

Petr Lachout

Convergence criterion for multiparameter stochastic processes

Commentationes Mathematicae Universitatis Carolinae, Vol. 28 (1987), No. 4, 783

Persistent URL: <http://dml.cz/dmlcz/106587>

Terms of use:

© Charles University in Prague, Faculty of Mathematics and Physics, 1987

Institute of Mathematics of the Academy of Sciences of the Czech Republic provides access to digitized documents strictly for personal use. Each copy of any part of this document must contain these *Terms of use*.



This paper has been digitized, optimized for electronic delivery and stamped with digital signature within the project *DML-CZ: The Czech Digital Mathematics Library* <http://project.dml.cz>

ANNOUNCEMENTS OF NEW RESULTS

(of authors having an address in Czechoslovakia)

CONVERGENCE CRITERION FOR MULTIPARAMETER STOCHASTIC PROCESSES

Petr Lachout (MFF UK, Sokolovská 83, 18600 Praha 8, Czechoslovakia), received 1.7. 1987)

Bickel and Wichura [1] extended the tightness criterion from processes on $D(0,1)$ (see Billingsley [2]) to processes on $D(0,1)^k$, $k > 1$. However, they impose an additional condition that the processes should vanish along the lower boundary of $\langle 0,1 \rangle^k$. This means that their criterion does not apply to many empirical processes of interest.

We shall provide an improved tightness criterion for processes in $D(0,1)^k$ without the above additional condition.

Definition: Let $k \in \mathbb{N}$, $d=1, \dots, k$, $j=0, \dots, k-d$, $\varphi: \langle 0,1 \rangle^k \rightarrow \langle 0,1 \rangle^k$ be a permutation of coordinates and $X=(X(t), t \in \langle 0,1 \rangle^k)$ be a random process. Define

$$(1) \Delta X(d,j,\varphi)(\prod_{i=1}^d \langle a_i, b_i \rangle) = \sum_{i=1}^d \delta_{i=a_i, b_i} (-1)^{\sum_{p=1}^i (\delta_p = a_p)} X \circ \varphi(\sigma_1, \dots, \sigma_d, 0, \dots, 0, \underbrace{1, \dots, 1}_{j \text{ times}})$$

for every $0 \leq a_i < b_i \leq 1$, $i=1, \dots, d$.

We shall prove the following theorem.

Theorem: Let $X=(X(t), t \in \langle 0,1 \rangle^k)$, $k \in \mathbb{N}$, be a random process right-continuous in every coordinate. Let $\mu_{d,j,\varphi}$, $d=1, \dots, k$, $j=0, \dots, k-d$ and $\varphi: \langle 0,1 \rangle^k \rightarrow \langle 0,1 \rangle^k$ being a permutation of coordinates, be a bounded measure with continuous marginals.

If there exists $\alpha, \beta > 0$ such that

$$(2) P(|\Delta X(d,j,\varphi)(A)| > y, |\Delta X(d,j,\varphi)(B)| > y) \leq y^{-\alpha} \mu_{d,j,\varphi}(A \cup B)^{1+\beta}$$

holds for every $y > 0$, $d=1, \dots, k$, $j=0, \dots, k-d$ and every permutation φ and for all

$$A = \prod_{i=1}^d \langle a_i, b_i \rangle, B = \prod_{i=1}^d \langle g_i, h_i \rangle,$$

$A \cap B = \emptyset$, $\text{clo } A \cap \text{clo } B \neq \emptyset$, then there exist an absolute constant $Q > 0$ and a function $R: \langle 0,1 \rangle \rightarrow \langle 0,1 \rangle$, $\lim_{\varepsilon \rightarrow 0_+} R(\varepsilon) = 0$, such that

$$(3) P(\sup \{ \min |X \circ \varphi(t,u) - X \circ \varphi(s,u)|, |X \circ \varphi(s,u) - X \circ \varphi(v,u)| \} | 0 \leq t < s < v \leq 1, v-t \leq \varepsilon, u \in \langle 0,1 \rangle^{k-1}, \varphi \text{ is a permutation of coordinates} \} > y) \leq Q y^{-\alpha} R(\varepsilon) \text{ for every } \varepsilon \in \langle 0,1 \rangle.$$

If $k=1$ then the criterion (2) reduces to the criterion in Billingsley [2] (see Theorem 15.6) while it is an improvement of the criterion of [1] if $k > 1$.

References:

[1] Bickel P.J., Wichura M.S.: Convergence criteria for multiparameter stochastic processes and some applications, The Annals of Mathematical Statistics 42(1971), 1656-1670.
 [2] Billingsley P.: Convergence of Probability Measures, John Wiley, New York, 1968.