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Convergence criterion for multiparameter stochastic processes

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ANNOUNCEMENTS OF NEW RESULTS

(of authors having an address in Czechoslovakia)

CONVERGENCE CRITERION FOR MULTIPARAMETER STOCHASTIC PROCESSES

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Bickel and Wichura [1] extended the tightness criterion from processes on $D(0,1)$ (see Billingsley [2]) to processes on $D(0,1)^k$, $k>1$. However, they impose an additional condition that the processes should vanish along the lower boundary of $\langle 0,1\rangle^k$. This means that their criterion does not apply to many empirical processes of interest.

We shall provide an improved tightness criterion for processes in $D(0,1)^k$ without the above additional condition.

Definition: Let $k\in N$, $d=1,\dots,k$, $j=0,\dots,k-d$, $\varphi:\langle 0,1\rangle^k \to \langle 0,1\rangle^k$ be a permutation of coordinates and $X=(X(t),\ t\in\langle 0,1\rangle^k$ be a random process. Define

$$(1)\quad \Delta X(d,j,\varphi)\left(\mathop{X}_{i=1}^{d}\langle a_i,b_i)\right)= \sum_{i=1}^{d}\sum_{\delta_i=a_i,b_i}(-1)^{\sum_p I(\delta_p=a_p)} X\circ\varphi(\delta_1,\dots,\delta_d,0,\dots,0,\underbrace{1,\dots,1}_{j\times})$$

for every $0\le a_i<b_i\le 1$, $i=1,\dots,d$.

We shall prove the following theorem.

Theorem: Let $X=(X(t),\ t\in\langle 0,1\rangle^k)$, $k\in N$, be a random process right-continuous in every coordinate. Let $\mu_{d,j,\varphi}$, $d=1,\dots,k$, $j=0,\dots,k-d$ and $\varphi:\langle 0,1\rangle^k \to \langle 0,1\rangle^k$ being a permutation of coordinates, be a bounded measure with continuous marginals.

If there exists $\alpha,\beta>0$ such that

$$(2)\quad P(|\Delta X(d,j,\varphi)(A)|>y,\ |\Delta X(d,j,\varphi)(B)|>y)\le y^{-\alpha}\mu_{d,j,\varphi}(A\cup B)^{1+\beta}$$

holds for every $y>0$, $d=1,\dots,k$, $j=0,\dots,k-d$ and every permutation φ and for all

$$A=\mathop{X}_{i=1}^{d}\langle a_i,b_i),\quad B=\mathop{X}_{i=1}^{d}\langle g_i,h_i),$$

$A\cap B=\emptyset$, $\operatorname{clo}A\cap\operatorname{clo}B\neq\emptyset$, then there exist an absolute constant $Q>0$ and a function $R:(0,1\rangle\to\langle 0,1\rangle$, $\lim_{\varepsilon\to 0_+}R(\varepsilon)=0$, such that

$$(3)\quad P(\sup\{\min\{|X\circ\varphi(t,u)-X\circ\varphi(s,u)|,\ |X\circ\varphi(s,u)-X\circ\varphi(v,u)|\}\ |\ 0\le t<s<v\le 1,\ v-t<\varepsilon,\ u\in\langle 0,1\rangle^{k-1},\ \varphi \text{ is a permutation of coordinates}\} >y)\le Qy^{-\alpha}R(\varepsilon)\quad \text{for every } \varepsilon\in(0,1\rangle.$$

If $k=1$ then the criterion (2) reduces to the criterion in Billingsley [2] (see Theorem 15.6) while it is an improvement of the criterion of [1] if $k>1$.

References:

[1] Bickel P.J., Wichura M.S.: Convergence criteria for multiparameter stochastic processes and some applications, The Annals of Mathematical Statistics 42(1971), 1656-1670.

[2] Billingsley P.: Convergence of Probability Measures, John Wiley, New York, 1968.