorem (T), based on the theorem of Ramsey, which was very simple and provided a straightforward possibility of generalization to other structures. In the presented thesis some general conditions on commutative semigroups are given, under which the analogy of Theorem (T) holds.

We also examine the semigroups for which the analogy of Theorem (T) does not hold. In particular, we examine the commutative semigroups $S = (X, .)$ containing a set M such that the following condition holds: every element $n \in X$ can be expressed, except for the order, in exactly one way as the product $x \cdot y$, where $x, y \in M$.

A particular attention is devoted to the study of the cardinal multiplication of simple graphs. It is proved that for the cardinal multiplication of simple graphs the analogy of Theorem (T) holds. We also study some related problems, namely the representations of commutative semigroups by products of simple graphs and the structure of irreducible decompositions of simple graphs.

Eventually we examine the chromatic number of products of graphs and the distances in products of graphs.