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Finite element approximation of nonlinear elliptic problems
with discontinuous coefficients

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ANNOUNCEMENTS OF NEW RESULTS

(of authors having an address in Czechoslovakia)

COHERENCE IMPLIES CONGRUENCE-REGULARITY (A LOCAL VERSION)

Jaromír Duda (616 00 Brno 16, Kroftova 21, Czechoslovakia, received 8.8 1988)

D.Geigher proved that any variety of coherent algebras is congruence-permutable and congruence-regular. D.M.Clark and I.Fleischer gave a local version of the first implication. We state that the second one can be sharpened in a similar way:

Theorem. Let A be an algebra. If $A \times A$ is coherent then A is congruence-regular.

Remark. Congruence-regularity on an algebra A does not follow from the coherence of A . Counterexample: any set of at least three elements all of which are constants.

FINITE ELEMENT APPROXIMATION OF NONLINEAR ELLIPTIC PROBLEMS WITH DISCONTINUOUS COEFFICIENTSM.Feistauer, V.Sobotíková (MFF UK, Sokolovská 83, 186 00 Praha 8, Czechoslovakia, received 15.9.1988) Submitted to M^2AN .

We deal with the finite element approximation of the equation

$$-\sum_{i=1}^2 \frac{\partial}{\partial x_i} a_i(x, u(x), \nabla u(x)) + a_0(x, u(x), \nabla u(x)) = f(x) \text{ in } \Omega,$$

considered together with mixed Dirichlet-Neumann conditions on the boundary $\partial\Omega$. The functions a_i ($i = 0, 1, 2$) and f are discontinuous across common parts of the boundaries of subregions of Ω , representing different materials which form the domain Ω . Such boundary value problems are met e.g. in the study of stationary magnetic fields, in heat conductivity processes and in the nuclear physics.

In the discretization, the domain is approximated by a polygonal domain Ω , which is triangulated in a suitable way, conforming piecewise linear elements are used and integrals are evaluated by numerical quadratures.

We prove the solvability of the discrete problem and study the convergence of the method both in strongly monotone and pseudomonotone cases under the only assumption that the exact solution $u \in H^1(\Omega)$. We apply similar techniques as in [2]. Provided u is piecewise of class H^2 and the problem is strongly monotone, we get the error estimate $O(h)$. In this case we use the "triple application of application of Green's theorem", proposed in [1].

REFERENCES

- [1] Feistauer M., *On the finite element approximation of a cascade flow problem*, Numer.Math. **50** (1987), 655-684.
- [2] Feistauer M., Ženíšek A., *Compactness method in the finite element theory of nonlinear elliptic problems*, Numer.Math. **52** (1988), 147-163.