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ADEQUATE FAMILIES OF SETS AND FUNCTION SPACES

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Abstract: Let X be an Eberlein-Grothendieck's space possessing the only one nonisolated point. In this paper we show that the space X is completely described in terms of adequate families of sets. As an application it is pròved that for this space X the space $C_p(X)$ is K-analytic (a Lindelöf Σ-space) iff X satisfies some property ($\mathcal{A}$) (is a Lindelöf Σ-space). Other applications concerning the space $C_p(K)$, where K is a Corson compact, are obtained.

Key words: Eberlein-Grothendieck's space, adequate family of sets, $\mathcal{K}$-analytic space, Lindelöf Σ-space.

Classification: 54C40

Introduction. Recall the following definition [1],[6].

Definition. Let T be a set. A family $\mathcal{A}$ of its subsets is called adequate if it satisfies the following conditions:

i) $\mathcal{A}$ contains all one-point subsets of T;

ii) A subset A of T belongs to $\mathcal{A}$ iff every finite subset of A belongs to $\mathcal{A}$.

Put $X=X_{\mathcal{A}}=\{\chi_A : A\in\mathcal{A}\}\subset\mathcal{D}^T$,

where χ_A is the characteristic function of A. The space X is closed in $\mathcal{D}^T$, hence X is a compact. We shall call X an adequate compact.

Using this simple idea, the number of concrete examples of Corson compacts are obtained now. Namely, it is shown that all classes of Corson, Gul'ko, Talagrand, Eberlein and uniform Eberlein compacts are strictly different (cf. [1],[2],[3],[6]). Moreover, an adequate Corson compact which has no dense metrizable subspaces is constructed [7].

Thus, the notion of an adequate family of sets is applied as a source of counterexamples.

On the other hand, M. Bell [4] studied the inner topological properties of an arbitrary centered compact which is a continuous image of an adequate compact. He proved that many important properties of dyadic compacts are preserved for the class of centered compacts.

In this paper we show that adequate families of sets are arised naturally in the studying of Eberlein-Grothendieck's spaces X possessing the only one nonisolated point (Proposition 1). As an application it is proved that for this simplest space X the space $C_p(X)$ is $\mathcal{K}$-analytic (a Lindelöf Σ-space) iff X satisfies some property ($\mathcal{A}$) (is a Lindelöf Σ-space) (Theorems 2, 3).

Note that the set satisfying the property ($\mathcal{A}$) is similar to the classical coanalytic set.

Other applications are concerned with the space $C_p(K)$, where K is a Corson compact. In particular, if K is a Corson compact, then there exists a subspace $Y \subset C_p(K)$ which separates points of K and is described in terms of adequate families of sets (Proposition 4).

Terminology and notation. Our terminology is standard. The symbol ω stands for the set of natural numbers; R is the real line; $|T|$ denotes the cardinality of a set T; $I=[0,1]$ is the closed segment; $\mathfrak{D}=\{0,1\}$ stands for the two-point discrete space.

For spaces X, Y we denote by $C_p(X,Y)$ the space of all continuous functions on X to Y endowed with the pointwise topology. If Y=R, we use the symbol $C_p(X)$.

Recall that the Corson compact is a compact subspace of

$$\Sigma(R,T)=\{x \in R^T : |\text{supp } x| \leq \aleph_0\},$$

where supp $x=\{t \in T : x(t) \neq 0\}$.

The space X is called an Eberlein-Grothendieck's space (EG-space) if $X \subset C_p(Y)$ for some compact Y [9].

In this paper the symbol Σ stands for the set of all infinite sequences of natural numbers ω^ω ; $S=\omega^{<\omega}$ consists of finite sequences. For $s \in S$, $\sigma \in \Sigma$ we write $s \prec \sigma$ if s is an initial segment of σ.

A completely regular space Z is $\mathcal{K}$-analytic if for some compact $K \supset Z$ there exists the family of compacts $\{F_s : s \in S\}$, $F_s \subset K$ such that

$$Z=\bigcup_{\sigma \in \Sigma} \bigcap_{s \prec \sigma} F_s.$$

If Σ is replaced by any $\Sigma' \subset \Sigma$, we obtain the definition of a Lindelöf Σ-space.

We shall use the notion of the perfect class of spaces. The class $\mathcal{P}$ of spaces is $\aleph_0$-perfect if it is closed under the operations of countable products, continuous images and closed subspaces. Consequently, $\mathcal{P}$ is closed under countable unions and intersections [8].

Both classes of $\mathcal{K}$-analytic spaces and Lindelöf Σ-spaces are $\aleph_0$-perfect.

If $X=\bigcup\{X_n:n\in\omega\}$, where each X_n is a compact, then X has the type K_σ; if $X=\bigcap\{Y_n:n\in\omega\}$, where each Y_n has the type K_σ, then X has the type $K_{\sigma\delta}$.

Results. Throughout the paper $X=T\cup\{*\}$ is the space in which all points $t\in T$ are isolated. Put $J=\{F\subset T$: F is closed in X$\}$. Evidently, the topology of X is completely characterized by the ideal J. If the ideal J has a base which is an adequate family, then we shall say that X is a space generated by an adequate family of sets or X is an adequate space.

Proposition 1. Let $X=T\cup\{*\}$ be a space possessing the only one nonisolated point $*$. Then X is an EG-space if and only if the ideal J is a countable union of adequate families.

Proof: (if). By V.V. Uspenskii's theorem [9], X is an EG-space iff the space $C_p^0(X,\mathfrak{D})=\{f\in C_p(X,\mathfrak{D}): f(*)=0\}$ has the type K_σ. Assume that $J=\bigcup\{\mathcal{Q}_n:n\in\omega\}$, where each $\mathcal{Q}_n$ is an adequate family. Then $Y_n=\{\chi_A:A\in\mathcal{Q}_n\}$ is a compact and clearly $C_p^0(X,\mathfrak{D})=\bigcup\{Y_n:n\in\omega\}$.

(only if). Suppose that $C_p^0(X,\mathfrak{D})=\bigcup\{Y_n:n\in\omega\}$, where each Y_n is a compact. Without loss of generality we can assume that the compact $\{\chi_{\{t\}}:t\in T\}\cup\{0\}$ lies in each Y_n.

Put $\mathcal{Q}_n=\{A\subset T:\exists\,\chi_B\in Y_n,\ A\subset B\}$.

Obviously, $J=\bigcup\{\mathcal{Q}_n:n\in\omega\}$. To prove that $\mathcal{Q}_n$ is an adequate family it is enough to check the following condition: if $B\subset T$ is such that $M\in\mathcal{Q}_n$ for any finite $M\subset B$, then $B\in\mathcal{Q}_n$. For any finite $M\subset B$ put $U_M=\{f\in Y_n: f|_M\equiv 1\}$. Then U_M is a closed subspace of Y_n and since $\chi_{M'}\in Y_n$ for some $M'\supset M$, we conclude that the family $\xi=\{U_M:M\subset B, |M|<\aleph_0\}$ is centered. Therefore, $\bigcap\xi\neq\emptyset$. If $\chi_C\in\bigcap\xi$, then $B\subset C$, i.e. $B\in\mathcal{Q}_n$. The proof is finished.

Theorem 2. Let $X=T\cup\{*\}$ be an EG-space possessing the only one nonisolated point $*$. Then $C_p(X)$ is $\mathcal{K}$-analytic if and only if X satisfies the following property ($\mathcal{A}$):

there exists a countable family of subsets $\{T_s:s\in S\}$, $T_s\subset T$ such that

i) $T_{s_1} \subset T_{s_2}$ if $s_1 \prec s_2$;

ii) $\bigcup\{T_s : s \prec \sigma\} = T$ for any $\sigma \in \Sigma$;

iii) if U is a neighbourhood of $*$ in X, then $|T_s \cap (X \setminus U)| < \aleph_0$ for some $\sigma \in \Sigma$ and every $s \prec \sigma$.

Proof: First, we show the necessity. It is easy to check that the space $C_p(X)$ is homeomorphic to the following space

$Y = C_p^0(X,(-1,1)) = \{f \in C_p(X,(-1,1)) : f(*) = 0\}$.

Y lies naturally in compact I^T, consequently, by the $\mathcal{K}$-analyticity of Y, there exists a countable family $\{F_s : s \in S\}$ consisting of compacts $F_s \subset I^T$ such that $F_{s_2} \subset F_{s_1}$, if $s_1 \prec s_2$ and $Y = \bigcup_{\sigma \in \Sigma} \bigcap_{s \prec \sigma} F_s$ (cf. [10]). Denote by $U_t = \{f \in I^T : |f(t)| < 1\}$. Put $T_s = \{t \in T : F_s \subset U_t\}$.

Then the family $\{T_s : s \in S\}$ is as desired. The condition i) is evidently fulfilled. Let us prove ii).

By the definition, $\bigcap\{F_s : s \prec \sigma\} \subset Y \subset U_t$ holds for any $\sigma \in \Sigma$, $t \in T$. Since U_t is open in I^T, and F_s, $s \in S$ are compact in I^T, we get that $F_s \subset U_t$ for some $s \prec \sigma$, so $t \in T_s$.

To show iii) assume the contrary: there exists a neighbourhood U of $*$ in X such that for any $\sigma \in \Sigma$ there exists $s(A) \prec \sigma$ for which the set $T_{s(A)} \cap A$ is infinite, where $A = X \setminus U$. It follows easily that the set $\pi_t(F_s) = \{f(t) : f \in F_s\}$ is a compact lying in $(-1,1)$, therefore $\pi_t(F_s) \subset (-\varphi(t,s), \varphi(t,s))$ for some $\varphi(t,s) \in (0,1)$. Renumbering all T_s, for which $|T_s \cap A| \geq \aleph_0$ holds, by $C_1, C_2, \ldots$, we put $A_n = C_n \cap A$. Applying the infinity of A_n, choose by the induction the sequence $\{t_n : n \in \omega\} \subset T$ such that $t_1 \in A_1$, $t_n \in A_n \setminus \{t_1, \ldots, t_{n-1}\}$ for every $n \in \omega$. As it has been noted, for any $n \in \omega$ there exists $\varphi(n,s) \in (0,1)$ such that $|f(t_n)| < \varphi(n,s)$ for every $f \in F_s$, where $C_n = T_s$. Since A is a closed discrete subset of X, and $B = \{t_n : n \in \omega\} \subset A$, then the function f, defined by $f(t_n) = \varphi(n,s)$, $f|_{X \setminus B} \equiv 0$, is contained in Y.

Clearly, $f \in \bigcap\{F_s : s \prec \sigma\}$ for some $\sigma \in \Sigma$. By assumption, there exists $s = s(A) \prec \sigma$ such that the set $T_s \cap A$ is infinite, i.e. $T_s = C_n$ for some $n \in \omega$. Finally, since $t_n \in T_s$, it follows that $|f(t_n)| < \varphi(n,s)$, which is a contradiction.

Remark 1. We emphasize that the assumption that X is an EG-space is not used in this reasoning, so this assumption may be omitted in the direct implication.

Let us prove the converse implication. By Proposition 1 there exists a sequence of adequate families $\{\mathcal{A}_n : n \in \omega\}$ such that any subset $A \subset T$ is closed in X iff $A \in \mathcal{A}_n$ for some $n \in \omega$. We can assume that $\mathcal{A}_n \subset \mathcal{A}_{n+1}$ for every $n \in \omega$.

Once more note that $C_p(X) \approx Y = C_p^0(X,(-1,1))$. Put

$Y_n = \{f \in Y : \text{supp } f \in \mathcal{A}_n\}$, $Z = \bigcup\{Y_n : n \in \omega\}$.

Then Z is uniformly dense in Y in the following sense: for any $f \in Y$, $\varepsilon > 0$ there exists $g \in Z$ such that $|f(t)-g(t)| < \varepsilon$ for each $t \in T$.

Indeed, $f(*)=0$ and the set $A = X \setminus f^{-1}(-\varepsilon,\varepsilon)$ lies in T and is closed in X, therefore $A \in \mathcal{A}_n$ for some $n \in \omega$ and $g = f \times \chi_A \in Y_n$ is as required.

It suffices to prove that Z is a $\mathcal{K}$-analytic space. To prove this claim we shall use a reasoning of A. Arhangel'skii [8]. Consider the set

$M_n = \{f \in I^T : \exists g \in Z, |f(t)-g(t)| \leq \frac{1}{n} \ \forall t \in T\}$.

Then M_n is a continuous image of $Z \times [-\frac{1}{n}, \frac{1}{n}]$. On the other hand, since the limit of the uniformly converging sequence of continuous functions is a continuous function, the uniform density of Z in Y yields that $Y = \bigcap\{M_n : n \in \omega\}$. Thus, by $\aleph_0$-perfectness of the class of $\mathcal{K}$-analytic spaces, we conclude the claim.

Finally, our proof will be finished if we show that each Y_n is $\mathcal{K}$-analytic.

Put $K_n = \{f \in I^T : \text{supp } f \in \mathcal{A}_n\}$.

Then K_n is a compact. Indeed, let $g \in I^T \setminus K_n$ and C=supp g. Since $C \notin \mathcal{A}_n$, applying the definition of the adequate family, we get that $B \notin \mathcal{A}_n$ for some finite $B \subset C$. Consider $U_B = \prod_{t \in T} U_t$, where $U_t = I \setminus \{0\}$, if $t \in B$ and $U_t = I$, if $t \notin B$. Then U_B is an open neighbourhood of g, and $U_B \subset I^T \setminus K_n$ i.e., K_n is a closed subspace of I^T.

Let $T^* = T \cup \{*\}$ be the adequate space generated by the adequate family $\mathcal{A}_n$. It is clear that the topology of T^* is contained in the topology of X, therefore T^* satisfies the property ($\mathcal{A}$). We assume that the sequence $\{T_s : s \in S\}$ is a witness of this fact. For each $s \in S$ and $m \in \omega$ we define

$F_{s,m} = \{f \in K_n : |f(t)| \leq 1 - \frac{1}{m} \ \forall t \in T_s\}$, $F_s = \bigcup\{F_{s,m} : m \in \omega\}$.

Obviously, $F_{s,m}$ is a compact; F_s has a type K_σ, so it is enough to prove that $Y_n = \bigcup_{\sigma \in \Sigma} \bigcap_{s \triangleleft \sigma} F_s$.

For $f \in Y_n$ there exists $\sigma \in \Sigma$ such that $|\operatorname{supp} f \cap T_s| < \aleph_0$ for each $s \prec \sigma$. Consequently, $f \in F_{s,m(s)}$ for some $m(s) \in \omega$, and $f \in \bigcap\{F_s : s \prec \sigma\}$. If $g \in K \setminus Y_n$ then $|g(t)|=1$ for some $t \in T$. For any $\sigma \in \Sigma$ there exists $s \prec \sigma$ such that $t \in T_s$, hence $g \notin F_{s,m}$ for each $m \in \omega$ and $g \notin \bigcup_{\sigma \in \Sigma} \bigcap_{s \prec \sigma} F_s$. The proof is finished.

It is easy to see that the space X satisfying the property ($\mathcal{A}$) is a Lindelöf Σ-space. Moreover, the slight modification of the previous proof allows to obtain the following result.

Theorem 3. Let X be an EG-space possessing the only one nonisolated point. Then $C_p(X)$ is a Lindelöf Σ-space if and only if X is a Lindelöf Σ-space.

Remark 2. As in Theorem 2, the necessity is valid without assumption that X is an EG-space.

Example 1. There exists a space X such that $C_p(X,I)$ has the type $K_{\sigma\delta}$ but $C_p(X)$ is not even a Lindelöf Σ-space.

Let $X=T \cup \{*\}$ be any adequate space generated by the adequate family $\mathcal{O}$. Denote by $\mathcal{O}_n$ the adequate family consisting of all finite less than n unions of elements of $\mathcal{O}$. Put $Y_n = \{f \in C_p(X,I) : T \cap \operatorname{supp} f \in \mathcal{O}_n\}$, $n \in \omega$.

Repeating the reasoning of Theorem 2 we can prove that each Y_n is a compact and $Y=\bigcup\{Y_n : n \in \omega\}$ is uniformly dense in $C_p(X,I)$. Therefore, $C_p(X,I)$ has the type $K_{\sigma\delta}$. But taking any X which is not a Lindelöf Σ-space (cf. [6]), applying Theorem 2, we obtain that $C_p(X)$ is not a Lindelöf Σ-space, too.

There exists an adequate space which satisfies the property ($\mathcal{A}$) but is not $\mathcal{K}$-analytic.

Example 2. We shall use the example of M. Talagrand [2]. Let Φ be the set of all finite strictly increasing sequences on ω. We define on Φ the usual order: $s \leq t$ iff $n \leq m$ and $s_i = t_i$ for all $i \leq n$, where $s=(s_1,\dots,s_n)$, $t=(t_1,\dots,t_m)$.

Denote by T_0 the set of all trees $X \subset \Phi$ which satisfy the following property: if $t \in X$ and $s \leq t$ then $s \in X$. We shall identify T_0 with the set of all characteristic functions of its elements. It is easy to check that T_0 is closed in $\mathfrak{D}^{\Phi}$, hence T_0 is a compact metric space. Let $T_1 \subset T_0$ be the set of trees containing an infinite branch. It is known that T_1 is an analytic set [2].

Given a tree X we denote by $V_n(X)$ the set of trees Y such that $X \cap \Phi_n =$

$=Y \cap \Phi_n$, where Φ_n is the set of finite increasing sequences of integers less than or equal to n. The sets $V_n(X)$ form a basis of neighbourhoods of X.

Let $\mathcal{A}_0$ be the set of finite subsets $B \subset T_0$ which are of the following type: B can be expressed as $\{Y_1^0, \dots, Y_n\}$, where for some $X \in T_0$ and $(s_1, \dots, s_n) \in X$, we have $Y_i \in V_{s_i}(X)$ for all $i \leq n$.

We denote by $\mathcal{A}_1$ the smallest adequate family which contains $\mathcal{A}_0$. Finally, $T = T_0 \setminus T_1$, $\mathcal{U} = \{A \subset T : A \in \mathcal{A}_1\}$.

It is shown in [2] that the adequate space T^* generated by the adequate family $\mathcal{U}$ is a Lindelöf Σ-space but is not $\mathcal{K}$-analytic.

We claim that T^* satisfies the property $(\mathcal{A})$. In order to prove this fact we shall use two lemmas, the first of them is proved by M. Talagrand [2].

Lemma 1. Let $A \in \mathcal{A}_1$. Then each limit point of A belongs to T_1.

Lemma 2. Let $A \in \mathcal{A}_1$ be an infinite set. Then A has the only one limit point.

Proof: Assume on the contrary that $A \in \mathcal{A}_1$ has two distinct limit points X and Y. Since the sequence $\{V_n(Z) : n \in \omega\}$ forms a basis of neighbourhoods of Z, there exists $m \in \omega$ such that $V_m(X) \cap V_m(Y) = \emptyset$. Let us note the next fact: for any point $Z \in T_0$, $V_m(X) \cap V_m(Z) \neq \emptyset$ and $V_m(Y) \cap V_m(Z) \neq \emptyset$ do not hold şimultaneously, otherwise, $Z \in V_m(X) \cap V_m(Y)$. Let $\{X_i : i \in \omega\} \subset V_m(X)$, $\{Y_i : i \in \omega\} \subset V_m(Y)$ be two sequences converging to X and Y respectively. Consider the set $C = \{X_1, Y_1, \dots, X_m, Y_m\}$. Since $C \in \mathcal{A}_1$, there exists $B \in \mathcal{A}_0$ such that $C \subset B$. If $B = \{Z_1, \dots, Z_k\}$, then by the definition of $\mathcal{A}_0$, there exist $Z \in T_0$ and an element $(s_1, \dots, s_k) \in Z$ such that $Z_i \in V_{s_i}(Z)$ for all $i \leq k$. One can assume that $Y_i = Z_{g(i)}$, $X_i = Z_{p(i)}$ for some $g(i), p(i) \leq k$.

Clearly, the collection $\{p(i), g(i)\}$ consists of pairwise different points. Consequently, applying the definition of $(s_1, \dots, s_k)$ as a strictly increasing sequence of integers, we conclude that there exist indexes i and j such that $s_{p(i)} \geq m$, $s_{g(j)} \geq m$. So,

$$X_i = Z_{p(i)} \in V_{s_{p(i)}}(Z) \subset V_m(Z);$$

$$Y_j = Z_{g(j)} \in V_{s_{g(j)}}(Z) \subset V_m(Z),$$

therefore, $V_m(Z) \cap V_m(X) \neq \emptyset$, $V_m(Z) \cap V_m(Y) \neq \emptyset$, which is impossible as it has been noted.

Let us prove that T^* satisfies the property $(\mathcal{A})$. We know that T_1 is analytic. Let $\mathcal{P}(T_1)_V$ be the set of all nonempty finite subsets of T_1 endowed by Vietoris topology. Clearly, $\mathcal{P}(T_1)_V$ is the continuous image of $\bigoplus_{n\in\omega} T_1^n$ under the mapping $j=\bigoplus_{n\in\omega} j_n$, where

$$j_n((x_1,\dots,x_n))=\{x_1,\dots,x_n\}.$$

The class of analytic sets is invariant under the operations of countable unions and continuous images, therefore $\mathcal{P}(T_1)_V$ is analytic. Moreover, from the classical results of the descriptive theory we conclude that there exists a countable family $\mathcal{P}(T_0)_V$ of open sets in $\{U_s:s\in S\}$ such that $U_{s_1}\subset U_{s_2}$ if $s_2\prec s_1$ and $\mathcal{P}(T_1)_V=\bigcup_{\sigma\in\Sigma}\bigcap_{s\prec\sigma} U_s$ [10]. One can assume that each U_s is of the standard form, that is, $U_s=\{B\in\mathcal{P}(T_0):B\subset\bigcup_{i=1}^{m} U_i,\ B\cap U_i\neq\emptyset\ \forall i\leq n\}$, where U_i is open in T_0. Put $V_s=\bigcup_{i=1}^{m} U_i$. Then V_s is open in T_0, too. It is easy to see that $T_1=\bigcup_{\sigma\in\Sigma}\bigcap_{s\prec\sigma} V_s$, besides $V_{s_1}\subset V_{s_2}$ if $s_2\prec s_1$, and for any finite $B\subset T_1$ there exists $\sigma\in\Sigma$ such that $B\subset\bigcap\{V_s:s\prec\sigma\}$.

We denote by $T_s=T\setminus V_s$ for each $s\in S$. Let us prove that the family $\{T_s:s\in S\}$ is as required. The condition i) of the property $(\mathcal{A})$ is evident; $T\cap\bigcap\{V_s:s\prec\sigma\}=\emptyset$ yields the condition ii). To show iii), suppose that $A\subset T$ is infinite and closed in T^*. It follows that $A=\bigcup_{i=1}^{k} A_i$, where $A_i\in\mathcal{A}$. Applying Lemmas 1, 2, we get that each A_i has the only limit point which belongs to the set T_1. Denote by B the set of all limit points of A. Then B is finite and $B\subset T_1$. There is $\sigma\in\Sigma$ such that $B\subset\bigcap\{V_s:s\prec\sigma\}$. The set V_s is a neighbourhood of B for any $s\prec\sigma$, therefore $A\setminus V_s$ is finite. This means that $|A\cap T_s|<\aleph_0$ for any $s\prec\sigma$.

The next problem is arised naturally.

Problem. Does there exist an adequate space which is a Lindelöf Σ-space but does not satisfy the property $(\mathcal{A})$?

G.A. Sokolov [5] proved that for any Corson compact K there exists a subspace $Y\subset C_p(K)$ which separates points of K and has some special structure. Our Proposition 1 gives a new more detailed information. The following result shows that any Corson compact in some sense can be studied by adequate compacts.

Proposition 4. Let K be a Corson compact. Then there exists a subspace $Y\subset C_p(K)$ which separates points of K and is of the following form:

$Y=\bigcup\{Y_n : n\in\omega\}$, where $Y_n^* = Y_n \cup \{0\}$ is closed in $C_p(K)$, Y_n consists of isolated points in Y_n^*, and there is a sequence of adequate families $\{\mathcal{A}_{n,m} : m\in\omega\}$ generating the topology of Y_n^* for each $n\in\omega$.

We shall establish an application of our results.

Theorem 5. Let K be a Corson compact. If $C_p(C_p(K))$ is a Lindelöf Σ-space, then $C_p(K)$ is a Lindelöf Σ-space, too.

Proof: First, we find the space $Y\subset C_p(K)$ and adequate families $\{\mathcal{A}_{n,m}\}$ as in Proposition 4. Then, by Theorem 3, we get that Y is a Lindelöf Σ-space, consequently $C_p(K)$ is a Lindelöf Σ-space, too.

Remark. All the results of this paper are obtained in the year 1984. Recently I was informed by O. Okunev that he had proved some strong strengthening of Theorem 3 and had shown that Theorem 5 is valid for any compact (to appear).

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