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$\{y \mid g^{-1}y \subset \cup \mathcal{U}_n\} = \cup\{\{y \mid g^{-1}y \subset U\} \mid U \in \mathcal{U}_n\}$.

3. <u>Theorem 3</u>. The following conditions on a space X are equivalent:

(2a) Some Čech complete subspace of $X \times \Sigma$ injectively projects onto X.

(2b) If X is a subspace of Z then $X \in \mathcal{S}_d(\mathrm{Borel}(Z))$.

(2c) X is obtained by the disjoint Suslin operation from locally compact subsets in some $Z \supset X$.

(2d) There exists a complete sequence $\{\cup\{\mathcal{M}_s \mid s \in \omega^n\} \mid n \in \omega\}$ of covers such that each $\mathcal{M}_s$ is an open cover of $M_s = \cup \mathcal{M}_s$, $M_s = \cup\{M_{si} \mid i \in \omega\}$ for each s, and if $\sigma \in \Sigma$, $M_n \in \mathcal{M}_{\sigma|n}$ then $\cap\{\overline{\cap\{M_i \mid i \leq n\}} \mid n \in \omega\} \in \cap\{M_{\sigma|n} \mid n \in \omega\}$.

A space satisfying the equivalent condition in Theorem 3 will be called Čech-Luzin. Any Čech-Luzin space X is absolutely bi-Suslin (Borel), and I do not know whether or not the converse holds.

The basic stability results follow easily from (ia) and the fact that any countable ($\neq 0$) power of Σ is homeomorphic to Σ.

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DISTINGUISHED SUBCLASSES OF ČECH-ANALYTIC SPACES

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This is a free continuation of $[F_3]$. Recall that if $\mathcal{F}$ is a set of families of subsets of X then a family $\{X_a \mid a \in A\}$ in X is called $\mathcal{F}$ σ-decomposable if there exist families $\{X_{an} \mid a \in A\}$ in $\mathcal{F}$, $n \in \omega$, such that $X_a = \cup\{X_{an} \mid n \in \omega\}$ for each a. So it is clear what is meant by discretely σ-decomposable. We shall call a family $\{X_a\}$ in a topological space uniformly discrete if it is discrete in the finest uniformity inducing the topology. A family $\{X_a\}$ is called isolated if it is discrete in $\cup\{X_a\}$.

Following $[F\text{-}H_1]$, if $\varkappa$ is an infinite cardinal then a space X is called $\varkappa$-analytic (or topologically $\varkappa$-analytic, abb. T $\varkappa$-analytic) if there exists an usco-compact correspondence from the metric space $\varkappa^\omega$ onto X such that the image of each discrete family (equivalently, discretely decomposable family) is uniformly discretely (or discretely, resp.) σ-decomposable. If the values are disjoint, then the space is called $\varkappa$-Luzin (or topologically $\varkappa$-Luzin, resp.), and if the values are singletons or empty then we speak about point-$\varkappa$-analytic etc. spaces. Analytic means $\varkappa$-analytic for some $\varkappa$, and similarly Luzin etc. The theory of analytic and Luzin spaces was developed in $[F\text{-}H_{1,2,3}]$. A discussion of topologically analytic spaces appeared in [H-J-R].

Theory of analytic spaces has two important advantages in comparison with that of topological analytic spaces:

(a) there is a nice description of analytic spaces as Suslin (closed) subsets of products $K \times M$ with K compact and M complete metric.

(b) Using the product $X \times \Sigma$ taken in uniform spaces then the projection $X \times \Sigma \longrightarrow X$ preserves uniformly discretely σ-decomposable families.

Lemma 1. If Y is a separable metric space then for any X the projection along Y preserves isolatedly σ-decomposable families.

Lemma 1 is the main point for introducing weakly topologically analytic (abb. WT analytic) spaces as images of complete metric spaces under usco-compact correspondences preserving isolatedly σ-decomposable families. Indeed we have the following characterization.

Theorem 1. Each of the following conditions is necessary and sufficient for X to be WT analytic:

(3a) Some paracompact Čech complete subspace of $X \times \Sigma$ projects onto X.

(3d) There exists a complete sequence of σ-isolated covers.

Of course, analytic or T analytic spaces are characterized by existence of a complete sequence of σ-uniformly discrete or σ-discrete covers, resp.

Theorem 2. Each of the following conditions is necessary and sufficient for X to be WT point-analytic:

(4a) Some completely metrizable subspace of $X \times \Sigma$ projects onto X.

(4d) There exists a complete sequence of σ-isolated covers with clusters of Cauchy filters being singletons.

(4e) X is Čech-analytic and there exists a σ-isolated network for X.

Using the main result of [F-H_1], we obtain

Theorem 3. In a WT point-analytic space X each point-finite completely $\mathcal{S}$(Borel(X))-additive family is isolatedly σ-decomposable. In WT analytic spaces X the result is true for Suslin (closed(X)) sets.

For the first separation principle the following kind of sets works. For each X let Isol Bo(X) be the smallest collection which contains open and closed sets of X, and which is closed under formation of countable intersections and σ-isolated unions.

There are many reasons for trying to understand whether or not the classes of all WT analytic or Čech analytic spaces are preserved by perfect maps. All I know is:

Theorem 4. The perfect image of a Čech analytic space is analytic if metrizable.

The proof depends on Lemma 2 from [F_2].

Note that analytic spaces are paracompact, T analytic spaces are subparacompact, and WT analytic spaces are σ-isolatedly refinable (also called weakly Θ-refinable spaces).

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